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Core Lab 57 Section 01
Homework 03
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HW 03 At time $t = 0$, the potential difference V_{in} switches suddenly from 0 to V_0. Calculate V_{out} as a function of time for $t > 0$ and sketch $V_{\text{out}}(t)$.

Applying Ohm's Law across the resistor R gives

$$V_{\text{in}} - V_{\text{out}} = IR$$

The same current I passes through the capacitor according to the relationship

$$\frac{dV_C}{dt} = \frac{dV_{\text{out}}}{dt} = \frac{1}{C}\frac{dQ}{dt} = \frac{I}{C}$$

Substituting and rearranging gives

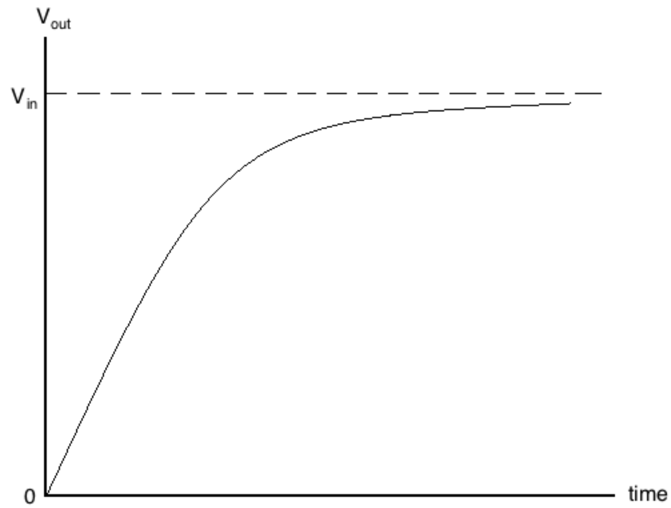
$$\frac{dV_{\text{out}}}{dt} + \frac{1}{RC}V_{\text{out}} = \frac{1}{RC}V_{\text{in}}$$

Solving for the homogeneous solution, we have

$$\begin{aligned}
\frac{dV_{\text{out}}}{dt} + \frac{1}{RC}V_{\text{out}} = 0 \\
\frac{dV_{\text{out}}}{dt} &= -\frac{1}{RC}V_{\text{out}} \\
\frac{dV_{\text{out}}}{V_{\text{out}}} &= -\frac{dt}{RC} \\
\int \frac{dV_{\text{out}}}{V_{\text{out}}} &= \int -\frac{dt}{RC} \\
\ln\left(V_{\text{out}}\right) &= -\frac{t}{RC} + C \\
V_{\text{out}} &= e^{-\frac{t}{RC}+C} \\
&= Ae^{-\frac{t}{RC}}
\end{aligned}$$

A particular solution is $V_{\text{out}} = V_{\text{in}}$ and the initial condition $V_{\text{out}}(0) = 0$ determines A. Therefore, we find the solution

$$\boxed{V_{\text{out}}(t) = V_{\text{in}}\left(1 - e^{-\frac{t}{RC}}\right)}$$



V_{out}
V_{in}
0
time

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