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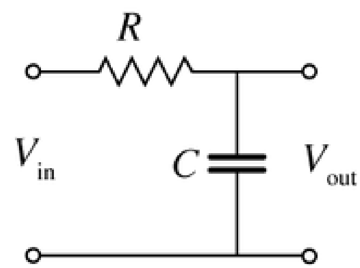
What Makes Things Tick?
Professor Saeta

Homework 3

At time $t = 0$ the potential difference V_{in} switches suddenly from 0 to V_0. Calculate V_{out} as a function of time for $t > 0$ and sketch $V_{\text{out}}(t)$.



Solution: The voltage out is equal to the voltage over the capacitor. We know that the voltage over the capacitor is $\frac{Q}{C}$ where Q is the charge on the capacitor and C is the capacitance of the capacitor. We start with

$$\begin{aligned}\frac{dV_C}{dt} &= \frac{1}{C}\frac{dQ}{dt} \\ &= \frac{I}{C}\end{aligned}$$

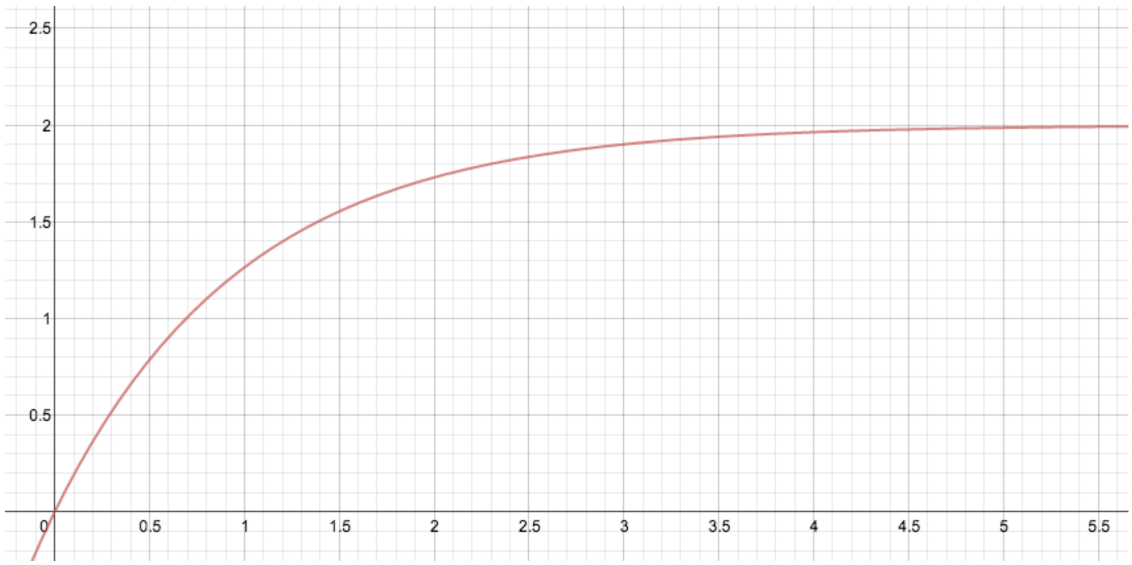
We know that $I = \frac{V_{in} - V_C}{R}$, so

$$\begin{aligned}\frac{dV_C}{dt} &= \frac{V_{in} - V_C}{RC} \\ \frac{dV_C}{V_{in} - V_C} &= \frac{dt}{RC} \\ \int \frac{dV_C}{V_{in} - V_C} &= \int \frac{dt}{RC} \\ -\ln(V_{in} - V_C) &= \frac{t}{RC} + C \\ V_{in} - V_C &= Ae^{-t/RC} \\ V_C &= V_{in} - Ae^{-t/RC}\end{aligned}$$

We know that initially, V_C is zero, so our final equation for V_{out} is (since $V_C = V_{out}$ and $V_{in} = V_0$):

$$V_{out} = V_0\left(1 - e^{-t/RC}\right)$$

A graph of this function yields the following result.



Notice that the voltage starts at zero, then exponentially approaches V_{in}, which was 2 V in this case. ■