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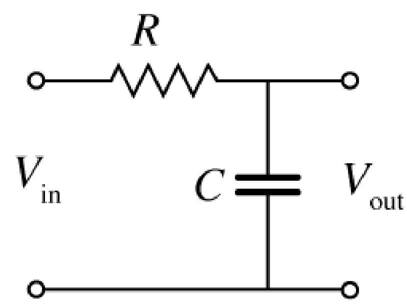
What Makes Things Tick
Homework 3
October 15, 2013

At time t=0 the potential difference V_{in} switches suddenly from 0 to V_0. Calculate V_{out} as a funtion of time for $t > 0$ and sketch $V_{out}(t)$.

Hints

1. See the top of *this page* for the relationship between charge Q and voltage across a capacitor, and between current $I = dQ/dt$ and voltage across a resistor.

2. See *this page* for a discussion of the method of separation of variables.



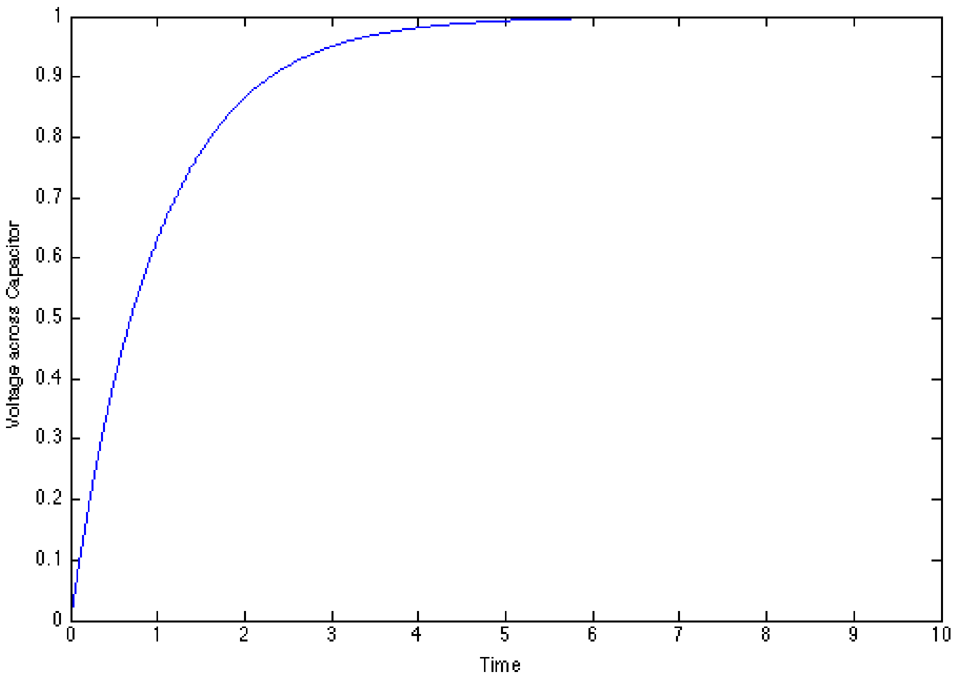
So after being confused by the unfamiliar notation for a bit and asking Tasha for clarification, I redrew this as a simple RC circuit in the form I know and love, with a voltmeter hooked up to measure the voltage difference around the capacitor. I already analyzed this for the capacitor lab, but I might as well recopy the work here. First I find an equation for the charge as a function of time.

$$\begin{aligned}
\Delta V_{throughCircuit} &= 0 \\
V_0 - V_{capacitor} - V_{resistor} &= 0 \\
V_0 - \frac{q}{C} - \frac{dq}{dt}R &= 0 \\
V_0 - \frac{q}{C} &= \frac{dq}{dt}R \\
V_0C - q &= \frac{dq}{dt}RC \\
\frac{1}{RC} &= \frac{1}{V_0C - q}\frac{dq}{dt} \\
\int_0^t \frac{1}{RC}dt &= \int_0^q \frac{1}{V_0C - q}dq \\
\frac{t}{RC} &= -\ln(V_0C - q)_0^q \\
-\frac{t}{RC} &= \ln(V_0C - q) - \ln(V_0C) = \ln\left(\frac{V_0C - q}{V_0C}\right) \\
e^{-\frac{t}{RC}} &= \frac{V_0C - q}{V_0C} = 1 - \frac{q}{V_0C} \\
1 - e^{-\frac{t}{RC}} &= \frac{q}{V_0C} \\
V_0C(1 - e^{-\frac{t}{RC}}) &= q
\end{aligned}$$

With this expression for charge as a function of time, we know that the voltage across the capacitor is the ratio of charge and its capacitance, so we can get V_C:

$$C = \frac{q}{V_C} = \frac{V_0 C(1 - e^{-\frac{t}{RC}})}{V_C}$$
$$1 = \frac{V_0(1 - e^{-\frac{t}{RC}})}{V_C}$$
$$V_C = V_0(1 - e^{-\frac{t}{RC}})$$

Here's a sketch for V_C with V_0, R and C all 1. $V_C = 1 - e^{-t}$



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