Astronomy 62 | Ann Esin
Introduction to Astrophysics | 24 Apr 2012

Problem Set 12

(Optional) Due: 4 May 2012

1. **Growth of Supermassive Black Holes by Accretion**

   Let's assume that all supermassive black holes found in the centers of galaxies started out as $10M_\odot$ objects very early in the history of the Universe, which subsequently grew by feeding on the interstellar gas and, later on, stars. But the black hole luminosity, and therefore its accretion rate, is fundamentally limited by the Eddington value – when the luminosity exceeds $L_{\rm Edd}$, the accreting gas will simply be blown away by radiation pressure. In HW#11 we showed that the mass accretion rate corresponding to the Eddington luminosity, $L_{\rm Edd}$, is given by

$$\dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{\eta c^2} = \frac{4\pi G M m_p}{\eta c \sigma_T} = 1.8 \times 10^{15} \left(\frac{M}{M_\odot}\right) \,{\rm kg\,s^{-1}}, \qquad (1)$$

   where η is the efficiency with which the black hole is converting accreting matter into radiation. For a non-rotating black hole we set $\eta \simeq 0.08$.

   (a) Assuming that black holes accrete matter at the Eddington rate, write down a differential equation describing the time evolution of the black hole mass. (Hint: Don't forget that $\dot{M} = dM/dt$.) Solve this equation to obtain the black hole mass as a function of time, $M(t)$.

   (b) How long will take to assemble the Sgr A* black hole with a mass $3.7\times10^6 M_\odot$? Comment on your result.

   (c) Repeat your calculation for a $10^9 M_\odot$ black hole. Such black holes are believed to be powering the most luminous quasars. The quasar with the highest redshift found so far has $z = 6.42$. Can its black hole be assembled in time through accretion? (In your calculations, assume that the Universe is flat and ignore the cosmological constant.)

2. **Measuring a Distance to NGC 2639**

   NGC 2639 is an Sa galaxy with a measured maximum rotational velocity of $324\,{\rm km\,s^{-1}}$ and an apparent magnitude of $B = 12.22\,{\rm mag}$ (after making corrections for extinction).

   (a) Determine the distance to NGC 2639 from the appropriate Tully-Fisher relation (Eq. 25.5 in C&O).

   (b) The H_α line in the spectrum of NGC 2639 appears at $\lambda = 666.8\,{\rm nm}$. Calculate the corresponding redshift and estimate the Hubble constant based on this measurement.

   (c) Even a Hubble telescope cannot produce reliable measurements for stars dimmer than about $V = 28\,{\rm mag}$. Use Figure 14.5 in C&O to determine whether we can measure the distance to NGC 2639 using classical Cepheid variables.

3. **Hot Gas in Clusters of Galaxies**

X-ray observations in the late 1970's showed that clusters of galaxies are filled with hot ionized gas that emits copious amounts of high-energy radiation. The total X-ray luminosity of the Virgo cluster (the nearest cluster to the Milky Way Galaxy) is $L_x = 1.5 \times 10^{36}\,\mathrm{J\,s^{-1}}$. Assume that this cluster has radius $R \approx 1.5\,\mathrm{Mpc}$ and radial velocity dispersion $\sigma_r = 666\,\mathrm{km\,s^{-1}}$.

(a) Estimate the total mass of the Virgo cluster (in solar units). Calculate its characteristic mass-to-light ratio if its visual luminosity is $L_V = 1.2 \times 10^{12} L_\odot$. Comment on your result.

(b) Estimate the temperature of the gas filling the Virgo cluster, assuming that the average kinetic energy of each particle (and therefore its thermal energy) obeys the virial theorem in the gravitational potential of the cluster. Take $R/2$ as a characteristic distance of a particle from the cluster center and assume that the corresponding enclosed mass is a quarter of the total cluster mass. You will find that electrons and protons have different *virial* temperatures, but since they are thermally coupled, a good estimate of the gas temperature is an average of your two results.

(c) The X-ray emission is due to bremsstrahlung radiation (free electrons accelerating due to the Coulomb forces from the protons), which has the luminosity density (energy emitted per unit time per unit volume, integrated over all frequencies) given by the equation:

$$\mathcal{L}_{\mathrm{vol}} = 1.42 \times 10^{-40} n_e n_p T^{1/2} (\mathrm{J\,s^{-1}\,m^{-3}}), \tag{2}$$

where n_e and n_p are electron and proton number densities (in units of $\mathrm{m^{-3}}$) and T is the temperature of the gas. (**Note**: Eq. 27.19 in C&O is incorrect!)

Assuming that the gas is mostly ionized hydrogen and the emission is optically thin, estimate the number density and the total mass of the hot gas in the Virgo cluster. Compare your answer to the virial mass of the cluster. Is there still need for dark matter?

(d) Assuming that the gas has no energy source and that it is losing energy via thermal bremsstrahlung at a constant rate L_x, estimate how long it will take for the gas to lose all of its energy. Compare your answer to the Hubble time $t_{\mathrm{H}} = 1/H_0$ and comment of the result.

4. **The Evolution of Λ Universe**

(a) Starting with the energy conservation equation appropriate for a Universe with non-zero cosmological constant Λ,

$$\frac{mv^2}{2} - \frac{GM_r m}{r} - \frac{1}{6}\Lambda m c^2 r^2 = -\frac{1}{2}k[r(t_0)]^2 mc^2, \tag{3}$$

show that the evolution of the scale factor $R(t)$ is given by the equation

$$\left(\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho R^2 - \frac{1}{3}\Lambda c^2 R^2 = -kc^2. \tag{4}$$

(b) Argue that for a constant and positive Λ, the potential energy term due to normal matter will be larger than the term containing the cosmological constant at early times, but will eventually become negligible at late times.

(c) Assume that our Universe is flat ($k = 0$) and solve Eq. (4) in the regime when the cosmological constant dominates the potential energy of the Universe, and so the matter term can be ignored. Comment on your result.