
Problem Set 1

Due: 29 Jan 2010

Please staple 1+2 and 3+4.

1. Distances to Sun and Moon

In this course we are mostly concerned with post-Newtonian astronomy. But it would be unfair to ancient astronomers to ignore them completely, since they managed to figure out quite a lot about our Solar system. Greek astronomy in particular is very interesting, since they were the first (as far as we know) scientists in the modern sense of the word: they made observations, developed theories to explain what they saw and made appropriate conclusions. This problem describes one of the first recorded astronomical measurements in human history.

(a) To determine the ratio of distances to the Moon and the Sun, Aristarchus of Samos used the moment when the Moon was precisely at half-phase. Draw a picture showing the relative positions of the Earth, the Moon and the Sun when the Moon is at half-phase. What is the ratio of distances to the Moon and to the Sun in terms of the angle between them in the sky?

(b) Perhaps because Aristarchus was more interested in illustrating a cute geometry problem than doing a real astronomical measurement (a typical theorist!), he did not bother to measure this angle himself. He simply assumed that it is equal to 87° . What is the ratio of distances he obtained? How does it compare to the real value?

2. Distance to Mars (problem 3.1 in C&O)

In 1672, an international effort was made to measure the parallax angle of Mars at the time of opposition, when it was closest to Earth; see Fig. 1.16 (in C&O).

(a) Consider two observers who are separated by a baseline equal to Earth's diameter. If the difference in their measurements of Mars's angular position is $33.6''$, what is the distance between Earth and Mars at the time of opposition? Express your answer both in units of m and in AU.

(b) If the distance to Mars is to be measured to within 10%, how closely must the clocks used by the two observers be synchronized? *Hint:* Ignore the rotation of the Earth. The average orbital velocities of the Earth and Mars are 29.79 km s^{-1} and 24.13 km s^{-1} , respectively.

3. α Centauri System

Proxima Centauri is the closest star to the Sun and is a part of a triple star system (the other two companions are called α Centauri A and B). It has epoch 2000.0 coordinates

$(\alpha, \delta) = (14^{\text{h}}29^{\text{m}}43^{\text{s}}, -62^{\circ}40'46'')$ while the brightest star in the system, α Centauri A, is located at $(\alpha, \delta) = (14^{\text{h}}39^{\text{m}}36^{\text{s}}, -60^{\circ}50'8'')$.

- (a) What is the angular separation between Proxima Centauri and α Centauri A?
- (b) If the trigonometric parallaxes for Proxima Centauri and α Centauri A are measured to be $0.758''$ and $0.741''$, respectively, calculate the total distance between the two stars?

4. Properties of Orbits

- (a) Consider the family of elliptical orbits with the same value of angular momentum, and show that the circular orbit has the most binding energy (the largest absolute value of the total energy).
- (b) Consider the family of elliptical orbits with the same value of total energy, and show that the circular orbit has the most angular momentum.
- (c) Now consider a system in a circular orbit with energy E . Show that for such orbits $K = -V/2$, where K and V are the kinetic and potential energies respectively. This is an example of the *virial theorem*, which turns out to hold for any gravitationally bound system in a time-averaged sense.