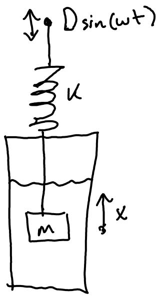
24 September 2013
09:34

Homework 2 9/24/2013

Consider:



Forces acting on mass m:
(assume x=0 for rest position)

$$F_d = -b\dot{x}$$
$$F_K = -Kx$$
$$F_{drive} = D\sin(\omega t)$$

$$m\ddot{x} = -b\dot{x} - Kx + D\sin(\omega t)$$ is our system eq.

More neatly,

$$\boxed{\ddot{x} + \frac{b}{m}\dot{x} + \frac{K}{m}x = D\sin(\omega t)}$$

Now let us solve this dif-eq, starting without drive.

Given $\ddot{x} + \frac{b}{m}\dot{x} + \frac{K}{m}x = 0$, guess $x = e^{i\omega t}$ ...

$$-\omega^2 e^{i\omega t} + i\omega\frac{b}{m}e^{i\omega t} + \frac{K}{m}e^{i\omega t} = 0$$

$$\left(-\omega^2 + i\frac{b}{m}\omega + \frac{K}{m}\right)\cancel{e^{i\omega t}} = 0$$

By quadratic formula,

$$\omega = \left(-\frac{b}{m}i \pm \sqrt{-\left(\frac{b}{m}\right)^2 + 4\frac{K}{m}}\right)\Big/-2$$

$$\omega = \frac{b}{2m}i \mp \frac{1}{2}\sqrt{\left(2\sqrt{\frac{K}{m}}\right)^2 - \left(\frac{b}{m}\right)^2}$$

In a desperate attempt to neaten this,
let $\frac{b}{m} = \varsigma$, and let $\omega_0 = 2\sqrt{\frac{K}{m}}$

$$\omega = \frac{1}{2}\varsigma i \mp \frac{1}{2}\sqrt{\omega_0^2 - \varsigma^2}$$

Our x func, then, is

$$\boxed{x(t) = c_1 e^{-\frac{1}{2}\varsigma t} e^{\frac{-i}{2}\sqrt{\omega_0^2 - \varsigma^2}\,t} + c_2 e^{-\frac{1}{2}\varsigma t} e^{\frac{i}{2}\sqrt{\omega_0^2 - \varsigma^2}\,t}}$$

Now let us consider our driving function, $D\sin(\omega t)$

Now let us consider our driving function, $D\sin(\omega t)$

I screwed up, and forgot ω wasn't fresh,

so, acknowledging $\omega \neq \omega$ as above....

(it has been evaluated, so, no issues will arise beyond "wait, what?")

Consider the forced solution as having $A\cos(\omega t)$ as form:

$$-A\omega^2\cos(\omega t) + A\zeta\omega\sin(\omega t) + \tfrac{1}{4}\omega_0^2 A\cos(\omega t) = D\cos(\omega t)$$

$$A\,\tfrac{1}{4}\omega_0^2 - \omega^2 = D$$ (taking $t=0$, s.t no sin contribution)

$$\left[x(t) = D/\left(\tfrac{1}{4}\omega_0^2 - \omega^2\right)\cos(\omega t)\right]$$ is forced sol.

Our final solution, then, takes the form

$$x(t) = \mathrm{Re}\left\{Z e^{-\frac{1}{2}\zeta t} e^{-i\frac{1}{2}\sqrt{\omega_0^2-\zeta^2}\,t} + D/\left(\tfrac{1}{4}\omega_0^2-\omega^2\right)\cos(\omega t)\right\}$$

* I eliminated the secondary term of x_{Hom}, which works, so long as $Z = a+bi$ is complex.

Using $m=0.2\,kg$, $K=10\,N/m$, $b=0.1\,kg/s$, $D=0.02\,m$ ....

[and thusly, $\zeta = \frac{1}{2}\frac{1}{s}$, $\omega_0 = 10\sqrt{2}\,\frac{1}{s}$]

$$x(t) = \mathrm{Re}\left\{Z e^{-5\sqrt{2}t} e^{-i\frac{1}{2}\sqrt{199.75}\,t} + \frac{0.02}{(50-\omega^2)}\cos(\omega t)\right\}$$