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Core Lab: What Makes Things Tick?
Homework 1
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You make five measurements for the time of flight of a projectile when launched at 37° from the horizontal, finding **0.366, 0.343, 0.367, 0.359, 0.368** (all values in seconds).

(a) What value and uncertainty should you report for the time of flight t?
(b) What is the **relative uncertainty** of the time value of the previous problem? (that is, the value is good to what percent?)
(c) The projectile was launched at some unknown velocity v_0 and the time of flight covered a trajectory with zero net vertical displacement. Consequently, the time of flight is given by the expression

$$t = \frac{2v_0 \sin\theta}{g}$$

If the uncertainty in the launch angle is 0.5°, find the value and uncertainty of the launch velocity v_0. Be sure to round properly at the end of the calculation. You may take $g = 9.796\frac{\text{m}}{\text{s}^2}$ with negligible uncertainty.

(a) The value I report should be the average of all my measurements, or $\boxed{0.3606\text{s}}$. The uncertainty I should report is plus or minus one standard deviation, or ± 0.0104547s, rouded to $\boxed{\pm 0.01045\text{s}}$.

(b) Expressed as a percentage of 0.3606, 0.01045 is $\boxed{2.9\%}$.

(c) The expression for launch velocity is

$$v_0 = \frac{tg}{2\sin\theta}$$

Because this depends on two quantities with appreciable uncertainty, t and θ, I'll need to find propagated uncertainties in v_0 from both of these quantities, then add them in quadrature. To find the uncertainty in v_0 due to uncertainty in t, I first find that $\frac{\partial v_0}{\partial t} = \frac{g}{2\sin\theta}$. Now multiplying this derivative by the uncertainty in t, I find that the uncertainty in v_0 due to t is

$$\frac{\partial v_0}{\partial t} * \Delta t = \frac{9.796\frac{\text{m}}{\text{s}^2}}{2\sin(37^\circ)} * 0.01045\text{s} \approx 0.08505\frac{\text{m}}{\text{s}}$$

The uncertainty in v_0 due to θ is

$$\frac{\partial v_0}{\partial \theta} * \Delta\theta = -\frac{tg\cos\theta}{2\sin^2\theta} * \Delta\theta = -\frac{0.3606\text{s} * 9.796\frac{\text{m}}{\text{s}^2}\cos(37^\circ)}{2(\sin(37^\circ))^2} * 0.5^\circ \approx -0.0339871\frac{\text{m}}{\text{s}}$$

Adding these two in quadrature, I find that the total uncertainty in v_0 is

$$\Delta v_0 = \sqrt{(0.08505)^2 + (0.0339871)^2} \approx 0.091589 \frac{\text{m}}{\text{s}}$$

Rouding that off to a reasonable two significant figures (significant figures after two are totally dwarfed by the uncertainty value itself), I find that

$$\boxed{\Delta v_0 \approx 0.092 \frac{\text{m}}{\text{s}}}$$

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