Physics 117 | Ann Esin
Statistical Mechanics & Thermodynamics | 22 Nov 2006

Problem Set 11

Due: 29 Nov 2006

Please staple problems 1 & 2 and 3 &4.

1. **Properties of Van der Waals Gas**

   (a) Find the entropy of an ideal van der Waals fluid.

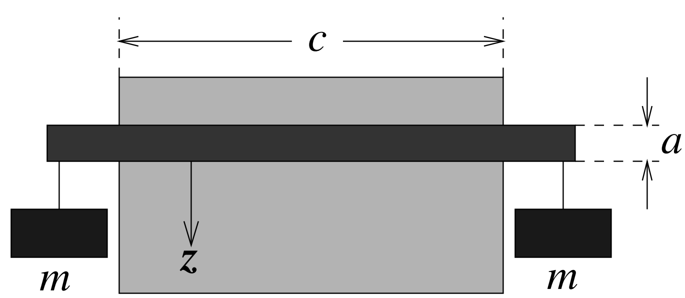
   (b) Find the relationship between the volume and temperature of an ideal van der Waals fluid in an adiabatic expansion.

2. **Drop Nucleation**

   Schroeder, 5.46

3. **Solid-Liquid Equilibrium**
   A long vertical column is closed at the bottom and open at the top; it is partially filled with a particular liquid and cooled to -5°C. At this temperature the fluid solidifies below a particular level, remaining liquid above this level. If the temperature further lowered to -5.2°C, the solid-liquid interface moves upward by 40 cm. The latent heat (per unit mass) is 8.4×10^3 erg/g and the density of the liquid phase is 1000 kg/m^3. Find the density of the solid phase. Neglect thermal expansion of all materials. (*Hint:* Start by drawing a correct picture of "before" and "after" states of your system. What is the sign of the slope of the solid-liquid coexistence curve for this substance?)

4. **Cutting the Ice** (based on Reif, 8.8)

   A steel bar of rectangular cross section (height a and width b) is placed on a block of ice (width c) with its ends extending a trifle as shown in the figure. A weight of mass m is hung from each end of the bar. The entire system is at $T = 0$°C. As a result of the pressure exerted by the bar, the ice melts beneath the bar and refreezes above the bar. Heat is therefore liberated above the bar, conducted through the metal, and then absorbed by the ice beneath the bar. (We assume that this is the most important way in which heat reaches the ice immediately beneath the bar in order to melt it.) Find an approximate expression for the speed with which the bar sinks through the ice. Take the latent heat of fusion per gram of ice to be ℓ, and the densities of ice and water to be ρ_i and ρ_w, respectively.

(a) Let's say the bar sinks a distance Δz in time Δt. Calculate ΔU, the amount of energy required for this to happen.

(b) The energy calculated in part (a) must pass through the bar via the process called thermal conduction, described by the following equation

$$F = -\kappa \frac{d\tau}{dz} \simeq -\kappa \frac{\Delta \tau}{a}. \tag{1}$$

Here F is the heat flux, i.e. total energy crossing unit area per unit time, constant κ is the coefficient of thermal conductivity, and $\Delta\tau$ is the difference between the temperatures under and above the bar. Note that F is proportional to the negative of the gradient of temperature (as expected, since heat flows from high to low τ). Use Eq. (1) and your result from part (a) to calculate the speed of the bar, $v = dz/dt$, as a function of $\Delta\tau$ and other given quantities.

(c) Finally, eliminate $\Delta\tau$ from your result in part (b) to obtain the final expression for v:

$$v = \frac{2mg\kappa\tau}{abc\ell^2\rho_i}\left(\frac{1}{\rho_i} - \frac{1}{\rho_w}\right) \tag{2}$$