
Problem Set 3

Due: 10 Feb 2012

Please staple problems 1+2 and 3+4.

1. Extended Source

Consider a roughly circular extended celestial source with an angular radius $\theta = 30^\circ$.

(a) Calculate the solid angle subtended by this source.

(b) If the intensity I_λ is uniform over the source, calculate the specific flux measured by a detector pointed directly at the center of it.

2. Properties of the Planck Function

(a) Show that the Planck function reduces to the classical Rayleigh-Jeans law

$$B_\lambda = \frac{2ckT}{\lambda^4} \quad (1)$$

in the limit when $hc/\lambda \ll kT$. Explain why Planck's constant h is not present in this equation.

(b) Derive Wien's displacement law

$$\lambda_{\max} = \frac{0.0029 \text{ m K}}{T}, \quad (2)$$

where λ_{\max} is the wavelength at which B_λ reaches its peak. (You will get an equation that has to be solved numerically. Simple iterative method on a hand calculator should work.)

(c) Starting with the wavelength form of the Planck function, B_λ (Eq. 3.22 in C&O), derive its frequency form, B_ν .

(d) Prove that the frequency ν_{\max} at which B_ν reaches its peak is given by:

$$\nu_{\max} = \frac{2.82kT}{h}. \quad (3)$$

Is $\nu_{\max} = c/\lambda_{\max}$? Why or why not?

(e) Integrate the Planck function over all wavelengths (or frequencies) and find the expression for the Stefan-Boltzmann constant σ in terms of other fundamental constants. Use the fact that

$$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}. \quad (4)$$

3. Sirius

Sirius A (the brighter companion in the Sirius binary star system) is one of our closest stellar neighbors and is the brightest star as seen from Earth. Its surface temperature is close to 9910 K and it is 23.5 times more luminous than our Sun. The parallax of Sirius has been measured to be $0.38''$. Find the following properties of Sirius A:

- (a) Radius in solar units.
- (b) Absolute bolometric magnitude. (Hint: Use the fact that $M_{\text{bol}} = 4.76$ for the Sun.)
- (c) Peak wavelength. Is your result consistent with the fact that Sirius appears to be a blue/white star?
- (d) Let's assume that the sensitivity functions of the standard filters can be approximated as step functions. Let's also assume that the filters are so narrow that specific flux from a star does not vary significantly over the width of a filter. Compute the apparent magnitude of Sirius in the V band. Use the fact that a star with $m_V = 0.0$ has specific flux $F_\lambda(V) = 3.8 \times 10^{-12} \text{ J s}^{-1} \text{ m}^{-2} \text{ \AA}^{-1}$ near the peak of the V-band. (*Hint*: Be careful with units!) Compare your answer to the value given in Appendix F (page A-7 in your textbook), but do not expect perfect agreement. Remember, stars are *not* perfect black bodies.
- (e) Compute the absolute magnitude of Sirius in the V-band.

4. Thermal Balance of a Human Body

Consider a person standing in an enclosed room. The person loses energy through blackbody radiation and also absorbs energy emitted by the walls of the room.

- (a) Ignoring the effect of clothes, calculate the rate of energy absorption and emission by this person. State all your assumptions clearly. Does the person gain or lose energy? Does the shape of the room affect your result?
- (b) Estimate the daily energy loss through radiation. Compare your answer to the average daily caloric intake. Keep in mind that food calories are actually kilo-calories, i.e. a 300 cal hamburger actually contains 3×10^5 cal. ($1 \text{ cal} = 4.184 \text{ J}$). Compare your result to your answer in part (a). What are your conclusions?
- (c) Explain why night-vision goggles, which allow you to see IR radiation, are good for spotting people in the dark.