1. **Two Orbital Boson System**

Consider a system of \( N \) bosons of spin 0, with orbitals at the single particle energies 0 and \( \epsilon \). The chemical potential is \( \mu \), and the temperature is \( \tau \). Find \( \tau \) (in terms of \( N \) and \( \epsilon \)) such that the thermal average population of the lower orbital is twice the population of the orbital at \( \epsilon \). Assume that \( N \gg 1 \) and make what approximations are reasonable.

2. **Properties of Degenerate Boson Gas**

Consider a gas of \( N \) noninteracting bosons of spin zero confined to a volume \( V \).

(a) Find the expression as a function of temperature in the region \( \tau < \tau_c \) for the energy of the system. Apply the same technique we used in class. Put the definite integral in dimensionless form, which can then be evaluated numerically.

\[
\int_0^\infty dx \frac{x^{3/2}}{e^x - 1} = 1.006\pi^{1/2} = 1.783
\]  

(b) Find the heat capacity, \( C_V \), of the system in the same temperature range. Sketch \( C_V/N \) as a function of \( \tau \).

(c) Explain what value \( C_V \) must approach in the high temperature limit and complete your plot of \( C_V/N \) for \( \tau > \tau_c \). Compare your result with the heat capacity of ideal Bose gas shown in Schroeder (Figure 7.37). Do you see the same discontinuous change in the slope of \( C_V(\tau) \) at \( \tau_c \)?

(d) Calculate the entropy of the boson gas for \( \tau < \tau_c \).

(e) Calculate the Helmholtz free energy and pressure of the boson gas for \( \tau < \tau_c \). Note that the pressure is independent of volume. How can this be?

3. **Heat Capacity of Liquid He**

Schroeder, 7.61

4. **Properties of Ferromagnets**

Schroeder, 7.64