Astron. Astrophys. 333, 338–342 (1998)

ASTRONOMY AND ASTROPHYSICS

Wavelength dependency of the Solar limb darkening

D. Hestroffer[1] **and C. Magnan**[2,3]

[1] Astrophysics Division, Space Science Department of ESA, ESTEC, Postbus 299, 2200 AG Noordwijk, The Netherlands
[2] GRAAL, Université de Montpellier II, F-34095 Montpellier Cedex 5, France
[3] Collège de France, Paris

Received 19 September 1997/ Accepted 27 October 1997

Abstract. A single parameter model of the mean Solar limb-darkening is presented. This empirical law has the advantage to represent the limb darkening over a large spectrum at least as well as a quadratic or logarithmic law. Since it is less sensitive than high degree polynomial laws, it is recommended for subsequent analysis of average limb-darkening variations and comparisons.

Key words: limb darkening – center-to-limb variation – Sun: atmosphere

1. Introduction

The intensity profile $I(\mu)$ across the solar disc is determined by the temperature distribution with optical depth in the photosphere. Moreover the intensity distribution, or limb darkening, is wavelength dependent, and one expects it to be a smooth function of λ except at particular lines. Limb darkening data are important for the construction and verification of model atmospheres, or the comparison of disk-integrated spectra of solar type stars with disk-center spectra of the Sun. Studies of limb-darkening variation are usually made by an analysis of the coefficient of an empirical law fitted to the observational data. Introduction of many parameters provides a better fit to the normalised intensity function by minimizing the residuals. On the other hand, global comparison between two different fits becomes difficult. Also a simple view of the limb-darkening at different wavelength, or analysis of wavelength dependency is complicated by the dimension of the vectorial space of the parameters, which has then to be performed for various linear combinations. Moreover, although different polynomial approximations gives a reasonable fit to the data, the comparison of the coefficients obtained for various wavelength shows scatter as large as a factor 3 (Neckel & Labs 1994). Fig. 1 shows the solutions for two of the 5 degrees polynomial derived by Pierce & Slaughter (1977, hereafter PS) and Neckel & Labs (1994, hereafter NL), and their corresponding residuals. The polynomials are taken for $\lambda = 579.88$ nm, and are of the form:

$$P_5(\mu) = \sum_{k=0}^{k=5} a_k\, \mu^k \quad ; \quad \sum a_k = 1$$

where $\mu = \cos\theta = (1-r^2)^{1/2}$. The coefficients and their differences are given in Table 1. Although the polynomials of PS and NL are completely independent, the agreement – in terms of approximation to the normalised intensity profile – is excellent for $0 \le r \le 0.9$. However the fit by a polynomial function at the limb ($\frac{\partial I}{\partial r}{}_{(r=1)\to\infty}$) is difficult to achieve from a numerical as well as on an observational point view. Not surprisingly the coefficients differ in a sensitive manner, reflecting the fact that although the polynomial functions are close in a restricted range of the variable μ, they are actually completely different. Hence it makes any wavelength dependency of the higher degrees coefficients less reliable.

Consider another empirical law for the model of the normalised brightness distribution across the disc:

$$I(\mu) = 1 - u\,(1-\mu^{\alpha}) \quad ; \quad (u,\,\alpha) \in \mathbb{R}^2 \qquad (1)$$

This simple law has the property of yielding the shape of the normalised intensity with only a few parameters. The fit to the observational data given by Petro et al. (1984, their Table 2B) is shown in Fig. 2. The approximation is better than $\pm 1\%$, and hence is competitive with for instance the logarithmic law:

$$I(\xi) = \sum_{k=0}^{k=2} a_k\, \xi^k \quad ; \quad \xi = \ln\mu$$

or a quadratic law of the form:

$$I(\mu) = \sum_{k=0}^{k=2} a_k\, \mu^k$$

For a given value of u and observed intensity distribution, we have $\alpha \ln\mu = \ln(1+(I-1)/u)$, and put $\Delta\alpha = \alpha - \ln(1+(I-1)/u)\,/\ln\mu$. The smaller the value $\Delta\alpha$, the better is the representation with constant exponent α. This characteristic value is also given in Fig. 2.

The empirical power law model is not intended to provide the best fit to a particular drift curve. Nevertheless, among the

Send offprint requests to: D. Hestroffer (dhestrof@astro.estec.esa.nl)

Table 1. Coefficients of the polynomial fits in Fig. 1

$\lambda = 5798.8$ nm	a_0	a_1	a_2	a_3	a_4	a_5
PS	0.30505	1.13123	-0.78604	0.40560	0.02297	-0.07880
NL	0.28392	1.36896	-1.75998	2.22154	-1.56074	0.44630
$\Delta\, a_k$	-0.021113	0.23773	-0.97394	1.81594	-1.58371	0.52510

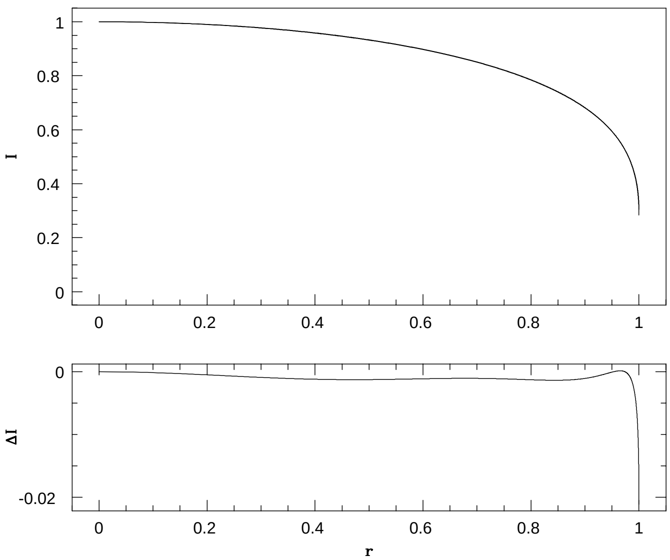

Fig. 1. Normalised intensity following NL and PS, indistinguishable on the graph (top), and residuals NL-PS (bottom). The agreement between the two representations is excellent for $r \leq 0.9$. This is not reflected by the coefficients of the polynomial functions (see Table 1)

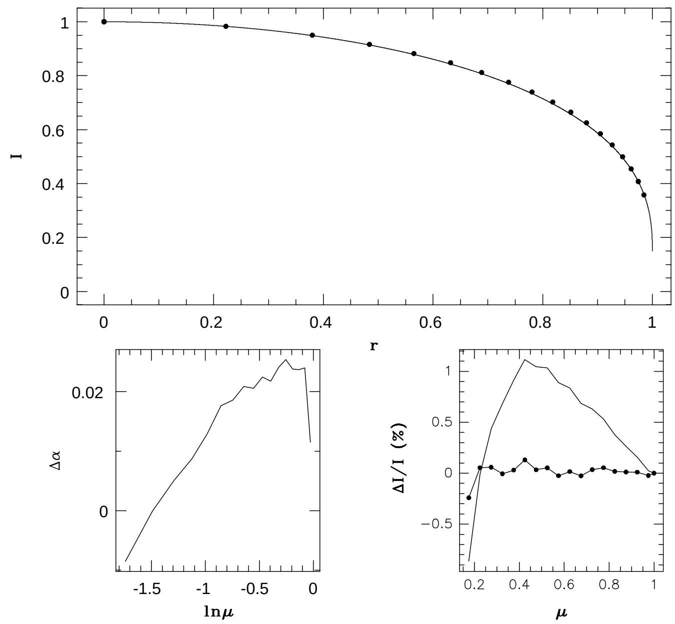

Fig. 2. Average drift scan of the Sun fitted by a power law of Eq. (1) with $u = 0.85$ and $\alpha = 0.8$. Filled circles correspond to observations of Petro et al. (1984). The residuals are given on the lower right panel for the 6 parameters approximation of Petro et al. and the fit of this paper. See text for an explanation of the characteristic quantity $\Delta\alpha$

various simple parameterization of the center to limb variation, it provides the best reproduction of the global behaviour of the centre to limb brightness depth. This holds not only for the Sun but also for other stars, and hence it can be used for general application in observations by interferometric techniques (Hestroffer 1997). Moreover comparison of different limb-darkening is possible and easier, and one can analyse its wavelength dependency in a more straightforward manner.

2. Fit to the data

The $P_5(\mu)$ polynomial model of NL, PS and Pierce et al. (1976, hereafter PSW) provide, in a compact manner, a good approximation of the observed intensity distribution, except near the limb (see Sect. 1), and will be taken as the basis of the following study. Putting $u = 1$ in Eq. 1, a single-parameter power law fit is performed by minimising the residuals in total energy over the range $0 \leq r \leq r_o = 0.9$, which corresponds to 81% of the total disc and avoids the disagreement of the polynomial approximations near the limb. By definition the exponent $\bar{\alpha}$ is such that:

$$\int_{\mu_o}^{1} P_5(\mu)\, d\mu - \int_{\mu_o}^{1} \mu^{\bar{\alpha}}\, d\mu = 0 \quad ; \quad \mu_o = (1 - r_o^2)^{1/2} \qquad (2)$$

hence it always exists and is a root of the equation:

$$A = \sum \frac{a_k}{k+1}\,(1 - \mu_o^{k+1})$$
$$\mu_o^{\bar{\alpha}+1} + (\bar{\alpha}+1)\,A - 1 = 0 \qquad (3)$$

which is solved by a numerical method.

A uniformly bright disc corresponds to $\bar{\alpha} = 0$, while a linearly fully-darkened one yields $\bar{\alpha} = 1$. The values of the exponent $\bar{\alpha}$ are given as a function of wavelength for both solutions[1] in Table 2. It is worth noting that in the present case a good approximation, useful for direct evaluation, can be given by the more simple expressions:

$$\bar{\alpha} \sim \frac{\ln P_5(\mu_o)}{\ln \mu_o} \quad \text{or} \quad \bar{\alpha} \sim P_5'(\mu = 1) = \sum_k k\, a_k \qquad (4)$$

showing by a different way the consistency of the power law fit.

[1] The result of PSW at $\lambda = 1527.4$nm has been rejected since the published coefficients a_k are such that $\sum a_k \sim 0$ instead of the expected value 1.

Table 2. Exponent of the limb-darkening function versus wavelength for PS, PSW and NL data

PS		NL	
λ [nm]	α	λ [nm]	α
303.327	0.967	303.327	0.939
306.982	0.942		
310.843	0.904	310.843	0.914
320.468	0.867	320.468	0.871
329.897	0.816	329.897	0.817
338.953	0.779		
349.949	0.753	349.947	0.763
356.352	0.749		
362.650	0.714		
365.875	0.684	365.875	0.695
374.088	0.751	374.086	0.750
377.992	0.739		
385.202	0.808		
390.928	0.790	390.915	0.791
395.425	0.782		
398.815	0.744		
401.970	0.741	401.970	0.751
406.944	0.731		
411.723	0.693		
416.320	0.712	416.319	0.724
421.905	0.705		
427.930	0.670	427.930	0.682
431.645	0.643		
443.885	0.652	443.885	0.649
445.125	0.648	445.125	0.646
454.355	0.633		
456.792	0.620		
457.345	0.631	457.345	0.628
461.510	0.614		
468.306	0.600		
471.900	0.582		
477.435	0.588	477.427	0.594
481.157	0.580		
483.075	0.569		
490.560	0.575		
492.905	0.569	492.905	0.570
498.090	0.573		
503.800	0.554		
510.210	0.539		
519.930	0.529	519.930	0.538
526.535	0.542		
533.460	0.522		
541.760	0.511	541.760	0.514
552.200	0.503		
559.950	0.502	559.950	0.496
569.560	0.482		
579.880	0.473	579.880	0.477
587.430	0.465		
601.015	0.464		

PS		NL	
λ [nm]	α	λ [nm]	α
610.975	0.452	610.975	0.447
620.590	0.434		
632.600	0.429		
640.970	0.420	640.970	0.428
649.250	0.413		
660.400	0.401		
669.400	0.396	669.400	0.407
679.140	0.397		
691.600	0.388		
700.875	0.383	700.875	0.386
710.425	0.368		
719.925	0.375		
729.675	0.372		

PSW		NL	
λ [nm]	α	λ [nm]	α
740.460	0.359		
748.710	0.348	748.710	0.361
770.820	0.342		
789.900	0.324		
811.760	0.325	811.760	0.333
828.410	0.315		
847.510	0.307		
869.600	0.303	869.600	0.311
903.380	0.284		
948.850	0.280	948.850	0.292
997.920	0.270		
1046.700	0.262	1046.600	0.274
1098.950	0.246	1098.950	0.260
1158.350	0.242		
1197.750	0.231		
1251.500	0.220		
1299.000	0.219		
1307.400	0.194		
1338.100	0.208		
1339.400	0.194		
1402.000	0.196		
1430.100	0.163		
1457.900	0.183		
1493.100	0.166		
1500.100	0.172		
1564.200	0.146		
1580.300	0.165		
1595.100	0.143		
1622.200	0.162		
1642.000	0.137		
1659.800	0.154		
1670.000	0.145		
1686.000	0.126		
1695.700	0.138		
1704.300	0.151		
1719.800	0.155		
1748.300	0.146		
1789.100	0.136		
1798.400	0.150		
1813.800	0.134		
1894.500	0.146		
1925.000	0.151		
1969.200	0.125		
2022.000	0.132		
2100.200	0.136		
2185.500	0.133		
2216.300	0.127		
2312.800	0.127		
2401.800	0.123		

3. Wavelength dependency

Since the model is built with a single parameter, inter-comparison of the limb-darkening at different lines and study of its wavelength dependency are easier. Not surprisingly the agreement of the centre-to-limb-darkening wavelength relation between PS and NL is good (see Fig. 3). There is however a small systematic deviation for wavelength larger than 641 nm. Hence the large disagreement or scatter found by NL or by Neckel (1996) from different analysis is in part due to the choice of the parameters.

Combining all the data of Table 2, we find the average relations, accurate to a few ± 0.02, in function of λ^{-1} given in units of $\mu\mathrm{m}^{-1}$:

$$\alpha \sim \begin{cases} -0.023 + 0.292\,\lambda^{-1} & \text{if } \lambda^{-1} \lesssim 2.4\,\mu\mathrm{m}^{-1} \\ -0.507 + 0.441\,\lambda^{-1} & \text{if } \lambda^{-1} \gtrsim 2.8\,\mu\mathrm{m}^{-1} \end{cases} \quad (5)$$

As can be seen on Fig. 3, there are two evident discontinuities in this function, also present when a similar fit is done on the observations of Mitchell (1981). The first discontinuity corresponds to the position of the Balmer limit. The other ($\lambda \sim 390\,\mathrm{nm}$) does not correspond to the same 'Balmer jump' at $\lambda \sim 410\,\mathrm{nm}$ of Neckel (1996) or NL obtained from a different data analysis. This however is not due to the particular power-law fit made here; similar jumps appear in the energy given by:

$$E = \int_0^1 I(\mu)\; d\mu$$

or in the ratio of the disc-averaged to disc-centre intensities (see Fig. 4):

$$F/I_o = 2\int_0^1 I(\mu)\,\mu\,d\mu$$

The same remark applies for the value of the intensity at selected value of μ (see e.g. PSW). The second strong discontinuity is

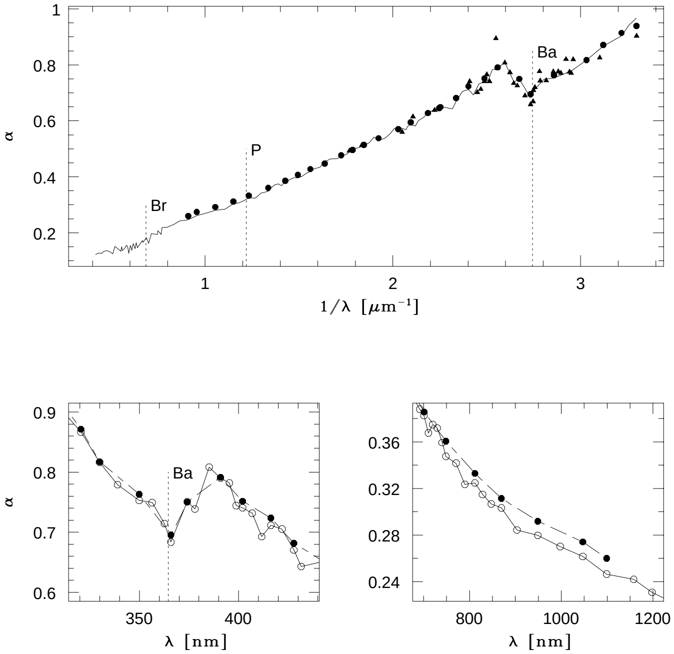

Fig. 3. Center to limb darkening as a function of wavelength. Upper panel, the curve corresponds to the data of PS(W), filled circles correspond to the data of NL, filled triangles correspond to the data of Mitchell (1981) given for comparison. Lower panels, filled circles correspond to the data of NL, open circles to the data of PS(W). The dotted lines indicate the positions of the Balmer, Paschen and Brackett limits

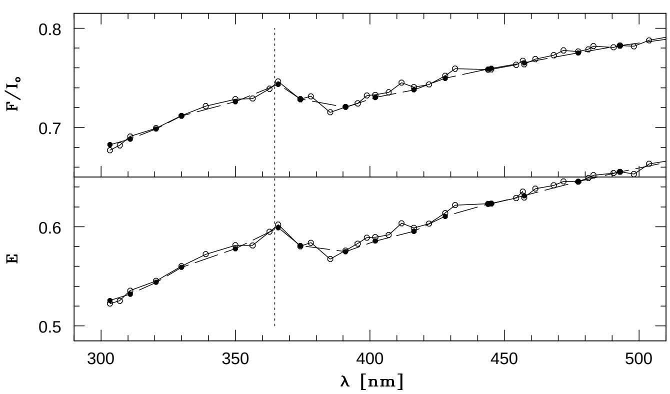

Fig. 4. Energy and ratio of the disc-averaged to disc-centre intensities as a function of wavelength. The dotted line marks the positions of the Balmer limit $\lambda = 364.6$ nm

more likely related to the H_ε instead of the H_δ line, which is in accordance with the limb-darkening derived by van Hamme (1993) from atmosphere models for solar chemical composition stars. Hence the limb-darkening profile reflects the globally increasing opacity of the centre with increasing wavelength while the absorption depth at the limb remains null; and shows a strong variegation of the opacity at $\lambda \sim 365$ and $\lambda \sim 390$ nm between the Balmer and Paschen continuum.

4. Discussion

From the above discussion it is clear that the total energy E (i.e. the surface under the normalised-intensity curve) or the ratio F/I_o of the disc-averaged to disc-centre intensities (as used by e.g. Greve & Neckel, 1996) are other simple parameters describing the limb darkening.

Another parameter that is also efficient for analysing the wavelength dependency of the centre-to-limb darkening, is the intensity depth at a given radius from the centre of the Solar disc. Since all average normalised-intensities are of the general form μ^α, the best suited position, in term of sensitivity to any limb-darkening variation, is at the limb (where the variation with respect to α is maximal). This is however not convenient, and one may prefer the locus $r_o = 0.9$. The depth will range between 0 for an uniformly bright disc, to 0.9 for a linearly darkened one.

The limb-darkening coefficient $\bar{\alpha}$ derived here from PS(W) data agree in general with the one derived from NL data, except for $\lambda \gtrsim 641$ nm. For higher wavelengths, the limb-darkening derived from NL data (observations made during 1986/1987) is systematically more pronounced than for the PS(W) data (observations made during 1975), one find $\alpha_{NL} \sim \alpha_{PS(W)} + 0.01$. As noted by Neckel (1996) and NL, this can be due to the differing periods of observations entering in the different data reductions.

The scatter in the plots of the various parameters α, E and F/I_o is higher for the PS(W) data than the NL approximation. This can be due to different methods of reduction applied by the authors for the observations within $7''$ from the limb. While PS(W) consider special corrections for seeing or blurring for points within the limb, NL excluded the signals nearest the limb. The PS(W) 5^{th} order polynomial fit shows a higher sensitivity to the less secure observations near the limb. As long as an averaged limb-darkening profile is foreseen, observations nearest the limb can be avoided.

5. Conclusion

Analysis of the wavelength dependency of the centre to limb darkening of the Sun or a star should not be made on the basis of each coefficient of a polynomial fit. We advocate utilization of a power law fit, or quantities related to the moments of the normalised intensity for such analysis. Following the power-law fit to the data of Pierce & Slaughter (1977), Pierce et al. (1977) and Neckel & Labs (1994), mean values for the average limb-darkening have been derived as a function of wavelength for $303.3 \leq \lambda \lesssim 357$ nm and $416 \lesssim \lambda \leq 1099$ nm. Analysis of these data shows strong variations of the limb-darkening at the Balmer limit and at $\lambda \sim 390$ nm.

Acknowledgements. D. Hestroffer is supported by a research grant from the European Space Agency.

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