

Rutherford Scattering

By 1911 general agreement existed that atoms contain a small number of electrons with most of the atomic mass associated with positive charge. The problem was to determine how the positive charge and mass are distributed. Two extreme views were proposed by J. J. Thomson and Ernest Rutherford. Thomson considered the atom to be made of a space filling sphere of positive charge in which the electrons were embedded—the “plum pudding” model. Rutherford considered the positive charge and mass to be contained within a central, very dense nucleus—the “nuclear atom” model.

The test of these views was suggested by Rutherford and carried out by H. Geiger and E. Marsden in 1913. The experiment is the prototype for a great many contemporary “particle experiments” of the so-called “scattering” type. Experiments by Hofstadter, *et al.*, on the special distribution of charge within the nucleus itself are of this type. The experimental procedure is to send known particles (known mass, charge, etc.) with a given momentum into a thin target of the material under investigation and to observe the scattering (the change of momentum) of the emergent beam. Given any model of the target such that the forces arising between the particle and the target are known, the expected scattering can be calculated. The observed scattering then serves to eliminate those models for which the predictions disagree with experiment. Rutherford’s particles were alpha particles of relatively low energy arising in natural radioactive decay. Since only electromagnetic forces are significant in this case, the experiments served to eliminate models of the positive charge distribution in an atom. The plum pudding model was definitely crossed off. The nuclear atom model, on the other hand, predicted results in very good agreement with the data.

1. The Rutherford model with which the results of this experiment are compared is that of a positive charge distribution which is represented as a point charge of magnitude Ze , where Z is the atomic number of the target material. The mass distribution was considered to be the same as that of the charge or, at any rate, the center of mass was assumed rigidly attached to the point charge. The predicted angular distribution of particles of mass m and charge $Z'e$ scattered from an incident beam of particles with velocity v by atoms of atomic number Z and mass M initially at rest is

$$\sigma(\theta) = \left(\frac{ZZ'e^2}{8\pi\epsilon_0\mu v_0^2} \right)^2 \frac{1}{\sin^4(\theta/2)} \quad \mu = \frac{mM}{m+M} \quad (1.1)$$

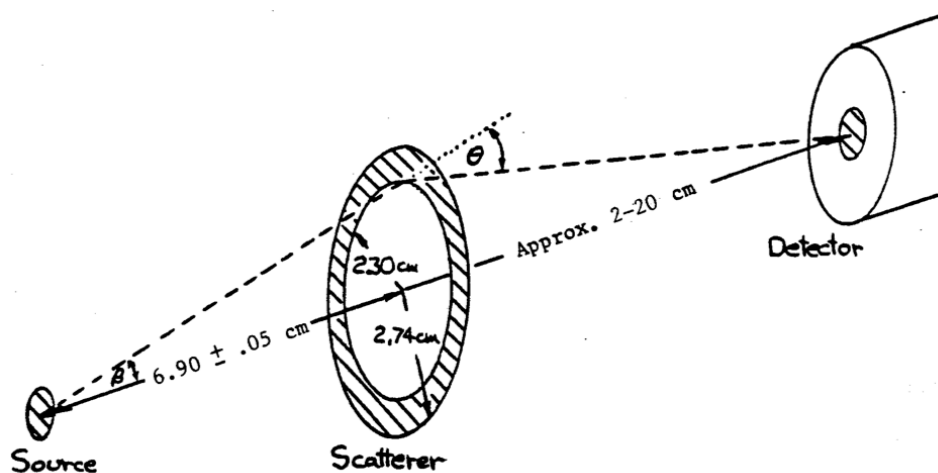


Figure 1.1: Scattering geometry.

The “cross section,” $\sigma(\theta)$, is a measure of the probability that an incident particle will be scattered in a single collision into the angle θ to $\theta + d\theta$ measured with respect to the direction of the initial velocity.

A sketch of the relation of the source, scatterer, and detector of the alpha particles in the laboratory apparatus is shown in Fig. ???. The apparatus may be disassembled at the flanged end by removing the four knurled nuts. **First**, however, read the following description. The source and scatterer are mounted together in a movable cage such that the angle β is fixed. The source is radioactive americium ^{241}Am , which decays primarily by emitting a 5.29-MeV alpha particle.¹ The scatterer is an annulus of gold foil about $3.5\text{ }\mu\text{m}$ thick. Neither the americium source nor the gold foil may be touched, for obvious reasons.

The detector is a solid-state device consisting of a silicon wafer with a thin ($0.02\text{ }\mu\text{m}$) gold surface covering on one side and an aluminum surface on the other side. A potential difference of 30 V is placed across this “sandwich.” When an ionizing particle passes through the silicon, electrons are ejected by collision with the particle from the filled band to the empty conduction band of the silicon semiconductor. Both the electrons in the conduction band and the “holes” left in the valence band move under the applied field: the electron to the gold surface, the holes to the aluminum. Hence a pulse of charge is collected, with size proportional to the number of electrons injected into the conduction band, and thus to the energy loss of the ionizing particle in the silicon. You will count this pulse of charge with a scaler after it is amplified. Further details are given in the appendix to this experiment.



Do not touch the detector! The gold coating is fragile, the silicon can be ruined by contamination, and static electricity could damage the detector irreversibly.

The distance from the scattering foil to the detector may be varied from about 1 to 20 cm by means of the vacuum sealed plunger attached to the source cage and extending outside the

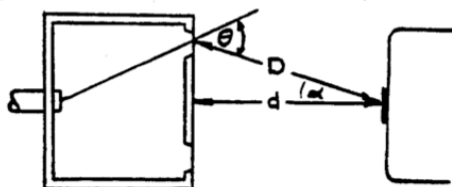
¹A thin cover over the radioactive source reduces the energy of the alpha particle somewhat.

apparatus. The scattering angle θ may thus be varied from about 27° to 90° .

Since the range of the alpha particles in air at normal pressure is only a few centimeters, it is necessary to evacuate the entire apparatus. The brass vacuum chamber is closed at one end by the sliding plunger and flange. The other end is closed by the mounting bracket of the detector seated against an O-ring seal.

Current pulses from the silicon detector generate voltage pulses in the amplifier circuit. These pulses are counted by a scaler. The experiment consists in determining the number of counts registered by the scaler in a measured time interval as the source cage plunger is moved in or out to vary the scattering angle θ .

2. Carefully study the apparatus prior to its evacuation. You will be given the minimum value of d (i.e., when the plunger is in as far as possible) for your apparatus. You will need this value together with your measurements of the external position of the plunger to compute the scattering angle θ and to correct for changes in the detector solid angle (see below). Begin collection of data with the plunger withdrawn as far as possible to measure the counting rate for the smallest scattering angle. Record the time necessary to accumulate at least 100 counts at all scattering angles.² The standard deviation for N counts is $\sqrt{N}$ so that 10% statistics are obtained with 100 counts. The counting rate at minimum scattering angle will probably be of the order of 30 counts per minute, falling to some 4 counts per minute at the largest angles.



The counting rate must be corrected for the change in the solid angle subtended by the detector at the gold annulus. The apparent size of the detector as seen from the annulus is a function of their separation, d . Ignoring the finite size of the detector and the annulus width, this correction consists of two factors. First, the detector size would vary as $1/d^2$ were it viewed “head on” from the annulus. This is very nearly the case when d is much greater than the radius of the annulus. For small separations, however, the projection of the detector into the line of sight from the annulus must be taken into account. The projected area goes as $\cos \alpha$, or as d/D . Combining these two factors, the apparent size of the detector varies as $\frac{d}{D} \frac{1}{D^2}$.

The counting rate is multiplied by the reciprocal of this factor to obtain a counting rate proportional to that which would have been measured with a detector whose size appeared always the same to the scattering annulus. The counting rate corrected for solid angle is proportional to the cross section $\sigma(\theta)$.

To compare your results with the predictions of the Rutherford model, plot the logarithm of the corrected counting rate vs. the logarithm of $\sin(\theta/2)$. (What should this plot look like

²In taking data, choose intermediate plunger positions in light of the plot you will be making. (See below)

according to the Rutherford model?) Enter your data in this plot with bars to indicate the standard deviations resulting from counting statistics.

1. The EGG Ortec Silicon Charged Particle Detector

Silicon is a semiconductor with a gap of 1.1 electron volts between the top of the filled band and the bottom of the (nearly empty) conduction band. At any temperature above absolute zero, some electrons will have enough thermal energy to reach the conduction band; for the detector you use, with 30 volts potential difference across the silicon wafer, this gives rise to a “dark current” of about 300 nA or 1×10^{12} electrons/second. (Incidentally, since the silicon wafer is about $150 \mu\text{m}$ thick, the electric field is $30 \text{ V} / 1.5 \times 10^{-4} \text{ m} = 200,000 \text{ V/m}$.)

When there is no voltage across the silicon wafer, the Fermi energies (see Eisberg and Resnick, Chapter 13, pp. 507 *et seq.*) of the electrons in the gold, aluminum, and silicon are equal; electrons move between these layers to change the potential of these layers until this equality is reached. The particular silicon wafer we use has donor impurities (see E & R, p. 507), so the Fermi energy in the silicon lies 0.16 eV below the bottom of the conduction band. There are, accordingly, thermally injected electrons in the conduction band. Once the 30 V power supply is turned on, these electrons are swept away, giving rise to the “dark current.”

When an α particle enters the silicon, it collides with electrons in the silicon lattice, giving many of them enough energy to reach the conduction band. The average energy lost by the α -particle to create an electron-hole pair is measured to be 3.6 eV. Thus a 5 MeV α -particle, completely stopped in the silicon, gives rise to $5 \times 10^6 / 3.6 = 1.4 \times 10^6$ electron-hole pairs, or 2.2×10^{-13} coulombs. The capacitance of the detector is 70 picofarads ($7 \times 10^{-11} \text{ F}$), so collecting this charge causes a voltage change $\Delta V = (2 \times 10^{-13} \text{ C}) / (7 \times 10^{-11} \text{ F}) = 3 \text{ mV}$. The detector voltage is supplied through a 20-M Ω resistor, so the recovery time is $RC = (7 \times 10^{-11} \text{ F}) 2 \times 10^7 \Omega = 1.4 \text{ ms}$.

Figure ?? shows a circuit diagram of the detector, its power supply, and the first (preamplifier) stage of amplification. Here R_L is the “equivalent resistance” of the silicon wafer; since the “dark current” is about $3 \times 10^{-7} \text{ A}$ for a potential of 30 V, $R_L = 100 \text{ M}\Omega$. The Model 109A preamplifier set at $10\times$ gain gives a pulse of 150 mV/MeV for a Si detector. This preamp also reduces the pulse width to approximately $50 \mu\text{s}$. The amplifier following the preamp further reduces the pulse width

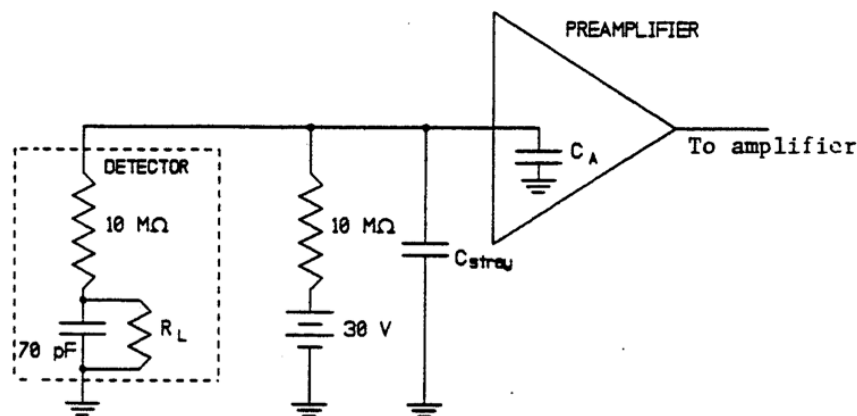


Figure 1.2: Circuit

and increases the peak voltage.

2. Addendum

Since the laboratory notes were written, the brass cylinder has been replaced by a plastic (Lexan) cylinder, which is semitransparent and slightly shorter. Accordingly, the angle of scattering of the alpha particles must be calculated with the new dimensions. The dimension you need to know is d Page 3-3 of the notes. d the distance from the plane of the gold scattering foil to the surface of the detector. It is also the distance between the two knurled knobs, one the handle on the plunger and the other the one the rod slides through, less 0.08 cm. We have placed a sleeve of length 2.00 cm on the plunger rod (to keep a bump on the can carrying the americium source and the foil from striking the detector), and therefore the shortest d available to you is 1.92 cm.

Why the Lexan cylinder, and these changes?

For reasons I did not understand, this experiment usually produced an exponent in the range of -4.3 to -4.5 instead of the -4 which Professor Rutherford had in mind. Mark Chalice, '94, asked me two years ago if any of the alpha particles which go through the foil undeflected might then strike the brass cylinder wall and be scattered there. Indeed, most of the alpha particles that strike the foil do go through essentially undeflected, having lost some energy by many collisions with electrons, thus ionizing gold atoms. These alpha particles enter the brass. Most spend out their range losing energy in more electron collisions, but a few of them may indeed be scattered by the copper and zinc nuclei of atoms which make up brass. From the cross-section equation on Page 3-1 of the notes, we see that the scattering cross section depends on Z^2 . For gold, $Z = 79$; for copper, $Z = 29$, and for zinc, $Z = 30$. Accordingly, these brass nuclei are only about 14% as effective as gold in Rutherford scattering, but the path length in the brass can be considerable. From the same equation on Page 3-1, we also learn that as the alpha particle slows down, the probability of scattering increases. Thus the cylinder walls in front of the gold foil may constitute a significant second scatterer. The angle of scattering at which the alpha particle is detected is greater for these brass-scattered alphas, and hence they are no longer much detected as the foil nears the detector. Accordingly, we are led to believe that the power law is greater than 4.

The solution to the problem is not to use brass, but a plastic, for which the atoms in the wall are predominately carbon, hydrogen and oxygen. These have at most 1% of the scattering cross section of gold. Essentially all the alpha particles striking the wall lose their kinetic energy through electron collisions and are not scattered.

J. B. Platt, January 1994