Problem Set 2
Due: 13 Sept 2006

Please staple problems 1+2 and 3+4.

1. **Einstein Brick**

(a) Compute the multiplicity of an Einstein solid in the “low-temperature” limit, \( q \ll N \).

(b) Find the relationship between the total energy and temperature of the brick. Compare your answer to the high-temperature limit expression we derived in section, namely \( U = N \tau \). What are your conclusions?

(c) Calculate the heat capacity of an Einstein solid in both low-and high-temperature limits. Make a sketch of \( C_V \) as a function of \( \tau \).

For the rest of this problem consider two identical Einstein bricks in the high-temperature limit, each containing \( N \gg 1 \) oscillators. Initially they are isolated from each other and the environment and are separately in equilibrium. Brick A is at \( T_A = 0^\circ C \) and brick B is at \( T_B = 100^\circ C \).

(d) The bricks are placed in thermal contact with each other (but still isolated from the environment) and are allowed to come to equilibrium. What is their final temperature?

(e) Calculate the entropy of the total system in the initial and final state (as functions of \( N \), temperatures and energy spacing \( \epsilon \)) and compute the total entropy change. Explain why the change in entropy does not depend on \( \epsilon \), even though both initial and final values of entropy do.

2. **Melting Ice cubes**

   Problem 3.10 in Schroeder

3. **Ferromagnetism** (Reif 4.4)

   A solid contains \( N \) atoms having spin \( 1/2 \). We can assume that at high temperatures, each spin is completely randomly oriented. But at temperatures below some value \( \tau_0 \) the interactions between the magnetic atoms cause them to exhibit ferromagnetism, with the result that all their spins become oriented in the same direction. A crude approximation suggests that the spin-dependent contribution to the heat capacity of this solid has an approximate temperature dependence given by

   \[
   C(T) = \begin{cases} 
   C_0 \left(2\tau/\tau_0 - 1\right), & \text{if } \tau_0/2 < \tau < \tau_0 \\
   0, & \text{otherwise.}
   \end{cases}
   \]  

   (1)
The abrupt increase in heat capacity as $\tau$ is reduced below $\tau_0$ is due to the onset of ferromagnetic behavior.

Using our results from lecture for the entropy of a 2-state paramagnet, find an expression for the maximum value of $C_0$ as a function of $N$.

4. **Curvature of Entropy**

We showed in lecture that the entropy $\sigma(U)$ for the Einstein brick was concave downward, i.e. $\frac{d^2\sigma}{dU^2} < 0$. Consider two systems for which this derivative is *positive* and show that they cannot achieve stable thermal equilibrium.