Astronomy 62 Ann Esin
Introduction to Astrophysics 2 April 2012

Problem Set 10

Due: 6 April 2012

Please staple problems 1 and 2 separately.

1. **40 Eridani** (C&O, problem 16.1)

The most easily observed white dwarf in the sky is in the constellation of Eridanus. Three stars comprise the 40 Eridani system: 40 Eri A is a 4th-magnitude star similar to the Sun; 40 Eri B is a 10th-magnitude white dwarf; and 40 Eri C is an 11th-magnitude red M5 star. This problem deals only with the latter two stars, which are in orbit around each other and are separated from 40 Eri A by 400 AU.

(a) The period of the 40 Eri B and C system is 247.9 years. The system's measured trigonometric parallax is $0.201''$ and the true angular extent of the semi-major axis of the reduced mass is $6.89''$. The ratio of the distances of 40 Eri B and C from the center of mass is $a_B/a_C = 0.37$. Find the masses of 40 Eri B and C in terms of the mass of the Sun.

(b) The absolute bolometric magnitude of 40 Eri B is 9.6. Determine its luminosity in terms of the luminosity of the Sun.

(c) The effective temperature of 40 Eri B is $16,900\,\mathrm{K}$. Calculate its radius, and compare your answer to the radii of the Sun, Earth and Sirius B (radius of the latter is $0.008 R_\odot$).

(d) Calculate the average density of 40 Eri B, and compare your result with the average density of Sirius B (with a mass $1.05 M_\odot$ the average density of Sirius B is $3 \times 10^9\,\mathrm{kg\,m^{-3}}$). Which is more dense, and why?

(e) Calculate the product of the mass and volume of both 40 Eri B and Sirius B. Is there a departure from the mass-volume relation? What might be the cause?

2. **Relativistic White Dwarfs** (based on C&O, problem 16.6)

When relativity cannot be ignored, kinetic energy of an electron is given by the equation

$$K = E - mc^2 = \sqrt{p^2c^2 + m^2c^4} - mc^2, \tag{1}$$

where E is the total energy and p is the momentum of an electron. The electron pressure is still proportional to the product of characteristic kinetic energy and number density of electrons, just like in the non-relativistic case. Use this to show that in the extreme relativistic limit (i.e. when $p \gg mc$) the degeneracy pressure has the form: $P \propto \hbar c n_e^{4/3}$.