Astronomy 62 Ann Esin
Introduction to Astrophysics 27 Jan 2012

Problem Set 2

Due: 3 Feb 2012

Please staple 1+2 and 3+4.

1. Planet Around HD 63454

HD 63454 is a $0.7M_\odot$ star known to have an extrasolar planet orbiting it in a *circular* orbit with an orbital period of 2.82 days. The distance to this system is 35.8 pc and the measured radial velocity curve for HD 63454 is shown in Figure 1. Use this information to determine:

(a) the minimum mass of the planet;

(b) the radii of the star's and the planet's orbits;

(c) the maximum astrometric shift of the star due to its orbital motion, expressed in arcseconds.

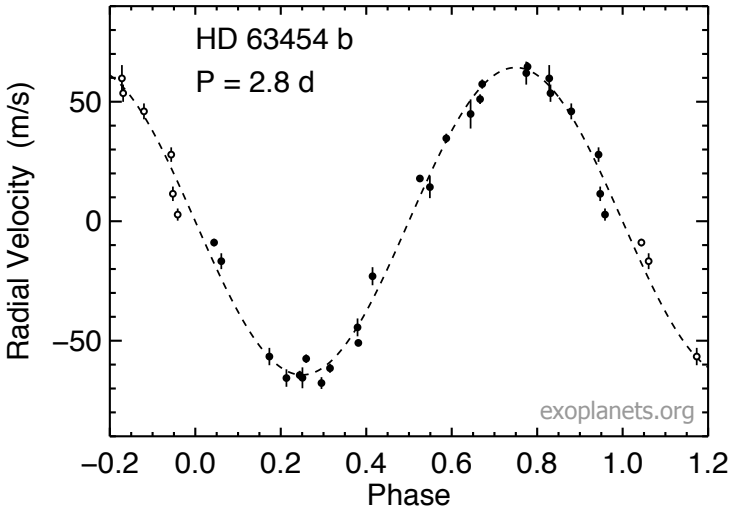

Figure 1: The radial velocity of HD 63454 plotted as a function of orbital phase. (Figure from Moutou et al. 2005, A&A, 439, 367.)

2. Sun-Jupiter System

(a) Assuming that the Sun interacts only with Jupiter, calculate the total orbital angular momentum of the Sun-Jupiter system. The semi-major axis of Jupiter's orbit is $a = 5.2\,\mathrm{AU}$, its orbital eccentricity is $e = 0.048$, and its orbital period is $P = 11.86\,\mathrm{yr}$.

(b) Starting with the cross product definition of angular momentum, prove that in a binary system

$$\frac{L_1}{L_2} = \frac{M_2}{M_1}, \tag{1}$$

where L_1 and L_2 are the contributions of the two masses to the total orbital angular momentum of the Sun-Jupiter system.

(c) Use your results from parts (a) and (b) to calculate the Sun's and Jupiter's contributions to the total orbital angular momentum of the Sun-Jupiter system.

(d) Recall that the moment of inertia of a solid sphere of mass m and radius r is given by $I = 2/5mr^2$, and that when the sphere spins on an axis passing through its center, its rotational angular momentum may be written as $L = I\omega$, where ω is the angular frequency measured in $\mathrm{rad\,s^{-1}}$. Assuming (incorrectly) that both Sun and Jupiter rotate as solid spheres, calculate approximate values for the rotational angular momentum of the Sun and Jupiter. Take the rotational periods of the Sun and Jupiter to be 26 days and 10 hours, respectively. The radius of the Sun is 6.96×10^8 m, and the radius of Jupiter is 6.9×10^7 m.

(e) What part of the Sun-Jupiter system makes the largest contribution to the *total* angular momentum that includes both orbital motion and rotation?

3. **Back to α Centauri System**

In HW#1 you determined that the present distances to α Centauri and Proxima Centauri are 1.32 pc and 1.35 pc, respectively, while the total distance between the two stars is about 12,000 AU.

(a) The apparent magnitudes of the three stars in this triple system are 0.0, 1.3, and 11.0 (α Centauri A, α Centauri B, and Proxima Centauri). What is the total apparent magnitude of the system?

(b) It turns out that A and B α Centauri are much closer to each other than to Proxima Centauri, so we assume they are located at the same distance from us. Compute the absolute magnitudes of the three stars.

(c) Estimate the orbital period of Proxima Centauri, if it's mass is about $0.1M_\odot$, while the masses of its two companions are $\sim 1M_\odot$ and $\sim 0.9M_\odot$. Please explain any assumptions you are making in answering this question.

4. **Kuiper Belt Objects**

Kuiper Belt, discovered in the early 1990's, is a collection of icy objects located in the plane of the ecliptic just beyond the orbit of Neptune, at distanced ranging from 30 AU to ∼1000 AU from the Sun. A discovery in 2003 of a new Pluto-sized Kuiper Belt member named Eris, sparked a bitter debate on whether there are now 10 planets, or whether Pluto should be demoted from its planetary status. (Ultimately, Pluto supporters lost and the Solar System is now considered to contain 8 bona fide planets and an unknown, but probably large, number of dwarf planets inhabiting the Kuiper Belt.) In this problem you will explore a potential scenario for the orbital evolution of Eris. Let's assume that it started its life in a circular orbit somewhere in the vicinity of the inner edge of the Kuiper belt and got kicked into its new orbit through a close encounter with Neptune.

(a) In its current orbit, the distance between Eris and the Sun varies between 38 to 97 AU. Calculate the semi-major axis and eccentricity of its orbit. Is your result consistent with a reported period of 560 years for this dwarf planet?

(b) For a plausible initial orbit of Eris, estimate the change in the planet's angular momentum and energy necessary to place it into its current orbit.

(c) Estimate the fractional change in Neptune's angular momentum and energy during the encounter. What can you say about the effect of this encounter on Neptune. *Hint:* The combined total energy and angular momentum of the two bodies must be conserved during the interaction.