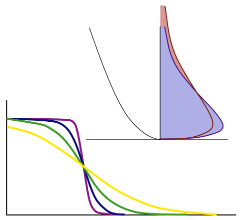

Statistical Mechanics

Homework Assignment 8

Due 6 November 2002

1. **Problem 8.1 – Gibbs-Duhem** A system obeys the two equations

$$T = \frac{3AS^2}{NV^x} \qquad p = \frac{AS^3}{NV^y}$$

where A, x, and y are constants.

(a) What must be the values of x and y? Explain.

(b) Find μ as a function of S and V.

(c) Find the fundamental relation, $E(S, V, N)$, for this system.

2. **Problem 8.2 – Triple Point (*Reif 8.3*)** A simple substance of molecular weight m has its triple point at the absolute temperature T_0 and pressure p_0. At this point the densities of the solid and liquid are ρ_s and ρ_l, respectively, while the vapor can be approximated by a dilute ideal gas. If at this triple point the slope of the melting curve is $\left(\frac{dp}{dT}\right)_m$ and that of the liquid vaporization curve is $\left(\frac{dp}{dT}\right)_v$, what is the slope $\left(\frac{dp}{dT}\right)_s$ of the sublimation curve of the solid?

3. **Problem 8.3 – Cooling Liquid He by Pumping** Liquid helium boils at a temperature $T_0 = 4.2$ K when its vapor pressure is equal to $p_0 = 1$ atm. The latent heat of vaporization per mole of the liquid is equal to L; this value is roughly independent of temperature. The liquid is contained within a dewar, which serves (approximately!) to insulate it thermally from the room temperature surroundings. Since the insulation is not perfect, the liquid is heated at a rate $\mathscr{P}$ from the environment, and this rate is essentially constant over the range of liquid temperatures we consider in this problem.

   To reach low temperatures (and to attain the superfluid transition at 2.17 K) one can reduce the pressure of the He vapor over the liquid by pumping it away with a pump at room temperature T_r. Such a pump is typically connected to the dewar with a long hose, so that as the helium vapor travels the length of the hose, it is warmed up to (close to) room temperature. The pump has a maximum pumping speed such that it can remove volume at a steady rate, given by $\dot{V}$. Furthermore, this rate is independent of the gas pressure over a wide range of pressures, since the mechanical pump simply sweeps out a fixed volume of gas per revolution.

   (a) Calculate the minimum vapor pressure p_m which this pump can maintain over the surface of the liquid for a heating rate $\mathscr{P}$ of the liquid.

   (b) If the liquid is thus maintained in equilibrium with its vapor at this pressure p_m, calculate its approximate temperature T_m.

4. **Problem 8.4 – Extracting Work from Einstein Bricks**  Three identical Einstein bricks are at temperatures 450 K, 450 K, and 150 K. You may assume that $k_{\mathrm{B}}T \gg \epsilon$ at these temperatures.

   (a) If no work or heat is supplied from the outside, show that the highest temperature to which any of these bodies can be raised is 600 K. *Hint: All these temperatures are divisible by 150 and you shouldn't need a calculator or computer.*

   (b) Describe a process for obtaining this maximum temperature. You may assume that you have available Carnot engines and a reversible work source that can store the work delivered to it.