Astronomy 62 Ann Esin
Introduction to Astrophysics 17 Feb 2012

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# Problem Set 5

Due: 24 Feb 2012

**Please staple problems 1 and 2 separately.**

1. **Opacity and Optical Depth**

According to the "standard model" of the Sun, its central density is $1.62 \times 10^5\,\mathrm{kg\,m^{-3}}$ and the opacity averaged over all wavelengths is $\kappa_{\mathrm{av}} = 0.116\,\mathrm{m^2\,kg^{-1}}$.

**(a)** Calculate the mean free path of a photon in the center of the Sun.

**(b)** If this mean free path remained constant for the photon's journey to the surface, calculate the average time it would take for the photon to escape from the Sun. (Ignore the fact that individual photons are constantly destroyed and created through absorption and emission.) If the Sun's energy generation processes suddenly stopped, this is roughly how long it would take for us to notice any change. Discuss whether your answer is likely to be an under- or overestimate. (*Hint*: Check the density profile inside the Sun, shown in Figure 11.6 in C&O.)

**(c)** The density of Earth's atmosphere is $1.2\,\mathrm{kg\,m^{-3}}$. Estimate how far you could see through Earth's atmosphere if it had the opacity similar to that of the Sun. Do you think the opacity of Earth's atmosphere is smaller or larger than that of the Sun? Justify your answer.

**(d)** Assume that roughly 90% of the visible flux reaches the Earth's surface. Calculate the optical depth of the Earth's atmosphere.

2. **A Model of the Stellar Atmosphere**

Let's model the stellar atmosphere as a plane-parallel slab of gas, as shown in Figure 9.16 (C&O). Suppose that the source function inside the slab has the form

$$S = a + b\tau_v, \tag{1}$$

where $\tau_v$ is defined to be the *vertical optical depth*, i.e. optical depth measured along the $z$-axis, perpendicular to the slab. To determine the intensity emerging at an angle $\theta$, we need to integrate the radiative transfer equations along the path of the light ray. The optical depth, $\tau$, measured along the ray is related to the vertical optical depth as $\tau = \tau_v / \cos\theta = \tau_v \sec\theta$. You may assume that the total optical depth through the slab is infinite.

**(a)** Prove that the intensity emerging from the slab is equal to the source function evaluated at an optical depth $\tau_v = \cos\theta$:

$$I(\theta) = S(\tau_v = \cos\theta) = a + b\cos\theta. \tag{2}$$

**(b)** Show that the radiative flux at the surface of the slab is determined by the value of the source function at $\tau_v = 2/3$:

$$F = \pi S(\tau_v = 2/3). \tag{3}$$

This result is called the **Eddington-Barbier relation**.