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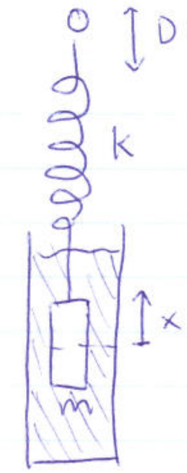
HW#2

CL HW 2



$$m\ddot{x} = -kx - b\dot{x} - mg + D\sin\omega t$$
$$\omega = 2\pi f$$

$$m\ddot{x} + b\dot{x} + kx = D\sin\omega t - mg$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{D}{m}\sin\omega t - g$$

consider Euler's identity: $e^{i\theta} = \cos\theta + i\sin\theta$
and $z(t) = x(t) + i\,y(t)$
↖ we want the imaginary part

solve ~~homogenous solution~~

$$\cancel{\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0} \qquad \cancel{\text{let } x = e^{st}}$$
$$\cancel{s^2 e^{st} + \frac{b}{m} s e^{st} + \frac{k}{m} e^{st} = 0}$$
$$\cancel{s^2 + s\left(\frac{b}{m}\right) + \frac{k}{m} = 0} \qquad \cancel{s = \frac{-\frac{b}{m} \pm \sqrt{\frac{b^2}{m^2} - 4\frac{k}{m}}}{2} = -\frac{b}{2m} \pm \frac{1}{2}\sqrt{\frac{b^2}{m^2}}\sqrt{1 - \frac{4km}{b^2}}}$$

we want $z = \cancel{x + iy}\ ix + y$ s.t.

$$\ddot{z} + \frac{b}{m}\dot{z} + \frac{k}{m}z = \frac{D}{m}e^{i\omega t} - \cancel{m}g \qquad \text{define } 2\beta = \frac{b}{m},\ \omega_0^2 = \frac{k}{m}$$

$$\ddot{z} + 2\beta\dot{z} + \omega_0^2 z = \frac{D}{m}e^{i\omega t} - g$$

let $z = e^{i\Omega t}$ $\cancel{\rightarrow -\Omega^2 z + 2\beta\Omega i z + \omega_0^2 z = (-\Omega^2 + 2i\beta\Omega + \omega_0^2)z = 0}$

↳ $\boxed{\text{solve homogeneous solution}}$

$$-\Omega^2 z + 2\beta\Omega i z + \omega_0^2 z = 0 \qquad (z \neq 0)$$
$$-\Omega^2 + 2\beta\Omega i + \omega_0^2 \qquad \Omega = \frac{-2\beta i \pm \sqrt{-4\beta^2 + 4\omega_0^2}}{2}$$
$$= -\beta i \pm \sqrt{\omega_0^2 - \beta^2} \qquad \text{let } \omega_1 = \sqrt{\omega_0^2 - \beta^2}$$
$$\Omega = -\beta i \pm \omega_1$$

$$\boxed{z_h = C_+ e^{-\beta t} e^{i\omega_1 t} + C_- e^{-\beta t} e^{-i\omega_1 t}}$$

guess $z_p = Ae^{i\omega t} + Bi$

$$-\omega^2 A e^{i\omega t} + 2\beta i\omega A e^{i\omega t} + \omega_0^2 A e^{i\omega t} + \omega_0^2 B i$$
$$= \frac{k}{m} D e^{i\omega t} - gi$$

$\omega_0^2 B = -g$ $\quad B = \frac{-g}{\omega_0^2}$

$$-\omega^2 A + 2\beta i\omega A + \omega_0^2 A = \frac{k}{m} D$$

$$\boxed{A = \frac{kD}{m(-\omega^2 + 2\beta i\omega + \omega_0^2)}}$$

steady state

$$z = C_+ e^{-\beta t} e^{i\omega t} + C_- e^{-\beta t} e^{-i\omega t} + \frac{kDe^{i\omega t}}{m(+\omega^2 + 2\beta i\omega + \omega_0^2)} - \frac{g}{\omega_0^2} i$$

could be complex

$\text{Im}\{z\} = x$

$$x = C_+ e^{-\beta t} \sin\omega t + C_- e^{-\beta t} \sin(-\omega t) + \frac{kD \sin\omega t}{m(-\omega^2 + 2\beta i\omega + \omega_0^2)} - \frac{g}{\omega_0^2}$$

$$\boxed{x(t) = \sin\omega t \left[e^{-\beta t}(C_+ - C_-) + \frac{kD}{m(-\omega^2 + 2\beta i\omega + \omega_0^2)} \right] - \frac{g}{\omega_0^2}}$$

steady-state:

$$x(t) = \frac{kD}{m(-\omega^2 + 2\beta i\omega + \omega_0^2)} \sin\omega t - \frac{g}{\omega_0^2}$$

$$\boxed{\text{amplitude } A = \frac{kD}{m(-\omega^2 + 2\beta i\omega + \omega_0^2)}}$$ for $\omega = 2\pi f$

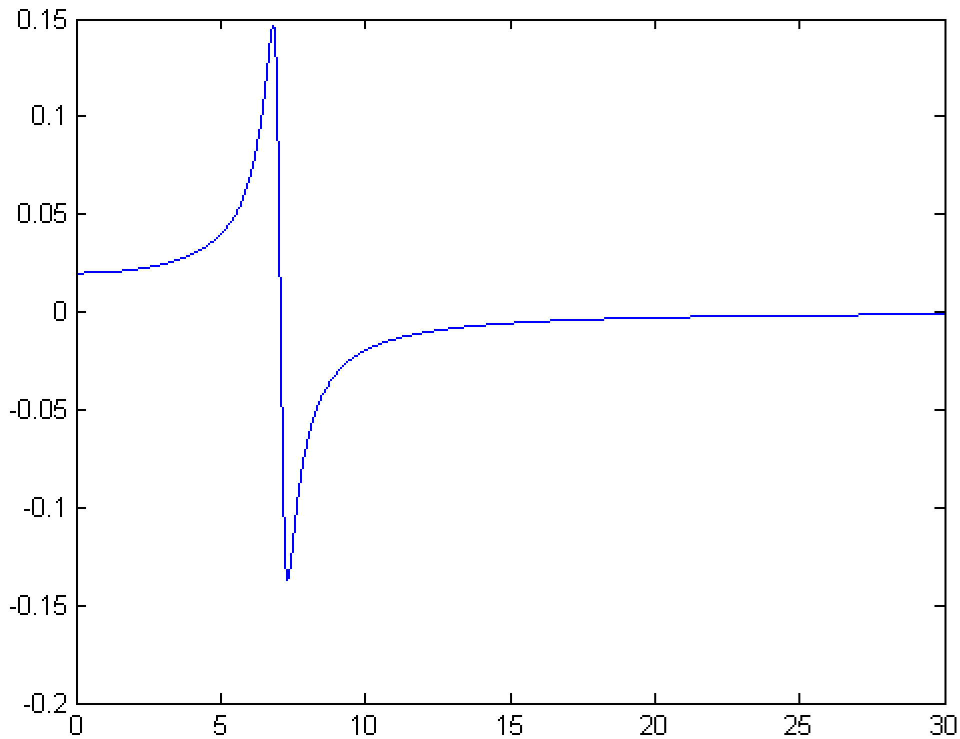
$$\frac{kD}{m(\omega^2 + 2\beta i\omega + \omega_0^2)} \cos\omega t + i\left[\frac{kD}{m(\omega^2 + 2\beta i\omega + \omega_0^2)} \sin\omega t - \frac{g}{\omega_0^2}\right]$$

what is the phase?

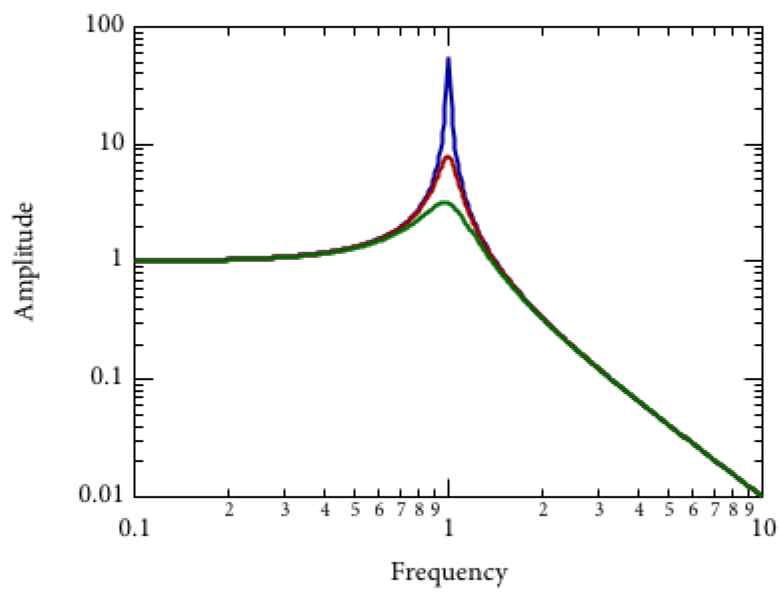
$$z_{steady} = A\cos\omega t + i\left(A\sin\omega t - \frac{g}{\omega_0^2}\right) \qquad A_z = \sqrt{A^2\cos^2\omega t + \left(A^2\sin^2\omega t - \frac{g}{\omega_0^2}\right)^2}$$

I plotted my result for amplitude with MATLAB.

Amplitude of steady-state oscillation (meters) vs. Frequency (Hz)



Ok, so the final amplitude graph is completely wrong. I should obtain a graph that looks similar to this:



(source: http://www.physics.hmc.edu/courses/p057/pmwiki/pmwiki.php/Physics/Theory4)

Possible Mistakes I made on What Makes Things Tick? Lab HW#2:

1. Should have converted the sinusoidal forcing function into a phase-shifted cosine function. May have made math easier to understand since then we'd be finding the real part of a function z.
2. May have followed the example on the page incorrectly.
3. Found the real part of my answer for x(t) incorrectly. This mistake is the most likely, and if I had done it correctly, I would probably have a graph for amplitude that was the right shape.

I did not understand the math enough to find an expression for the *phase* of the bob in terms of frequency. I know that the phase of the driving force was 0 (or -pi/2 if I had converted the sine to a cosine), so if the bob had a different phase, then in the complex solution, there would be an

$$e^{i(\omega t+\phi)}$$

factor in the steady-state solution for x(t), where phi would be the phase.