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SIMPLE HARMONIC OSCILLATOR

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Day 1

Objective: Measure the effect of mass on period.

Materials:

· 2 springs (10 N/m ±0.05) . We used the red springs from the ”Series/Parallel Spring Set”.

Prediction: Using the measured mass of the cart, we predict that the period of oscillation will be

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ &= 2\pi\sqrt{\frac{0.5035\text{ kg}}{(10\text{ N/m})}} \\ &= 1.41\text{ s } \Rightarrow 1.4\text{ s} \end{aligned}$$

Procedures:

1. **Verify and Measure:** Measured the mass of cart, springs, and additional mass blocks using a mass balance. Before use, we verified the integrity of the mass balance using pre-measured weights.

   Mass of Cart: 503.5 g
   Mass of both springs: 27.5 g
   Mass of both aluminum blocks: 172.5 g
   Mass of Iron block 1: 492.5 g
   Mass of Iron block 2: 494.0 g

   NOTE: Error bars for mass measurements are ± 0.05 g

   We also verified that the track was leveled using a level.

2. **Measuring the Period:** Using a stopwatch, we measured the amount of time it took for the cart to complete one oscillation. We displaced the edge of the cart about 20 centimeters. After release, we let the cart complete one cycle and then started the timer. After 5 cycles, we noted down the time.

   NOTE: We realize that we are not working with perfect conditions. The track is not frictionless and the springs are not massless. Thus this will affect our results such that they differ from our prediction.

3. **Measuring the Period - Varying Mass:** We repeated *Step 2* for different masses. First we had one aluminum block at a time, then both aluminum blocks. Next we measured the period of one block of Iron then the other, then both. We then measured the period for a combination of one iron block and one aluminum block.

Day 2

Objective: Measure the effect of mass on period.

Materials:

· Outboard aluminum plate

· Two magnets attached to the rail

· Motion sensor

Prediction: Assuming that the car begins with an initial velocity v_0, we assume that the damping force is proportional to the cart's velocity. So

$$m\,\frac{d\mathbf{v}}{dt} \propto -\mathbf{v}$$

or

$$\frac{dv}{dt} = -\beta v$$

for some constant damping coefficient β. Integrating this differential equation, we get

$$\begin{aligned}
\frac{dv}{dt} &= g\sin\theta - \frac{b}{m}v \\
\frac{dv}{dt} &= -\frac{b}{m}\left(v - \frac{mg\sin\theta}{b}\right) \\
\frac{dv}{\left(v - \frac{mg\sin\theta}{b}\right)} &= -\frac{b}{m}dt \\
\int_{v_0}^{v} \frac{dv'}{\left(v' - \frac{mg\sin\theta}{b}\right)} &= \int_0^t -\frac{b}{m}dt' \\
\ln\left(v - \frac{mg\sin\theta}{b}\right)\bigg|_{v_0}^{v} &= -\frac{b}{m}t \\
\ln\left(\frac{v - \frac{mg\sin\theta}{b}}{v_0 - \frac{mg\sin\theta}{b}}\right) &= -\frac{b}{m}t \\
\frac{v - \frac{mg\sin\theta}{b}}{v_0 - \frac{mg\sin\theta}{b}} &= e^{-\frac{b}{m}t} \\
v - \frac{mg\sin\theta}{b} &= e^{-\frac{b}{m}t}\left(v_0 - \frac{mg\sin\theta}{b}\right) \\
v &= v_0 e^{-\frac{b}{m}t} - \frac{mg\sin\theta}{b}\left(1 - e^{-\frac{b}{m}t}\right)
\end{aligned}$$

Procedures:
Follow procedures from **Day 1**, except this time, attach an outboard to the cart.