5. **Conclusion** — Has the author accomplished the investigation described in the introduction? Are limitations with the experiment discussed, along with possible extensions or lines of follow-up research?

TBD

6. **Abstract** — Does the abstract concisely summarize the whole paper? Is it as quantitative as possible?

Make sure you talk about the equation you got - exponential decay / envelope

Could use more description

7. **Mechanics** — Is the prose easy to read; does it follow logically? Are terms defined adequately? Are citations used correctly? Are there spelling, punctuation, or usage errors? Is there a pattern to these errors? Have variables been italicized, units typeset properly, etc.

Citations! [see paper]

8. **Strengths** — What are at least two things you think are particularly strong in the paper?

Interesting project

Theory is well-described

9. **To Work On** — What are at least two **specific** suggestions for changes you'd like to see in a revision? Please focus on substance over mechanics.

Mathematica

Finish the analysis

Overall - diction - more professional sometimes
humor is good, but sometimes distracting

Technical Report Peer Review

Author: Alex Rich **Reviewed by:** Mo Zhao

1. **Introduction** — Does the introduction provide sufficient context for the experiment? Does it answer the "so what?" question? Does it motivate you to read further? Can you tell by the end of the introduction what question the experiment seeks to answer?

Should say two pendulums, both swinging parallel to cart's direction of movement

Brings up synch of metronomes on movable plank - good, but elaborate further

~~[illegible]~~ could be a little clearer. Add a sentence or 2 at end.

2. **Theory** — The theory section should relate the relevant theory. It need not (indeed should not) show all the algebraic steps, but any derivations should be set up well enough that someone competent in algebra could make it to the final reported result. Does the paper concisely describe the relevant theory? Is the geometry clear? Have all symbols been defined? Have any simplifying assumptions been stated and justified?

So far, mechanical derivation looks good

frictionless cart?

affects results?

3. **Experimental Methods** — Is there enough detail to permit an interested reader with access to the appropriate equipment to reproduce the experiment? Are any subtleties of the apparatus or data taking noted? Does it read too much like a recipe? (The author shouldn't issue commands to the reader, but should describe what was done.)

maybe? Think about this a little more?

[see paper]

4. **Results / Discussion** — Are the important data presented in one or more figures with appropriate captions, before the results are "spun" with an interpretation? Have they been carefully analyzed? Has the author claimed that something follows from the data without thoroughly justifying that claim? Are the results as quantitative as the data allow? Is it clear how uncertainties were estimated and what limits the precision of the results?

Figure 3 - talk about fit first, then ~~[illegible]~~ talk about steady-state

Analysis - TBA

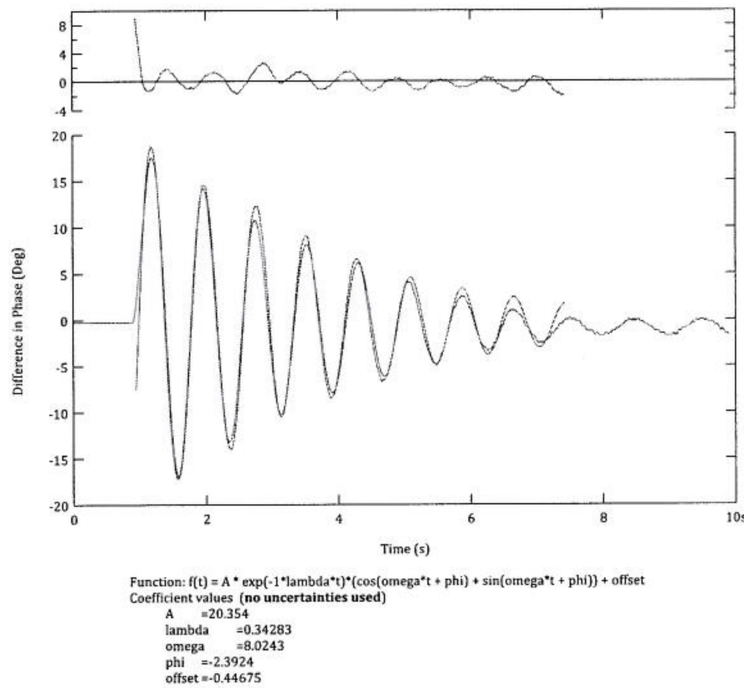

Figure 3: The difference in angular displacement. As steady state is reached, the pendulums are swinging exactly opposite (the way that ϕ and θ are defined say that when they are opposite the difference is zero. The fit is fairly close. The equation is $Ae^{-\lambda t}[\cos(\omega t + \phi) + \sin(\omega t + \phi)] + d$. Looking at the residuals, it seems that the fit is fairly good. However, because the residuals follow some sort of function, it seems that we could do better.

Why this fit? discuss in Results.

5 Conclusion

Also show fit values for $A, \lambda, \omega, \phi, d$

The mass of the cart seemed to influence the magnitude to witch the pendulums could pull it. For example, if the cart was very heavy, a pendulum would not be able to pull it one way for it to pull the other one. In the future, the experiment could be done with a lighter cart or heavier pendulum. Interestingly, although the pendulums were expected to finally reach synchronization, ~~they in fact somehow~~ reached exactly the opposite, being out of phase.

Just wondering, what would happen if pendulum weight were different?

Wait, what? Figure 3 points otherwise....

Initially, $\phi = 0$ (pendulum 2 was at rest) and θ was given a small displacement ($\theta_0 \approx \pi/8$). The two pendulums swung, and the recording stopped once the friction in the cart forced it to stop moving. At this point, the pendulums were swinging as if the pivot was stationary. Fig. 2 shows a sample run.

4 Results

Without results from Mathematica, it is impossible to compare these results to theory, however it kind of makes sense. This section will develop more later.

wording

Yes

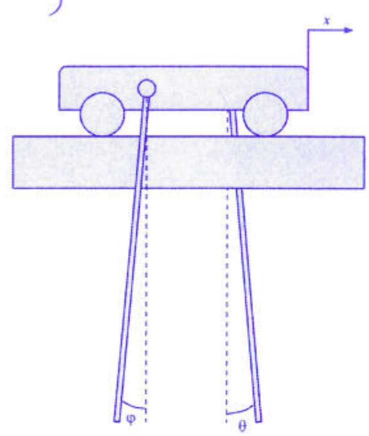

Figure 1: Apparatus to measure the relationship between the angular displacement of pendulum two (ϕ) and the angular displacement of pendulum one (θ) simultaneously. A rotary motion sensor was placed at the hinge of the pendulum to achieve this. The displacements were measured in opposite directions because, in the experiment, it became apparent that the two realized steady state where $\theta = \phi$.

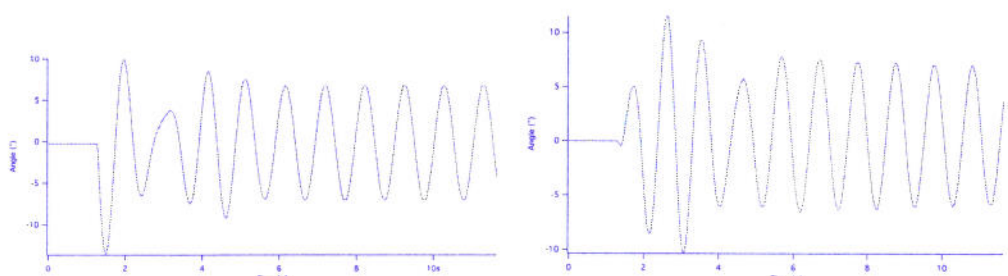

Figure 2: The left graph is pendulum 1 as a function of time, and the right is pendulum two. Note that at $t = 6$ s, both pendulums lapse into a more predictable swing.

where m is the mass of a pendulum (For simplicity, the pendulums were the same weight), ℓ is the length of each, and θ and ϕ are the angular displacements from the vertical of each pendulum.

The kinetic energy is slightly more complicated, since the pendulums have both vertical and horizontal components. The kinetic energy is

$$T(t) = \frac{1}{2}\left(m_c\dot{x}^2 + m\left[(\dot{x} + \ell\dot{\theta}\cos\theta)^2 + (\ell\dot{\theta}\sin\theta)^2 + (\dot{x} + \ell\dot{\phi}\cos\phi)^2 + (\ell\dot{\phi}\sin\phi)^2\right]\right) \tag{4}$$

where m_c is the mass of the cart and x is the position of the cart.

Using Euler's equations, we can find a function for q by solving the system

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} \tag{5}$$

Since we want to find an equation for ϕ, the angular displacement of the pendulum at rest initially, we let $q = \phi$. We find that

$$\frac{\partial L}{\partial \phi} = -m\ell(\dot{x}\sin\phi\dot{\phi} + g\sin\phi) \tag{6}$$

and

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} = 2\ell m\dot{\phi} + \ell^2\dot{\phi} + \ell^2 m\ddot{\phi} + m\dot{x}\cos\phi + \ell\dot{x}\cos\phi + \ell m\ddot{x}\cos\phi - \ell m\sin\phi\dot{x}\dot{\phi} \tag{7}$$

however, when we equate Eq. (6) and Eq. (7) in order to find ϕ, we have the function x and its derivatives. ~~So~~ then we need another equation.

We have the equation for x:

$$0 = m\left[m_c\ddot{x} + m(2\ddot{x} + \ell\cos\theta\ddot{\theta} + \ell\sin\theta\dot{\theta}^2 + \ell\cos\phi\ddot{\phi} + \ell\sin\phi\dot{\phi}^2)\right] \tag{8}$$

But this also depends on ϕ and θ. Notice that, since θ relates to a pendulum in the same way that ϕ relates, the equations for ϕ will be the same for θ, with all the ϕ's replaced with θ. Using mathematica, we find that the solution to these equations, with appropriate initial conditions, are

Mathematica hates me, but these equations will get here eventually. (9)

3 Experiment

The apparatus for the experiment is shown in Fig. 1. It was not feasible to measure the angular displacement of both pendulums in addition to the position of the cart. A rotary motion sensor was placed on the pivot point of each pendulum. A mass (not shown in the figure) was also placed at the bottom of each rod. These masses (m) were equivalent for simplicity, the mass of the cart (m_c), however was different.

somewhere mention cart on rail

Also say used ultrasonic sensor for position of cart, if you took data for this --

Interactions Between Two Non-Stationary Pendulums

Alexander Rich
Harvey Mudd College
23 November 2013

Abstract

Should two pendulums on a frictionless cart synchronize? It seems that if they are pulling the cart one way, the other one would like to do the same. This experiment finds that two pendulums will reach a steady state in which the pendulums are exactly out of phase. The results hopefully will match with the theory, but currently the theory hasn't been completely explained.

If I read this first, I would have no idea what you are testing... are the pendulums swinging in the same direction?

1 Introduction

This experiment sought to uncover the perplexing nature of two pendulums on a cart. Inspired by the synchronization of metronomes on a movable plank, it seemed reasonable to test out how and why the metronomes ~~synched up~~. It makes sense intuitively, ~~if~~ one considers the forces exerted by each metronome ~~on the plank~~. In this experiment, two pendulums were placed on a "frictionless" cart and one of them was given an initial angular displacement. What happened next was Science!

why is this an interesting question?

I think using "in phase" would be good here for wording

2 Theory

Intro to theory is too abrupt. Write something like "I will now try to derive a theoretical model..."

The simple pendulum is a well-understood system. It's behavior is sinusoidal, and if damped it will eventually stop swinging. The equation of motion can be written using $\sum \tau = I\ddot{\theta}$, where τ is the torque and I is the moment of inertia,

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0 \tag{1}$$

where g is the acceleration due to gravity and l is the length of the pendulum.

However, this assumes small angles and no unforced behavior. By putting the pendulum on a cart, there is now a forcing function on each function, the torque axis is not constant, so the method used above is not available. Instead, we can use the Lagrangian L:

$$L = T - u \tag{2}$$

where T is the kinetic energy and u the potential. For the double pendulum and cart system, we see that the potential energy is simply gravitational and held by the pendulums:

$$u(t) = \ell mg(2 - \cos\theta - \cos\phi) \tag{3}$$