Homework Assignment 2, *What Makes Things Tick?*

Problem 1 You make five measurements for the time of flight of a projectile when launched at 37° from the horizontal, finding **0.366, 0.343, 0.367, 0.359, 0.368** (all values in seconds).

(a) What value and uncertainty should you report for the time of flight t?

(b) What is the **precision** of the time value of the previous problem?

(c) The projectile was launched at some unknown velocity v_0 and the time of flight covered a trajectory with zero net vertical displacement. Consequently, the time of flight is given by the expression

$$t = \frac{2v_0 \sin\theta}{g}$$

If the uncertainty in the launch angle is 0.5°, find the value and uncertainty of the launch velocity v_0. Be sure to round properly at the end of the calculation. You may take $g = 9.796\ \mathrm{m/s^2}$ with negligible uncertainty.

Solution: (a) The value should be the mean and the uncertainty the standard deviation of the mean (also called the *standard error*). These are

$$t \pm \delta t = (0.361 \pm 0.005)\ \mathrm{s}$$

(b) The precision is the relative resolution of the answer:

$$\frac{\delta t}{t} = \frac{0.0047\ \mathrm{s}}{0.3606\ \mathrm{s}} = 1.3\%$$

(c) Solving for v_0, we have

$$v_0 = \left(\frac{g}{2}\right)\frac{t}{\sin\theta}$$

The value of $g/2$ is essentially without error, so we can focus our attention on the uncertainties arising from δt and $\delta\theta$. There is no reason a priori for these errors to be correlated. That is, we have no reason to suspect that a small mistake in the measurement of θ means we will also make a small mistake in measuring t with the same sign (or opposite sign) as the mistake in θ. Therefore, we add these errors in quadrature. Let $s \equiv \sin\theta$. Then

$$\frac{\delta v_0}{v_0} = \sqrt{\left(\frac{\delta t}{t}\right)^2 + \left(-\frac{\delta s}{s}\right)^2}$$

by the formula for generalized products on the cheat sheet. Since

$$\delta s = \frac{ds}{d\theta}\delta\theta = \cos\theta\,\delta\theta$$

we have

$$\frac{\delta v_0}{v_0} = \sqrt{\left(\frac{\delta t}{t}\right)^2 + \left(\frac{\delta\theta}{\tan\theta}\right)^2} = \sqrt{\left(\frac{0.0047}{0.3606}\right)^2 + \left(\frac{(0.5^\circ)(\frac{\pi}{180^\circ})}{\tan 37^\circ}\right)^2}$$
$$= \sqrt{(1.3\%)^2 + (1.2\%)^2} = 1.7\%$$

Since

$$v_0 = \frac{9.796\text{ m/s}^2}{2}\frac{0.3606\text{ s}}{0.6018} = 2.9348\text{ m/s}$$

we should quote the final value and uncertainty as

$$\boxed{v_0 = (2.93 \pm 0.05)\text{ m/s} \quad (1.7\%)}$$

A number of students have asked me why one must use radians for $\delta\theta$. Perhaps the easiest way to see this is to return to the definition of the derivative:

$$\frac{d\sin\theta}{d\theta} = \lim_{h\to 0}\frac{\sin(\theta+h) - \sin\theta}{h} = \lim_{h\to 0}\frac{\sin\theta\cos h + \cos\theta\sin h - \sin\theta}{h}$$

For small h, $\cos h$ strays negligibly from 1, so the first and third terms in the numerator wipe each other out. The remaining term, therefore is

$$\lim_{h\to 0}\cos\theta\frac{\sin h}{h}$$

If we use radian measure for h, then this limit is $\cos\theta$. If we choose to use degrees for h, then the numerator is unchanged, but the denominator has been multiplied $180^\circ/\pi$, so the value of the derivative is really $\pi\cos\theta/180 \approx 0.0175\cos\theta$.