Modeling Non-Linear Response of Analog Logarithmic Amplifiers

Mo Zhao
Harvey Mudd College
23 November 2013

Abstract

A logarithmic amplifier was given a sinusoidal input voltage, and the corresponding output voltage was studied. The logarithmic amplifier circuit was made with an operational amplifier, diode, and resistor circuit elements and the input and output voltages were studied with oscilloscopes. A fit function $V_{out} = -V_T \ln(A\sin(ft+\phi)+k)$ was made with the following coefficient values: $V_T = 0.05814 \pm 7.7 \times 10^{-5}$ V, $A = 13100 \pm 170$ V/A, $f = 1.256039 \times 10^{-6} \pm 7.9 \times 10^{-12}$ Hz, $\phi = 1.6357 \pm 0.00087$, and $k = 13800 \pm 180$ V/A. The χ^2-value for the graph was 1.78 per degree of freedom. The fit function modelled most of the data well except for periodic sharp peaks, resulting in a partially accurate model of the logarithmic output voltage.

Introduction

In this experiment, I studied a logarithmic amplifier, which produces a nonlinear output voltage that varies according to the natural logarithm of the input voltage. The logarithmic amplifier consists of two main important circuit components: an operational amplifier and a diode. Operational amplifiers are integrated circuits used in many circuit applications and can be used to perform many analog mathematical computations, such as multiplication and integration when coupled with different circuit elements. Diodes, a popular example of which is the LED (Light-Emitting Diode), are circuit elements that allow current to flow in one direction more easily than the other. A logarithmic amplifier can be created by arranging these two circuit elements together with a resistor, which dissipates energy. This experiment seeks to study the logarithmic amplifier's nonlinear behavior by modeling the output voltage as a function of a sinusoidal voltage input.

Theory

A simple logarithmic amplifier can be made following the diagram in Fig. 1.

The operational amplifier performs two basic functions: its inputs draw no current and its output ensures the two input voltages are equal.[1] Thus, when a voltage V_{in} is inputted, the current through the resistor (V_{in}/R by Ohm's Law) is equal to the current through the diode. Next, using the Shockley Diode equation ($I = I_s(e^{V_D/nV_{thermal}} - 1) \approx I_s(e^{V_D/nV_{thermal}})$ and the input current as V_{in}/R, we

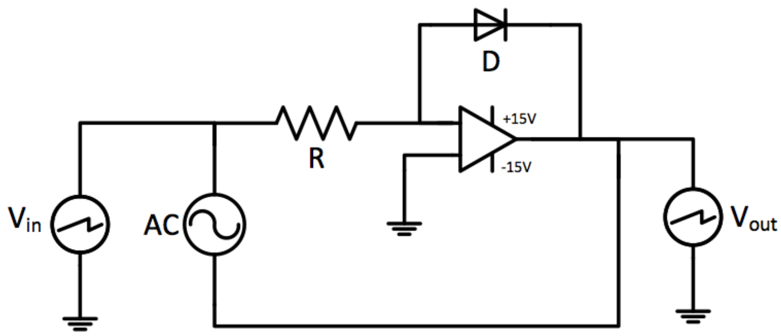

Figure 1: Circuit diagram of a logarithmic amplifier. The function generator generates an alternating voltage to supply the input voltage. The oscilloscope labeled V_{in} measures this input voltage with respect to ground. Charge then passes through the resistor (R) and then the diode (D). The output voltage is read by the second oscilloscope labelled V_{out} with respect to ground.

can derive the voltage across the diode (V_D) to be[2]

$$V_D = -nV_{thermal} \ln(\frac{V_{in}}{I_s R}) \tag{1}$$

The saturation current I_s, the thermal voltage $V_{thermal}$, and the ideality constant n characterize the diode. In the circuit drawn, the output voltage (V_{out}) is equivalent to the voltage across the diode (V_D) because the diode lies between virtual ground and the output voltage. Theoretically, substitute a time-varying V_{in} into this equation should yield V_{out} as a function of time. In this experiment, I will try to determine if this is true.

Experiment

The setup of the experiment is based on the circuit diagram described in Fig. 1.

In this experiment to test equation (1), a function generator was connected to supply a sinusoidal voltage versus time input with respect to ground. A sinusoid with a frequency of 20 Hz was used.

In the circuit, a 100 ohm resistor was used as the resistor. Also, an external $+15V$ and $-15V$ power supply was used to power the operational amplifier.

To collect my data, a LabJack U3-HV unit was used to take in numerical data for the input and output voltages with respect to ground (where the two oscilloscopes are labeled in the Fig. 1). In addition, an oscilloscope was attached across these same two voltages in order to view a live voltage vs. time plot. The sampling rate with the LabJack unit was 10,000 Hz, well above the input voltage's 20 Hz frequency to gather the maximum possible points within the LabJack's ability and also to prevent any possible aliasing. I took my data in approximately 2 second intervals.

In order for me to determine the instrumental error from my setup, I also took data using a square wave for an input voltage. This will be discussed further when I discuss the results of my experiment.

Results

To analyze my data, I first hypothesized that my output voltage was of the form $f(t) = -V_T ln(Asin(ft + \phi) + k)$. I then fit my data with this function in Igor.

This fit function is the original theoretical derivation in Eq. 1 with V_{in} substituted as a sinusoidal function of time $(-nV_{thermal} ln(\frac{A' sin(ft+\phi)+k'}{I_s R}))$ with a few modifications. I chose not to include the I_s, R, and n parameters because leaving the $I_s R$ and n terms in the fit function resulted in insanely high uncertainties for the fit coefficients, resulting in a meaningless result. However, the behavior of the I_s, R, and n parameters are still encapsulated by the parameters of the fit function. That is, $V_T = nV_{thermal}$, A is equal to the amplitude of the sinusoidal input voltage (A') divided by $I_s R$, and k is equal to the voltage offset of the sinusoidal input voltage (k') divided by $I_s R$. Figure 2 shows my best fit curve with this fit function.

To obtain uncertainties for my output voltage, I only took into account instrumental error. The instrumental error was found by using the output voltage data obtained from using the square wave input voltage. Since the output is also a square wave, I took one section of the output when all the output voltage values were approximately the same, which amounted to 250 data points, and took the standard deviation of those points. The resulting standard deviation was 0.0053, which was then used as the value for all the error bars.

The most notable feature of this fit function can be seen immediately through the residuals, which follow a pattern. Residuals are computed by calculating the difference between the expected value (obtained from the fit function) and the collected data value. The residuals thus show that our fit function is not reliable at modeling the behavior close to the high peaks.

Looking at the uncertainties for each of the fit coefficients, it can be seen that most of the uncertainties are several orders of magnitude smaller than the actual coefficient values, indicating that the fit function fits the data pretty well. The exception lies with the A and k values, whose uncertainties are both around two orders of magnitude below the actual value. This makes sense because these two values control how tall the fit function is, and since the fit function is not encapsulating the behavior at the sharp peaks of the data, it is reasonable to see that the uncertainties for A and k are larger than ideal.

Recall how the real parameters I_s, R, and n were eliminated under the claim

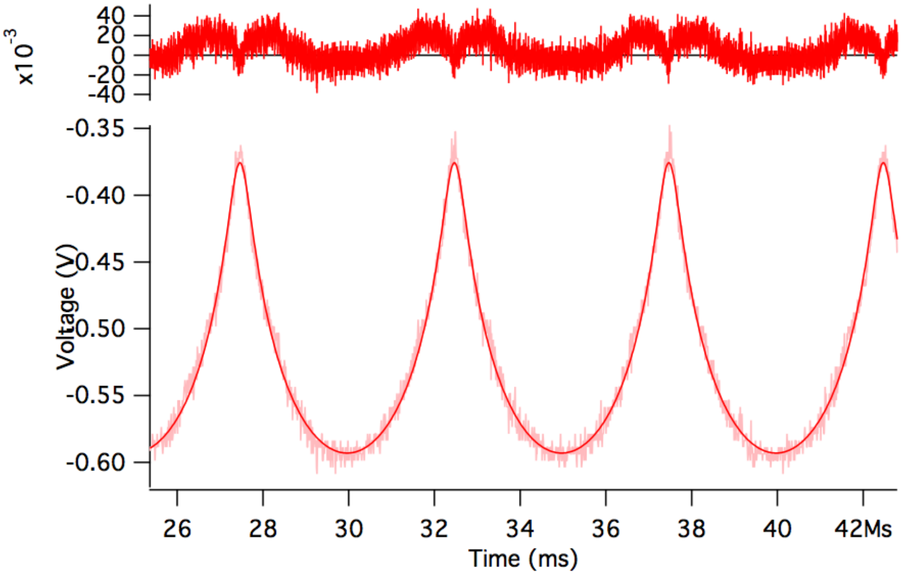

Figure 2: Output Voltage as a function of time. The fit curve has the form $V_{out} = -V_T ln(Asin(ft + \phi) + k)$ with the following values: $V_T = 0.05814 \pm 7.7 \times 10^{-5}$ V, $A = 13100 \pm 170$ V/A, $f = 1.256039 \times 10^{-6} \pm 7.9 \times 10^{-12}$ Hz, $\phi = 1.6357 \pm 0.00087$, $k = 13800 \pm 180$ V/A. The χ^2-value for this graph is 1.78 per degree of freedom. The upper panel shows residuals, which follow a pattern, showing that this fit does not fully describe the output voltage at the sharp peaks.
Note: Above graph is a zoomed-in image of all the collected data.

that their behavior would be encapsulated in the fit function's parameters. Now, we compare our value of our fit coefficients A and k with the expected values for $\frac{A'}{I_s R}$ and $\frac{k'}{I_s R}$, where A' and k' are the amplitude and voltage offset of the original input voltage respectively. We can do this by comparing $\frac{A}{k}$ and $\frac{A'}{k'}$, which we expect to be equal. After fitting the raw input voltage to a sine function, I obtained that $A' = 0.066662$ V and $k' = 0.13603$ V. However, because of how the circuit was arranged, there is an inherent voltage offset reading in k' as it is read through the LabJack unit. To find this offset, I let the LabJack unit take in data for about two seconds without turning on the function generator. I then took the input voltage data and obtained an average of 0.06486 V. Thus, the actual input voltage has a DC offset of $k' = 0.13603 \text{ V} - 0.06486 \text{ V} = 0.07117$ V. Having showed that $\frac{A}{k}$ and $\frac{A'}{k'}$ are approximately equal validates the elimination of the $I_s R$ parameter when we fit our function in Figure 2.

The χ^2-value is greater than 1, which means there are other sources of error I have not accounted for. This makes sense because, as mentioned before, my error bars only took into account instrumental error. Also, this fit function, as mentioned before, does not fit the sharp peaks well at all, contributing to a less-than-ideal χ^2-value as well because the χ^2-value measures how well the fit function fits the data. Nevertheless, I think this χ^2-value still implies a fairly good fit for the "valleys" of the graph.

Conclusion

As evidenced in Figure 2, the hypothesized fit function fits the data for most of the graph, the exception being that it does not model the behavior at the peaks very well. Besides the peaks, my function fits the data very well, as evidenced through the coefficient values I obtained and the reduced χ^2-value, which would be even closer to 1 (and therefore fit the collected data better) without the peaks. I think that my results are limited by the mathematical model I used to try to encapsulate the behavior of the circuit. It is also possible that at the peaks, the LabJack oscilloscope does not take in properly because of the sudden change. To improve upon this experiment, different mathematical models or even different input voltage functions can also be tested to possibly better model the behavior of this circuit.

Logarithmic amplifiers are used in real-world applications, such as signal processing.[3] As evidenced by my experiment, for some portion of the logarithmic amplifier's behavior, cannot simply substitute the input voltage as a function of time into Eq. 1. to accurately predict the amplifier's output. Thus, in this experiment, I have determined the equation that accurately models the output voltage for a logarithmic amplifier given a sinusoidal voltage input with the exception of the behavior at the peaks. In real-world applications, knowing this information can help avoid signal processing inaccuracies when using a logarithmic amplifier.

References

[1] "Op Amps." *What Makes Things Tick?*. 11 August 2011. Web. 23 November 2013.

[2] Peyton, Anthony, and Vincent Walsh. *Analog Electronics With Op Amps: A Source Book of Practical Circuits*. New York: Cambridge University Press, 1993. *Google Books*. Web. 10 December 2013.

[3] Signal Processing Group, Inc. "Log amplifier fundamentals." Web. 23 November 2013.