Modeling Non-Linear Response of Analog Logarithmic Amplifiers

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Abstract

A logarithmic amplifier was given a sinusoidal input voltage, and the corresponding output voltage was studied. A logarithmic amplifier circuit was made with an operational amplifier, diode, and resistor circuit elements and the input and output voltages were studied with oscilloscopes. A fit function $V_{out} = -V_T ln(Asin(ft + \phi) + k)$ was made with the following coefficient values: $V_T = 0.05814 \pm 7.7 \times 10^{-5}$ V, $A = 13100 \pm 170$ V/A, $f = 1.256039 \times 10^{-6} \pm 7.9 \times 10^{-12}$ Hz, $\phi = 1.6357 \pm 0.00087$, $k = 13800 \pm 180$ V/A. The χ^2-value for the graph was 1.78 per degree of freedom. The fit function fit most of the data well except for periodic sharp peaks in the data, resulting in a partially accurate model of the logarithmic output voltage.

Introduction

In this experiment, I decided to study a logarithmic amplifier, which produces a nonlinear output voltage that varies according to the natural logarithm of the input voltage. The logarithmic amplifier consists of two main important circuit components that I am interested in studying: an operational amplifier and a diode. Operational amplifiers are powerful circuit elements that amplify electronic voltage and are used in many circuit applications. Diodes, a popular example of which is the LED (Light-Emitting Diode), are circuit elements that allow current to flow in one direction more easily than the other. We can create a logarithmic amplifier by arranging these two circuit elements together with a resistor, which dissipates voltage. My experiment seeks to study the logarithmic amplifier's nonlinear behavior by modeling the output voltage as a function of an sinusoidal voltage input.

Theory

A simple logarithmic amplifier can be made following the diagram in Fig. 1.

The operational amplifier performs two basic functions: its inputs draw no current and its output makes sure the two input voltages are equal. [1] Thus, when a voltage V_{in} is connected, the current through the resistor (V_{in}/R by Ohm's Law) is equal to the current through the diode. Next, using the Shockley Diode equation ($I = I_s(e^{V_D/nV_t} - 1) \approx I_s(e^{V_D/V_T})$, where the diode is characterized by the saturation constant I_s, the thermal voltage V_T, and the ideality constant n) and the input

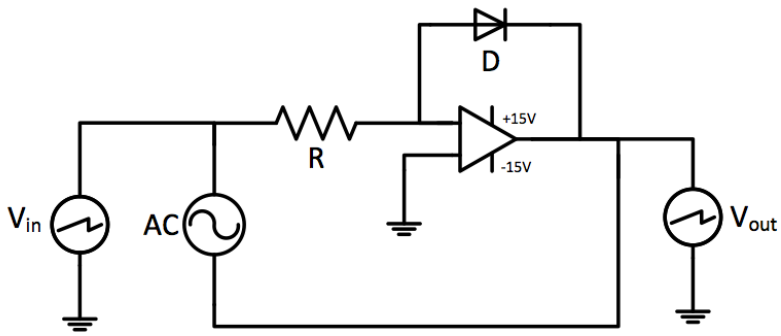

Figure 1: Circuit diagram of a logarithmic amplifier. The function generator generates an alternating voltage to supply the input voltage. The oscilloscope labeled V_{in} measures this input voltage with respect to ground. Charge then passes through the resistor (R) and then the diode (D). The output voltage is read by the second oscilloscope labelled V_{out} with respect to ground.

current as V_{in}/R, we can derive the output voltage (V_D) to be

$$V_D = -V_T ln(\frac{V_{in}}{I_s R}) \tag{1}$$

According to theory, we should be able to substitute a time-varying V_{in} into this equation to find V_D as a function of time. We now set to experimentally determine if this is true.

Experiment

The setup of the experiment is based on the circuit diagram described in Fig. 1. I used a 100 ohm resistor and used an external +15V and -15V power supply to power the operational amplifier.

To test if equation (1) is true, I decided to use a continuous and differentiable input voltage function on the function generator. Naturally, this led me to use a sinusoidal input voltage function. I used a sinusoid with a frequency of 20Hz

To collect my data, I used a LabJack U3-HV unit to take in numerical data onto my computer for the input and output voltages with respect to ground (where the two oscilloscopes are labeled in the figure). In addition, I also attached an oscilloscope to these same two voltages in order to view a live voltage vs. time plot. My sampling rate with the LabJack unit was 10,000 Hz, well above the input voltage's 20 Hz frequency to gather the maximum possible points within the LabJack's ability and also to prevent any possible aliasing. I took my data in approximately 2 second intervals.

In order for me to determine the instrumental error from my setup, I also took data using a square wave for an input voltage. This will be more relevant when I discuss the results of my experiment.

Results

To analyze my data, I first hypothesized that my output voltage was of the form $f(t) = -V_T ln(Asin(ft + \phi) + k)$. I then fit my data with this function in Igor. This fit function is the original theoretical derivation in Eq. 1 with V_{in} substituted as a sinusoidal function of time $(-V_T ln(\frac{Asin(ft+\phi)+k}{I_s R}))$ except without the I_s and R parameters. Leaving the $I_s R$ term in the fit function resulted in insanely high error bars for the fit coefficients, resulting in a meaningless result. Figure 2 shows my best fit curve after using several different guesses for the coefficients.

To obtain error bars for my output voltage, I only took into account instrumental error. I found the instrumental error by using the output voltage data I obtained from using the square wave input voltage. Since the output is also a square wave, I took one section of the output when all the output voltage values are the same, which amounted to 250 data points, and took the standard deviation of those points. I obtained a value of 0.0053 and then used that value for all my error bars.

The most notable feature of this fit function can be seen immediately through the residuals, which follow a pattern. Residuals are computed by calculating the difference between the expected value (obtained from the fit function) and the collected data value. The residuals thus show that our fit function is not reliable at modeling the behavior close to the high peaks.

Looking at the error bars for each of the fit coefficients, we can see that most of the error bars are several orders of magnitude smaller than the actual coefficient values, showing that the fit function fits the data pretty well. The exception lies with the A and k values, whose error bars are both around two orders of magnitude below the actual value. This makes sense because these two values control how tall the fit function is, and since the fit function is not encapsulating the behavior at the sharp peaks of the data, it is reasonable to see that the A and k error bars are larger than I would like.

Next, we compare our value of our fit coefficients A and k with the expected values for $\frac{A'}{I_s R}$ and $\frac{k'}{I_s R}$, where A' and k' are the amplitude and voltage offset of the original input voltage. We can do this by comparing $\frac{A}{k}$ and $\frac{A'}{k'}$. After fitting the input voltage to a sine function, I obtained that $A' = 0.066662$ V and $k' = 0.13603$ V. We can quickly see that $\frac{A'}{k'}$ is about half of $\frac{A}{k}$. Since this difference is very significant and would change the graph of V_{out} significantly, the only explanation that I have for this is perhaps I had forgot to connect one oscilloscope reading to be with respect to ground, resulting in a voltage offset (the k value) and thus, a source of systematic error.

The χ^2-value is greater than 1, which means there are other sources of error I have

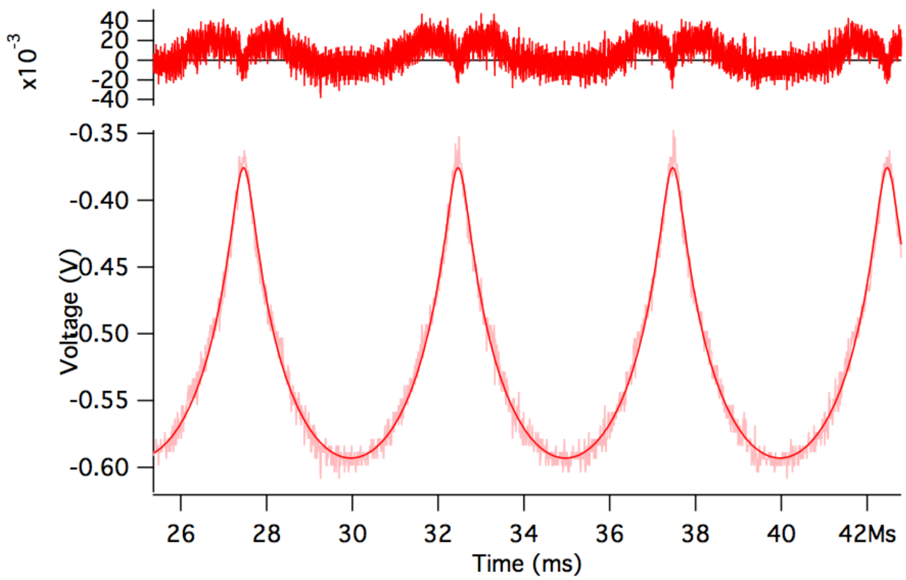

Figure 2: Output Voltage as a function of time. The fit curve has the form $V_{out} = -V_T ln(Asin(ft + \phi) + k)$ with the following values: $V_T = 0.05814 \pm 7.7 \times 10^{-5}$ V, $A = 13100 \pm 170$ V/A, $f = 1.256039 \times 10^{-6} \pm 7.9 \times 10^{-12}$ Hz, $\phi = 1.6357 \pm 0.00087$, $k = 13800 \pm 180$ V/A. The χ^2-value for this graph is 1.78 per degree of freedom. The upper panel shows residuals, which follow a pattern, showing that this fit does not fully describe the output voltage at the sharp peaks.
Note: Above graph is a zoomed-in image of all the collected data.

not accounted for. This makes sense because, as mentioned before, my error bars only took into account instrumental error. Also, this fit function, also mentioned before, does not fit the sharp peaks well at all, contributing to a less-than-ideal χ^2-value as well because the χ^2-value measures how well the fit function fits the data. Nevertheless, I think this χ^2-value still is a fairly good fit for the "valleys" of the graph.

Conclusion

As evidenced in Figure 2, my hypothesized fit function fits the data for most of the graph, the exception being that it does not model the behavior at the peaks very well. Besides the peaks, my function fits the data very well, as evidenced through the coefficient values I obtained and the reduced χ^2-value, which would be even closer to 1 without the peaks. To try to explain the behavior at the peaks, I think that I would need to explore different mathematical models; considering I had 500 data points per period and approximately 20,000 data points in total for my fit, this behavior cannot be attributed to any source of random error. Logarithmic amplifiers are used in real-world applications, such as RSSI (Received Signal Strength Indicator). [3] As evidenced by my experiment, for some portion of the logarithmic amplifier's behavior, we cannot simply substitute the input voltage as a function of time into Eq. 1. to accurately predict the amplifier's output. Essentially, I think that my experiment should caution anyone using non-linear circuits to experimentally determine what the circuit does as I have demonstrated that theoretical models can sometimes fail.

References

[1] "Op Amps." *What Makes Things Tick?*. 11 August 2011. Web. 23 November 2013.

[2] "Log amplifier", *Wikipedia*. 18 March 2013. Web. 23 November 2013.

[3] Signal Processing Group, Inc. "Log amplifier fundamentals." Web. 23 November 2013.

[Do not include references that are not specifically cited with a numerical endnote in the text.] - what does this mean?