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Problem Set 6

Due: 2 Mar 2012

Please staple problems 1+2 and 3+4.

1. **Pressure Inside the Stars**

Since photons have momentum, photons scattering off an electron produce a force. With many photons trapped inside the Sun the repeated scatterings generate radiation pressure that acts in addition to gas pressure. The expression for the pressure due to the blackbody radiation field is derived in chapter 9.1 of C&O (Eq. 9.11):

$$P_{\text{rad}} = \frac{4\sigma T^4}{3c}. \quad (1)$$

(a) Show that the radiation pressure inside the Sun is negligible compared to gas pressure. The temperature and density inside the present day Sun are  $T_{c,\odot} = 1.58 \times 10^7$  K,  $\rho_{c,\odot} = 1.62 \times 10^5$  kg m<sup>-3</sup>. Assume that the mass fraction of hydrogen is  $X = 0.34$  and of metals  $Z = 0.02$ .

(b) For the stars on the main sequence the central density is roughly inversely proportional to mass, i.e.  $\rho_c(M) \simeq \rho_{c,\odot}(M_\odot/M)$ , and the central temperature is increasing with mass as  $T_c \simeq T_{c,\odot}(M/M_\odot)^{0.5}$ . Find the mass of the star for which the radiation pressure in the center becomes comparable to gas pressure. Assume that the composition ( $X$ ,  $Y$  and  $Z$ ) remains the same for all masses.

2. **Structure of Earth's Atmosphere**

(a) Re-derive the equation for the equilibrium temperature of the Earth, balancing the heat inflow from the Sun and heat loss from the Earth's atmosphere. Evaluate your result taking the albedo to be  $a = 0.3$ .

(b) Use the equation for hydrostatic equilibrium to derive an equation describing how density in the Earth's atmosphere changes with altitude. Assume that the atmosphere is isothermal ( $T$  does not change with altitude) and can be treated as an ideal gas. Express your answer in terms of air density at the surface,  $\rho_0$ , and temperature  $T$ . (*Hint*: Since the thickness of the atmosphere is much smaller than the Earth's radius, you can assume  $g$  is constant.)

(c) Use your answer to parts (a) and (b) to find the altitude at which the atmospheric density falls to  $1/e$  of its surface value. Give both an algebraic and a numerical answer. This special altitude is called the pressure scale height.

### 3. Chemical Energy

Problem 10.3 in C&O.

### 4. Mass Loss from the Sun

The Sun is constantly losing mass by converting it to energy via nuclear reactions and through launching of the solar wind. Solar wind is a continuous stream of high-energy electrons and ions escaping from the Sun.

(a) Calculate the escape speed at the surface of the Sun. This gives us an order of magnitude estimate for the speed of the solar wind particles,  $v_{\text{sw}}$ .

(b) Assume that the solar wind is launched isotropically. Show that the rate at which the solar wind matter is crossing a spherical surface of radius  $r$  is equal to

$$\left(\frac{dM}{dt}\right)_{\text{sw}} = 4\pi r^2 \rho v_{\text{sw}}. \quad (2)$$

*Hint:* Start by calculating the mass contained in a thin shell of radius  $r$  and thickness  $v_{\text{sw}}dt$ . Evaluate the solar wind mass loss rate if the solar wind contains mostly protons and their number density at  $r = 1 \text{ AU}$  is  $n = 7 \times 10^6 \text{ m}^{-3}$ .

(c) Calculate  $(dM/dt)_{\text{nuc}}$ , the rate at which the Sun's mass is decreasing due to nuclear reactions.

(d) Compare your answers parts (b) and (c). Assuming that the mass loss rates remain constant, would either mass loss process significantly affect the total mass of the Sun over its main-sequence lifetime?