Problem Set 9

Due: 2 Apr 2012

Please staple problems 1+2 and 3+4.

1. **Planetary Nebula**

The Helix Nebula is a planetary nebula with an angular diameter of $15'$ that is located approximately 120 pc from Earth. Like all such objects, it is powered by the UV radiation from the compact white dwarf remnant of the original star, located at the center of the nebula. UV photons are absorbed by atoms in the expanding shell, and as excited electrons cascade back down to the ground state, many optical emission lines are produced.

(a) Calculate the diameter of the Helix Nebula.

(b) Assuming that the outer edge of the nebula is expanding away from the central star at a constant velocity of $20\,\mathrm{km\,s^{-1}}$ estimate its age.

(c) The evolution of a planetary nebula can be described as a free expansion, i.e. gravity is negligible, so the outward velocity of each clump remains constant with time. Then the material that was ejected with higher velocity soon overtakes slower moving clumps. Because of this, a snapshot of the nebula at any moment will show that the radial velocity of the moving material increases linearly with the distance from the central star. Let's say at time t_0 the nebula has an outer radius R_0, thickness ΔR_0, uniform number density of UV absorbers n_0, and optical depth for absorption of UV photons $\tau_0 \simeq \Delta R_0 n_0 \sigma_{UV}$. Derive an equation describing the time evolution of the optical depth of the nebula at times $t > t_0$. You can assume that the density remains uniform throughout the expansion, while its radius and thickness increase linearly with time. Start with calculating n as a function of time.

(d) The energy balance of the nebula requires that its luminosity is equal to the fraction of the white dwarf luminosity that it absorbs. Use your answer for part (c) to derive an equation for the total optical luminosity of the nebula as a function of the UV luminosity of the white dwarf, L_{UV}, and other relevant parameters.

2. **Supernova Light Curves**

The emission from a regular supernova is dominated for several months by the radioactive decay of heavy elements created in the explosion, as a the blast wave moves through the stellar envelope (e.g. $^{56}_{27}\mathrm{Co}$ or $^{56}_{27}\mathrm{Ni}$).

(a) Suppose we start with N_0 atoms of some radiactive material with a half-life of $\tau_{1/2}$. Starting with Eq. (15.9) in C&O, show that the amount of radioactive material remaining after time t is given by equation:

$$N(t) = N_0 e^{-\lambda t}, \tag{1}$$

where λ is a constant, given by

$$\lambda = \frac{\ln 2}{\tau_{1/2}}. \quad (2)$$

(b) Assuming that the light curve of a supernova is dominated by the energy released in the radioactive decay of an isotope that has a decay constant λ, show that the slope of the light curve is given by equation:

$$\frac{d}{dt}(\log_{10} L) = -0.434\lambda. \quad (3)$$

Thus, by measuring the slope of the light curve, astronomers can verify the presence of large quantities of a specific radioactive isotope.

Calculate the expected light curve slope from the decay of $^{56}_{27}$Co with a half-life of 77.7 days and compare your answer to the data shown in Figure 1

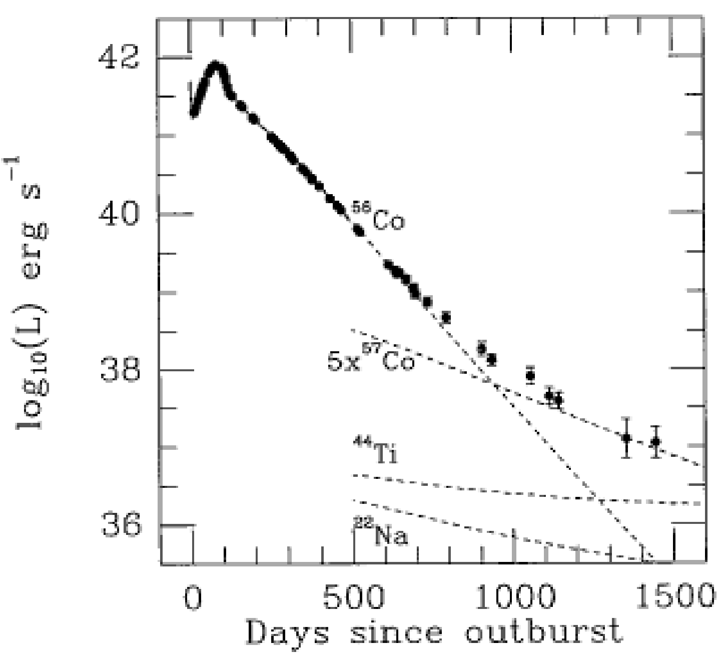

Figure 1: The bolometric lightcurve of SN 1987A. The dashed lines show the contributions expected from the radioactive isotopes produced by the shock wave. (Figure from Suntzeff et al., 1992. Also Fig 15.12 in C&O.)

(c) The energy released during the decay of one $^{56}_{27}$Co atom is 3.72 MeV. If $0.075 M_\odot$ of cobalt was produced by the decay of $^{56}_{28}$Ni following the explosion of SN 1987A, estimate the amount of energy released per second through the radioactive decay of cobalt one year after the explosion and compare your answer to the data shown in Figure 1.

3. **Stellar Clusters**

   Download the photometry for two clusters, 47 Tuc and M45 (Pleiades).

   (a) Use the data provided to construct the color-magnitude diagram of the two clusters. Please include your figure in you homework solution. Make sure to correctly orient and label your axes.

   (b) If the distance to M45 is 135 pc, use the technique of main-sequence fitting to estimate the distance to 47 Tuc. Do *not* use just *one* point on the main sequence!

   (c) The accepted distance to 47 Tuc is around 4.5 kpc. Compare thi s to your result in (b). If there is a discrepancy, discuss its likely causes. Hint: think of the metallicities of the two clusters.

4. **GRB Explosions**

(a) The neutrino fluence (the particle flux times the duration of the burst) from SN 1987A was estimated to be 1.3×10^{14} particles per m^2 at the location of Earth. If the average energy per neutrino was approximately 4.2 MeV, estimate the amount of energy released via neutrinos during the supernova explosion, if the distance to the SN is roughly 50 kpc.

(b) Estimate the gravitational binding energy of a neutron star with a mass $1.4M_\odot$ and a radius of 10 km. Compare your answer with the amount of energy released in neutrinos during the collapse of the iron core of Sk -69 202 (the progenitor of SN 1987A).

(c) A supernova in which the iron core exceeds the maximum allowed mass for a neutron star (believed to be about 3-5$M_\odot$) produces a Gamma-Ray Burst (GRB) event in which most of the total gravitational energy released in the collapse is emitted as γ-rays with energies > 50 keV, rather than neutrinos. If such an event were to occur in the vicinity of the solar system, it might have very real consequences for us. A fatal dose of radiation for a human body is roughly equivalent to $5\,\mathrm{J\,kg^{-1}}$ of absorbed γ-rays per unit mass. Estimate the total exposure per person ignoring the atmospheric absorption (you can assume that a γ-ray has a 100% probability of being absorbed by a human body) if Betelgeuse were to end its life as a GRB. Take the distance to Betelgeuse to be 150 pc.

(d) Fortunately, the optical depth of the atmosphere in the γ-ray energy range is on the order of 100. Revise your estimate in (c) to account for atmospheric absorption. What are your conclusions?