Problem 1 – Townsend 9.12  The wave function for a particle is of the form \( \psi(r) = (x + y + z)f(r) \). What are the values that a measurement of \( L^2 \) can yield? What values can be obtained by measuring \( L_z \)? What are the probabilities of obtaining these results? 

**Suggestion:** Express the wave function in spherical coordinates and then in terms of the \( Y_{l,m} \)'s.

Problem 2 – Townsend 9.19  The wave function of a rigid rotator with a Hamiltonian \( \hat{H} = \hat{L}^2/2I \) is given by

\[
\langle \theta, \phi | \psi(0) \rangle = \sqrt{\frac{3}{4\pi}} \sin \theta \sin \phi
\]

(a) What is \( \langle \theta, \phi | \psi(t) \rangle \)? 

**Suggestion:** Express the wave function in terms of the \( Y_{l,m} \)'s.

(b) What values of \( L_z \) will be obtained if a measurement is carried out and with what probability will these values occur?

(c) What is \( \langle L_x \rangle \) for this state?

(d) If a measurement of \( L_x \) is carried out, what result(s) will be obtained? With what probability?

Problem 3 – Townsend 9.21  Treat the ammonia molecule, \( \text{NH}_3 \), shown in Fig. 9.12 as a symmetric rigid rotator. Call the moment of inertia about the \( z \) axis \( I_3 \) and the moments about the pair of axes perpendicular to the \( z \) axis \( I_1 \).

(a) Express the Hamiltonian of this rigid rotor in terms of \( \hat{L}, I_1, \) and \( I_3 \).

(b) Show that \([\hat{H}, \hat{L}_z] = 0\).

(c) What are the eigenstates and eigenvalues of the Hamiltonian?

(d) Suppose that at time \( t = 0 \) the molecule is in the state

\[
|\psi \rangle = \frac{1}{\sqrt{2}} |0,0 \rangle + \frac{1}{\sqrt{2}} |1,1 \rangle
\]

What is \( |\psi(t)\rangle \)?

Problem 4 – Townsend 10.1  The position-space representation of the radial component of the momentum operator is given by

\[
\hat{p}_r \rightarrow \frac{\hbar}{i} \left( \frac{\partial}{\partial r} + \frac{1}{r} \right)
\]

Show that for its expectation value to be real: \( \langle \psi | \hat{p}_r | \psi \rangle = \langle \psi | \hat{p}_r | \psi \rangle^* \), the radial wave function must satisfy the condition \( u(0) = 0 \). 

**Suggestion:** Express the expectation value in position space in spherical coordinates and integrate by parts.
Problem 5 – Townsend 10.7  Show that there are no allowed energies $E < -V_0$ for the potential well

$$V = \begin{cases} -V_0 & r < a \\ 0 & r > a \end{cases}$$

by explicitly solving the Schrödinger equation and attempting to satisfy all the appropriate boundary conditions.