

Physics 54
Modern Physics Laboratory

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1. Schedule

You will conduct four two-week experiments and extend one of the experiments for your technical report. The experiments will be done in pairs and in a rotation to be established after the orientation meeting in the first week, in which you will be introduced to each experiment.

Currently, the following experiments are working and available:

1. the Cavendish experiment
2. barrier penetration
3. the Hall effect + the photoelectric effect (5)
4. thermal radiation
5. the photoelectric effect (paired with the Hall effect)
6. Rutherford scattering
7. gamma radiation interactions
8. chaotic motion

The Franck-Hertz experiment (Expt. 9) may replace the photoelectric experiment at some point during the semester, if the latter gets sick.

Week	Section 3 (Tues.)	Section 1 (Thurs.)	Section 2 (Fri.)
January 17–20	<i>Orientation</i>	<i>Orientation</i>	<i>Orientation</i>
January 24–27	1A	1A	1A
January 31–February 3	1B	1B	1B
February 7–10	2A	2A	2A
February 14–17	2B	2B	2B
February 21–24	3A	3A	3A
February 28–March 2	3B	3B	3B
March 7–10	4A	4A	4A
March 13–16	<i>Spring Break</i>	<i>Spring Break</i>	<i>Spring Break</i>
March 20–23	4B	4B	4B
March 27–30	Tech Report Data	Tech Report Data	<i>Cesar Chavez</i>
April 3–6	Tech Report Work	Tech Report Work	Tech Report Data
April 10–13	Tech Report Due	Tech Report Due	Tech Report Work
April 17–20	<i>Free</i>	<i>Free</i>	Tech Report Due
April 24–27	Revised TR Due	Revised TR Due	Revised TR Due

2. General Instructions

In preparing for the first laboratory meeting for each experiment, you should carefully read the instructions for the scheduled experiment before reporting to the laboratory meeting. In many cases, you will also find it useful to do some background reading on the subject, in your Physics 52 textbook, in various sources found in the library, or on the internet. (Beware of material on the internet. Some of it is very good, but there is no guarantee that the information is relevant, useful, or correct!) You are encouraged to take notes on your reading and attach these notes in your lab book. Be sure to include references to your sources. While you may find some details of the instructions will have been changed in the lab (as we are constantly tweaking the equipment), complete familiarity with the objectives and general procedures of the experiment before the laboratory period will help you in working efficiently in lab.

Please observe the precautions emphasized in the laboratory instructions and appendices and accord the research-type equipment the respect it deserves. Note that much of the equipment in this laboratory is one-of-a-kind, delicate, difficult to repair, and expensive! Much of it can be damaged if used incorrectly. If you have any questions about how to use the equipment, be sure to ask the instructor before turning it on or starting a new procedure. Report any damaged equipment to your instructor immediately.

Laboratory Notebook

Unless you keep careful records of what you do in the laboratory, your work is essentially useless. Almost all experiments require days, weeks, or years to set up and carry out. There's just no way to remember everything you need. Indeed, it is difficult to underestimate your ability to remember critical experimental details, or when you released a mistake and tweaked your procedure, or how you estimated the uncertainty in a measurement, ...the list is virtually endless. One missing value can undermine four hours of work or more.¹ Your records should be complete enough that you could return to the notebook after a year's hiatus and know just what went on and how you analyzed the data you took.

As regards lab notebooks, students in Profs. Eckert and Lynn's sections should use a traditional notebook: the brown-cover spiral Ampad #22-157, for instance. Students in Prof. Saeta's section have a choice. If you prefer to take notes on your computer, you may do so. In my experience, neither a paper notebook nor a computer notebook is entirely satisfactory. Each has its advantages and disadvantages. If you decide to go all-electronic, I suspect you will nevertheless have to scan in a diagram or an algebraic derivation from time to time. We'll work out a roughly convenient (or at least not hopelessly inconvenient) way to manage this, probably by putting a scanner in the lab. Note, however, that I (Prof. Saeta) will cut you no slack for missing sketches and other work that is easier to do by hand if you decide to go the electronic route.

¹I'm being excessively optimistic here. Most scientists have lost weeks if not months of work at some point because of poor record keeping. Do your best not to emulate them!

1. You will conduct the experiment in pairs. Each person needs to keep complete records. Can partners share a spreadsheet of data? Sure. But not a spreadsheet or Igor experiment file with all the analysis. That should be done individually. You may then compare results with your partner, track down the source of any discrepancies, and determine how to make any necessary corrections.
2. A notebook (paper or electronic) is not supposed to be perfect and ready for publication. It is a working document, with entries dated and made in chronological order, recording your thought process as you learn about the apparatus and system under study. Blemishes are par for the course. Don't work out a calculation on a piece of scratch paper, in case you do it wrong; that's *exactly* what the notebook is for.
3. Plot your data as it comes in and save often. Do not lapse into "data-taking mode," in which you mindlessly wait for the next data point to enter into a spreadsheet, figuring you will analyze later. Use your time in lab wisely and productively. Plotting and analyzing "while you go" will help you find silly mistakes and make sure that you understand what is going on.
4. There are several computers in the laboratory for your convenience, but if you have a laptop, you will find it more convenient than any of the computers except the one used for the chaotic motion and gamma radiation interactions experiments. Please do not tamper with the lab computers or alter their programs or operating systems.
5. Define algebraic symbols for relevant quantities early in your notebook entry for an experiment, then use these symbols consistently. Often the best place to define symbols is on a sketch, which is usually better than a photograph or a false three-dimensional representation. Sometimes, however, a photograph is just the thing. There is a camera in the lab that you may use.
6. Record qualitative observations as well as numbers and diagrams.
7. Enter computer-generated graphs as soon as they are made. Annotate them. A graph does not stand alone; it needs to be explicated in the text.

Summary of the Experiment At the end of an experiment you should type a summary of one to two pages containing a concise discussion of important points of the experiment, including the purpose, theoretical predictions you are testing (if appropriate), a brief outline of experimental methods, results, analysis, and conclusions. There is no need to put in detailed procedural discussions unless they bear directly on understanding some aspect of the data. In discussing results, you can make references (with page numbers) to specific entries—such as tables, figures, and calculations—in your lab notebook. Be sure to refer to or include in your summary well-documented graphs of important findings. Discuss the major sources of random and systematic errors, including possible methods

of reducing them. If you have a hunch about the source of a discrepancy, make some order of magnitude estimates, make some approximations, and check quantitatively to see if hunch could reasonably explain the discrepancy. If relevant, compare your results with theoretical predictions. Your results should be as quantitative and precise as possible. Your conclusions should also be as specific as possible, given your experimental results and analysis. This summary should be attached to the end of your lab notebook write-up; it is a significant part of the record of your experiment.

Honor System All work that is handed in for credit in this course, including laboratory reports, is regulated by the Harvey Mudd Honor Code, which is described in general terms in the student handbook. In application, this Code means simply that all work submitted for credit shall be your own. You should not hesitate to consult texts, the instructor, or other students for general aid in the preparation of laboratory reports. However, you must not transcribe another student's work without direct credit to him or her, and you must give proper credit for any substantial aid from outside your partnership. Again, remember that while you may discuss the experiment with your lab partner, your analysis of the experiment should be done individually.

The Cavendish Experiment



Do not attempt to align the torsion balance. If you have concerns about its alignment, ask your instructor.

1. Background

Isaac Newton's (1642–1727) theory of gravitation explained the motion of terrestrial objects and celestial bodies by positing a mutual attraction between all pairs of massive objects proportional to the product of the two masses and inversely proportional to the square of the distance between them. In modern notation, the law of universal gravitation is expressed

$$\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}} \quad (1.1)$$

where M and m are the masses of the two objects, r the distance separating them, and G is the universal constant of gravitation. Newton was not particularly concerned to evaluate the constant of proportionality, G , for two reasons. First, a consistent unit of mass was not in widespread use at the time. Second, he judged that since the gravitational attraction was so weak between any pair of objects whose mass he could sensibly measure, being so overwhelmed by the attraction each feels toward the center of the Earth, any measurement of G was impractical.

Notwithstanding Newton's pessimism, towards the latter half of the 18th century several scientists attempted to weigh the Earth by observing the gravitational force on a test mass from a nearby large mountain. These efforts were hampered, however, by very imperfect knowledge of the composition and average density of the rock composing the mountain. Spurred by his interest in the structure and composition of the interior of the Earth, Henry Cavendish in a 1783 letter to his friend, Rev. John Michell (1724–1793), discussed the possibility of devising

an experiment to “weigh the Earth.” Borrowing an idea from the French scientist Charles-Augustin de Coulomb (1736–1806), who had investigated the electrical force between small charged metal spheres, Michell suggested using a torsion balance to detect the tiny gravitational attraction between metal spheres and set about constructing an appropriate apparatus. He died, however, before conducting experiments with the apparatus.

The apparatus eventually made its way to Cavendish’s home/laboratory, where he rebuilt most of it. His balance was constructed from a 6-foot wooden rod suspended by a metal fiber, with 2-inch-diameter lead spheres mounted on each end of the rod. These were attracted to 350-pound lead spheres brought close to the enclosure housing the rod, roughly as illustrated in Fig. 1.1. He began his experiments to “weigh the world” in 1797 at the age of 67, and published his result in 1798 that the average density of the Earth is 5.48 times that of water. His work was done with such care that this value was not improved upon for over a century. The modern value for the mean density of the Earth is 5.52 times the density of water. Cavendish’s extraordinary attention to detail and to the quantification of the errors in this experiment has led C.W.F. Everitt to describe this experiment as the first modern physics experiment. In this experiment you will use a torsional balance similar to Cavendish’s to “weigh the Earth” by determining a value for G .¹

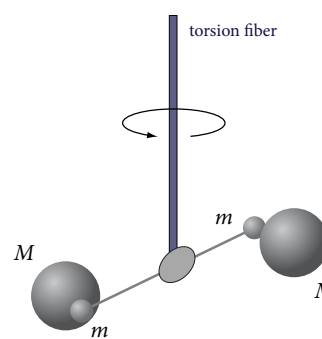


Figure 1.1: The Cavendish torsional balance.

2. Theory

Recent generations of students seem tempted to rush to the Internet for help in deciphering the mechanics of the torsion oscillator and the Cavendish experiment, rather than thinking for themselves. We trust that you are quite able to ask Google and Wikipedia for answers to a great variety of questions. However, please avoid the temptation with this experiment. Do not look outside this manual for theoretical assistance. If you cannot remember $\mathbf{F} = m\mathbf{a}$ or $\boldsymbol{\tau} = I\boldsymbol{\alpha}$, then either review your frosh physics notes or reread this sentence.

The Cavendish torsional balance is illustrated in Fig. 1.1. Two small metal balls of mass m are attached to opposite ends of a light, rigid, horizontal rod, which is suspended from a torsion fiber. When the “dumb-bell” formed by the rod and masses is twisted away from its equilibrium position (angle), the

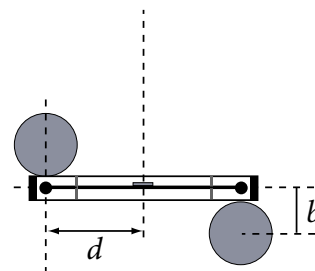


Figure 1.2: Top view.

¹In 1998 the accepted value for G was known to only 0.15% precision, a surprisingly crude number, reflecting the miniscule forces involved. In May 2000 in an experimental *tour de force*, Jens Gundlach and Stephen Merkowitz of the University of Washington improved on that precision by a factor of 100, finding a value of $(6.67390 \pm 0.00009) \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

fiber generates a restoring torque proportional to the angle of twist, $\tau = -\kappa\theta$. In the absence of damping, the dumbbell executes an oscillatory motion whose period is given by $T = 2\pi\sqrt{\frac{I}{\kappa}}$, where I is the rotational inertia of the dumbbell, $I = 2m\left(d^2 + \frac{2}{5}r^2\right)$. In this expression, r is the radius of the small masses m , d is the distance from the center of the rod to the center of one of the masses, and we have neglected the mass of the thin rod. Knowledge of m , d , and r , plus a careful measurement of the period of oscillation T , allows one to calibrate the torsion fiber, obtaining its torsional spring constant κ . From κ and a measurement of the twist caused by the large masses M you can deduce the gravitational force between the masses, and hence G .

2.1 Gravitational Torque

When the large metal spheres are positioned as shown in Fig. 1.2, the gravitational attraction between the large and small spheres produces a torque that rotates the dumbbell clockwise. Only the component of the force on each mass that is perpendicular to the horizontal bar produces a torque about the center of the rod. The magnitude of the torque between the two adjacent masses is given by $\tau_g = 2F_{\perp}d$, where the factor of 2 comes from the fact that the torque is equal on the two masses m . This torque displaces the equilibrium angular position of the dumbbell by an amount given by $\tau = -\kappa\theta_o$. Hence, if one can measure the equilibrium angle θ_o very carefully, one can deduce the gravitational force that produces the torque and finally G .

2.2 Light Lever

Cavendish mounted a finely ruled scale near the end of the dumbbell, which he could read with a telescope to one-hundredth of an inch. The telescope allowed him to remain outside the experimental chamber, thus eliminating air currents and his own gravitational influence on the oscillator.

We will take advantage of a light lever to magnify the dumbbell's tiny rotation into an easily observed displacement on a far screen. The light lever is produced by bouncing a laser beam off a mirror mounted to the dumbbell (see Fig. 1.3). When light bounces off a mirror, the angle the incoming beam makes with the normal (perpendicular) of the mirror is equal to the angle the outgoing beam makes with the normal. If the mirror rotates through a small angle α , the outgoing beam rotates through the angle 2α , since both the incoming and outgoing angles change by the same amount. By measuring the motion of the laser spot on a far screen, and knowing the distance between the mirror and the screen, you can determine the angle α , from which you can infer the rotation of the dumbbell, θ .

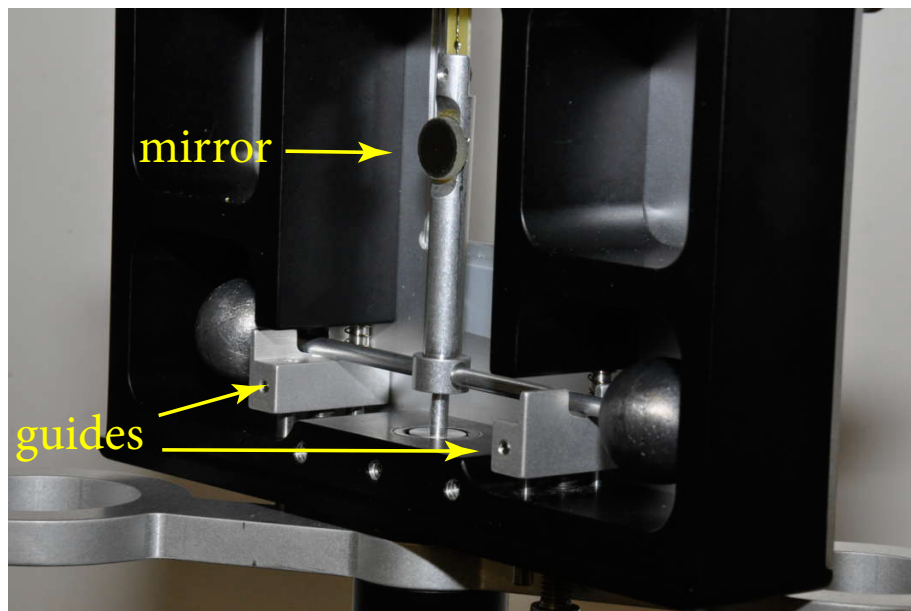


Figure 1.3: Inside the housing of the torsion balance. The guides are raised to support the dumbbell and settle the balance, then **gently** lowered to allow it to swing freely.

2.3 Damping

In the absence of damping, the motion of the dumbbell is a sinusoidal oscillation with the period given by

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \quad (1.2)$$

Viscous damping of the pendulum's motion caused by air resistance produces a drag torque proportional to the angular speed of the dumbbell. This causes the sinusoidal oscillation to decay exponentially, with a time constant τ that is long compared to the oscillation period (a healthy fraction of an hour). The equation of motion is therefore

$$I\ddot{\theta} = -\kappa\theta - b\dot{\theta} \quad (1.3)$$

for damping constant b . It is convenient to divide by I and rearrange to get

$$\ddot{\theta} + 2\beta\dot{\theta} + \omega_0^2\theta = 0 \quad (1.4)$$

where $2\beta = b/I$ and $\omega_0^2 = \kappa/I$.

You have probably encountered this differential equation before, but may not remember how to solve it. The standard approach is to guess some sort of exponential solution $e^{i\omega t}$,

substitute into the differential equation, and look for the values of ω that work. Since Eq. (1.4) is a second-order differential equation, there are two linearly independent solutions. To get a real solution, you will need to superpose the two solutions you obtain. You should end up with a damped sinusoid. By fitting to this function you can extract both ω and β , from which you can deduce the torsional spring constant κ and the value of G .

2.4 Parameters

According to the Pasco manual, the parameters of the apparatus include the following:

Variable	Value
m	$(38.3 \pm 0.2) \text{ g}$
r	9.53 mm
d	50.0 mm
b	46.5 mm

No uncertainties are given for the last three values. Since $m \propto r^3$, one could infer an uncertainty in r of 0.02 mm resulting from the uncertainty in m given above. Based on the precision of the value given for d one might reasonably assume an uncertainty of 0.1 mm in that value. The uncertainty in b depends on the accuracy of the horizontal alignment of the pendulum. You can make an estimate by observing any horizontal displacement of the spot from the mirror compared to the spot arising from reflection from the front window when the pendulum is at rest in equilibrium in the absence of the lead weights. If you feel that the apparatus is misaligned, **please ask your instructor to investigate.**

2.5 Safety



The laser pointer that forms the light lever for this experiment is a class III laser capable of damaging retinas. **Do not look directly into the beam.** Please ensure that nobody looks into the beam. Note that it is safe to look at the diffuse spot the beam produces as it reflects from an object, such as a meter stick.

3. Procedure



To measure the tiny gravitational torque, the fiber-dumbbell combination must be very sensitive. It picks up the least vibrations, so be extremely careful around the balance. Settling the dumbbell down after it has been jostled requires skill and patience; it may take an hour. Be careful!

1. The apparatus should be aligned for you, but you should confirm that the reflection from the front window and from the mirror are *approximately* on the same vertical line,

indicating that, in the absence of the lead weights, the dumbbell hangs symmetrically in its enclosure. **Do not attempt to align the pendulum yourself. Ask your instructor to investigate.**

2. If the apparatus is acceptably aligned and the balance is not rotating, it is probably wise to record the equilibrium position of the reflected laser beam.
3. Carefully weigh the large lead balls. Place a styrofoam tray on the electronic scale and tare it. Then gently place the lead ball into the tray. If the lead balls are dropped, they will become misshapen, which will severely compromise the accuracy of the experiment.
4. *Gently* place the large lead balls in the armature. The more you disturb the balance, the longer it takes to settle down so that you can acquire usable data. Rotate the armature until the balls just touch the sides of the enclosure. Gently lower the support screws until the dumbbell rotates freely. If it rotates so much that the dumbbell bounces off the sides of the enclosure, use the support screws to settle the motion.
5. Once the dumbbell oscillates freely, carefully slide the door on the enclosure closed to minimize air currents and temperature variations.
6. Measure the distance from the mirror to the laser spot on the far meterstick. Using this distance and measurements of the displacement of the spot on the meterstick, you will be able to deduce the angle of rotation of the balance and mirror. Is the small-angle approximation appropriate?

3.1 Option 1

1. Begin recording the position of the reflected laser spot as a function of time. Record at least two full periods.
2. Gently rotate the armature until the large balls once again touch the sides of the enclosure in the opposite orientation and record the position of the reflected laser spot as a function of time for at least another two periods.
3. If you have time you should finally return the armature to its original position and again record the position of the laser spot as a function of time for at least another two periods.

3.2 Option 2

1. If you have a satisfactory measurement of the equilibrium position before the lead weights were installed, you can measure how far the equilibrium shifts under the influence of the weights. That is, a single run might suffice.
2. Repeat if possible.

3.3 Option 3

1. If the balance is in an equilibrium configuration and the weights are present on the arm, reverse them smoothly and carefully.
2. From the initial angular acceleration of the balance, and a value of the torsional spring constant obtained from the period, you can determine the torque supplied by the weights.

In all cases, you can fit the appropriate damped sinusoid to your $\theta(t)$ data to determine the period and center of oscillation.

4. Further Options

1. There is a silicon position-sensing detector you may use to record the position of the laser spot as a function of time. It is a small device that puts out a voltage proportional to x position across the face of the detector. This voltage can be digitized using a LabJack or other analog-to-digital converter. You will have to calibrate the mapping between position and output voltage carefully to obtain useful data. For more information on analog-to-digital converters, see the course web site.
2. Some students have brought in small webcams and written Python scripts to analyze the images of the laser spot on the meterstick to measure the spot's position as a function of time.

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Revised 1/17/02 by Daniel C. Petersen
Revised 1/4/12 by Peter N. Saeta

Barrier Penetration

1. Introduction

A striking consequence of quantum mechanics is the prediction that a particle of total energy E located in a potential well of depth $V_0 > E$ has a finite probability of escaping if the walls of the potential well have a finite thickness. This phenomenon, known as *barrier penetration* or tunneling, is not uncommon at the atomic or subatomic scale; for example, α decay occurs via tunneling of the α particle through the Coulomb barrier of the radioactive nucleus (see Section 9.3 of Townsend). It is the basis of the scanning tunneling microscope, for which Gerd Binnig and Heinrich Rohrer were awarded the 1986 Nobel Prize. While barrier penetration is hardly commonplace on the macroscopic scale, it can be seen; in fact, **barrier penetration is a property of both classical and quantum-mechanical wave motion.**

An optical analog of barrier penetration, known as “frustrated total internal reflection,” is described formally by the same equations that describe quantum mechanical tunneling. In this phenomenon, a light beam traveling through glass (or any other transparent medium with an index of refraction $n > 1$) is incident on the glass-air interface. For sufficiently small angles of incidence, the light is partly reflected and partly transmitted into the air. But for angles of incidence greater than the “critical angle,” $\sin^{-1}(1/n)$, the beam is perfectly reflected back into the glass; no light is transmitted into the air. The oscillating electromagnetic field of the light does not stop precisely at the interface, however; it extends some distance into the air. If another piece of glass is brought close enough to the interface, this electromagnetic field can then propagate away from the interface (thus, the total internal reflection is “frustrated”). The trick is getting the second piece of glass close enough, to within about a wavelength of the interface. Unless the interface is very flat, the effect won’t occur; in any case, the gap is so small as to be invisible.

One can change the scale of electromagnetic radiation to the microwave region, where wavelengths are on the order of centimeters. Then this phenomenon can be easily observed. For radiation with wavelengths of a few centimeters, polyethylene becomes a good substitute

for glass; it is almost transparent to microwaves and has an index of refraction very similar to that of glass for optical frequencies. A microwave beam traveling through a polyethylene block and incident on the polyethylene-air interface at an angle of 45° undergoes total internal reflection, provided the interface is isolated. Again, there is an oscillating electromagnetic field extending into the air beyond the interface, as you will see. If another polyethylene block is brought close enough to the interface, it should allow a transmitted wave to propagate away from the interface. You will study this phenomenon.

In the experiment there are two 45° - 45° - 90° polyethylene prisms arranged so that the two hypotenuse faces can be brought close together. A microwave beam is incident on the first prism perpendicular to one base, travels through the prism and strikes the hypotenuse at 45° . If the perpendicular separation of the two prisms is d , then the fraction of the microwave radiation intensity that can penetrate the gap between the prisms (T) is given by

$$T = (1 + \alpha \sinh^2 \beta d)^{-1} \quad (2.1)$$

The form of this equation is identical to that seen for quantum mechanical barrier penetration (see Section 4.7 of Townsend). The coefficients α and β can be obtained from classical electromagnetic theory. For the geometry of this experiment one obtains¹

$$\alpha = \frac{(n^2 - 1)^2}{n^2(n^2 - 2)} \quad (2.2)$$

$$\beta = \frac{2\pi}{\lambda} \sqrt{(n^2 - 2)/2} \quad (2.3)$$

where n is the index of refraction of the polyethylene and λ is the wavelength in air.

2. Preliminary Measurements

To predict the intensity of the microwave radiation transmitted across the gap, you need to know the microwave wavelength and the index of refraction of polyethylene. And there is one more subtle question, namely, does the detector measure the intensity of the microwave radiation? That is, does the detector respond linearly to the square of the microwave electric field strength? You will perform some preliminary measurements to determine these three parameters. Keep your eye on the clock; you'll need to work efficiently here. Your goal should be to have *reasonable* (not "perfect") values for λ , n , and the detector response by the end of the first day. If you can also get in a preliminary measurements of $T(d)$, so much the better.

Note that there are available a number of foam pads of material that attenuates microwaves. You can use these to minimize stray reflections whenever it seems appropriate.

Transmitter and receiver The microwave transmitter operates at 10.5 GHz. The microwaves emitted from the horn are polarized parallel to the long axis of the Gunn diode (the

¹See, e.g., J. Strong, Concepts of Classical Optics (1958), Section 6-9.

slender shiny cylinder located at the base of the horn). While the transmitter can be rotated to change the polarization axis, for this experiment the Gunn diode should be kept vertical (0 on the scale). The microwave receiver consists of a detector diode mounted similarly at the base of the detector horn. The diode responds only to the component of the microwave signal that is parallel to the diode axis. There are four amplification ranges and a variable gain control on the receiver. **Always start at the least sensitive range ($30\times$) to avoid damaging the electronics.**

Wavelength A good way to measure the wavelength is to use a Fabry-Perot interferometer. Follow the procedures in the PASCO Instructions and Experiments Manual, Experiment 9, p. 26. Compare your result with the expected value for 10.5-GHz radiation.

Receiver Response You can check to what extent the meter reading on the microwave receiver is proportional to the intensity by seeing how the meter reading changes as the diode axis of the receiver is rotated relative to the polarization direction of the transmitted electric field (transmitter diode axis). If the meter responds directly to the electric field strength, then the meter reading should be proportional to $\cos \theta$, where θ is the angle of the detector diode relative to the E field. If the meter responds to intensity, then the meter reading should be proportional E^2 , hence to $\cos^2 \theta$. In fact, the detector diode is a nonlinear device, so that the way the meter responds can vary with the strength of the field. To test the meter in the relevant range, you should separate the transmitter and receiver by about 2.5 m, roughly the same distance you will be using in the barrier penetration experiment. (The goniometer arm on which the transmitter and receiver were mounted for the wavelength determination should be removed from the table to minimize spurious reflections.) Adjust the detector position to get a maximum signal when both transmitter and receiver diodes are vertical. Set the meter reading to full scale. Then rotate the receiver in 15° increments up to 90° and record the meter readings. Compare with $\cos \theta$ and $\cos^2 \theta$. The meter response should be quite close to $\cos^2 \theta$ (intensity) at this separation. If it is not, see your instructor.

Index of refraction Light (or microwave radiation) incident on a prism is refracted on en-

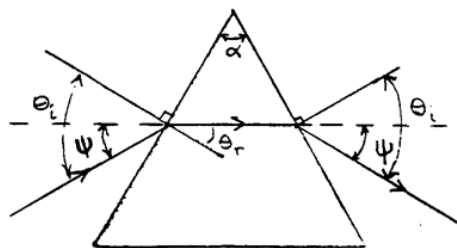


Figure 2.1: Symmetric propagation.

tering and leaving the prism. If the prism is oriented so that the angle with which the beam leaves the prism (where as usual the angle is measured relative to the normal to the surface it is exiting) is the same as the angle at which it enters (relative to the normal to the front surface)—see Fig. 2.1—then the index of refraction is given by

$$n \equiv \frac{\sin \theta_i}{\sin \theta_r} = \frac{\sin(\psi + \alpha/2)}{\sin(\alpha/2)} \quad (2.4)$$

where θ_i is the angle of incidence, θ_r is the angle of refraction, α is the apex angle of the prism, and ψ is defined as shown in Fig. 2.1.

(**Note:** As part of your writeup for this experiment, you should derive this formula.) Two rotating goniometer arms are attached to the platform supporting the fixed polyethylene prism. (You may have to roll back the second polyethylene prism to see the second goniometer arm.) The angle scales marked for each arm correspond to the angle ψ in the figure above. Position the transmitter on one arm and the receiver on the other. Rotate the arms symmetrically relative to the 45° apex angle of the prism (i.e., both angles must be the same) and locate the angle ψ where the receiver signal is a maximum. Use this to determine the index of refraction of the prism.

3. Barrier penetration measurement

Remove the transmitter and receiver from the goniometer arms and rotate the arms so that they are parallel to the base of the prism. Use the pegs provided to fix these arms in position. Now roll the second prism all the way up to the first prism. Place the receiver in the guides on the movable platform behind the second prism. (The separation of the prism and the receiver should be about 30 cm to the base of the receiver horn.) Place the source about 190 cm from the front face of the fixed prism and align it carefully to give a maximum reading on the receiver meter. Slide the receiver forward and backward a few centimeters in the guides to maximize the signal. Set the sensitivity for full-scale reading when the two prisms are touching. Now roll the prism back. You should see the meter reading drop quickly, with essentially 0 reading for a separation of several centimeters.

If the reading does not drop to at most 5–10% of the initial reading, you need to realign the source and detector and look for any causes of extraneous reflections. With the second prism still several centimeters away, adjust the position of the second receiver, located to detect the beam reflected from the hypotenuse of the fixed prism. This detector should be about 30 cm from the fixed prism. Adjust its position for a maximum meter reading and set this to 1 when the movable prism is “far” away. Now roll the second prism toward the fixed prism and note the meter readings on the two receivers. Describe qualitatively what you see.

Since the receiver monitoring the reflected beam can produce spurious reflections which affect the transmitted beam receiver, it should be removed for the rest of the experiment. **Be sure to turn off the receivers after you are finished with them, as their batteries**

run down quickly. Recheck the transmitter receiver for the extreme positions of the movable prism, and if necessary readjust and reset before taking quantitative measurements. Vary the movable prism location, measure the perpendicular separation d of the prisms, and plot the resulting transmission coefficient T . On the same graph, plot the predicted transmission coefficient. Compare the two and comment.

The Hall Effect

In 1879, American physicist Edwin Herbert Hall (1855–1938) observed a small potential difference across a conducting sample through which a current flowed perpendicular to an applied magnetic field. In this experiment you will study this phenomenon, called the Hall effect, in a semiconductor. Measurements of the Hall potential will yield the sign and density of the charge carriers in the semiconductor.

1. Theoretical Background

The charge carriers that conduct electricity in metals are electrons. If a metal strip is placed in a magnetic field and a current is established in the strip, then a small transverse electric field is set up across the strip. The resulting difference of potential is the Hall potential. Note that this potential is perpendicular to both the current flow and the magnetic field (see Fig. 3.1). When semiconducting materials are used in place of the metal, the Hall potential is generally much larger and may be of opposite sign. The change in sign implies that, in such cases, the charge carriers are positive and that a different conduction process is occurring than in metals.

Two quantities are of interest here. First, the sign of the Hall potential, which depends on the sign of the charge carriers. Second, the magnitude of the potential, from which may be deduced the density of charge carriers. This deduction is briefly the following: Let E_H be the transverse electric field generated in the strip carrying a current of density j in a field B in the geometry shown in

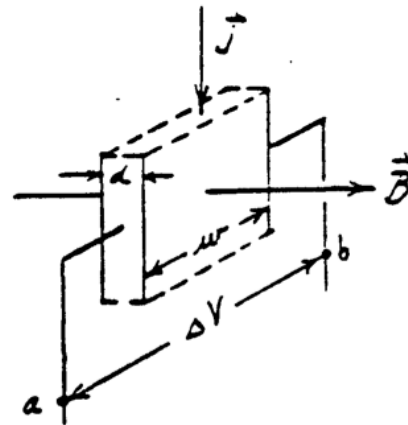


Figure 3.1: The Hall effect.

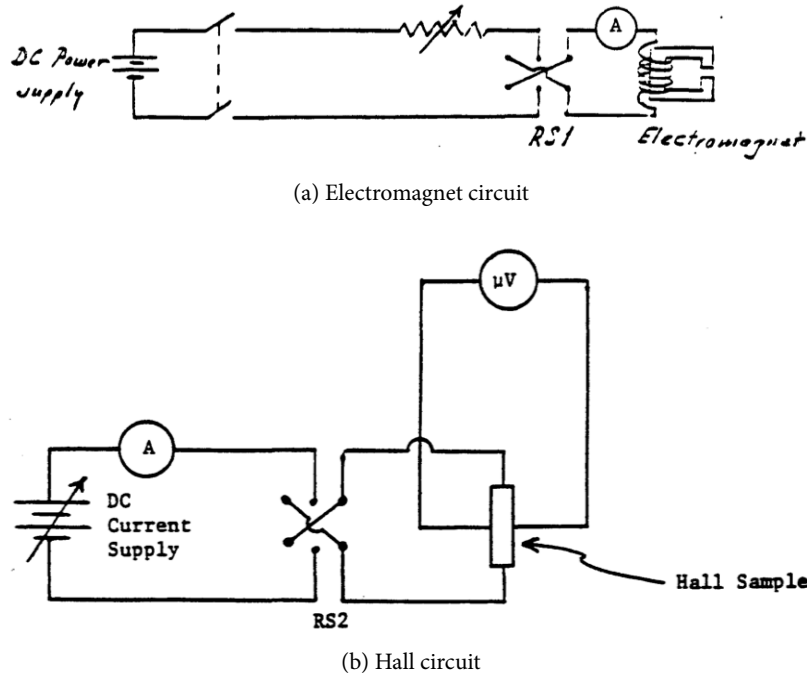


Figure 3.2: Circuits

Fig. 3.1. The quantity

$$R = \frac{E_H}{jB}$$

is called the Hall coefficient. In equilibrium, the transverse force of qE_H acting on the charge carriers of charge magnitude q must just compensate the Lorentz force of qvB acting upon these charges moving with drift velocity v . Since also $j = qnv$, where n is the density of charge carriers, we have

$$R = \frac{E_H}{jB} = \frac{vB}{qnvB} = \frac{1}{qn}$$

For both metals and semiconductors, it turns out that $|q| = e$, the magnitude of the charge of an electron. Thus n may be calculated from measured values of R . For a strip of width w and thickness d , carrying a uniform current density j , the conduction current is $I = jwd$. The Hall potential is $\Delta V = E_H w$. Thus,

$$R = \frac{\Delta V}{w} \frac{wd}{I} \frac{1}{B} = \frac{\Delta V}{IB} d \quad (3.1)$$

2. Measurement of the Hall Potential

Circuits Figs. 3.2a and 3.2b show the circuits used in obtaining the Hall potential. Fig. 3.2a shows the circuit used to produce the required magnetic field and Fig. 3.2b shows the Hall effect circuit itself.

Concerning these figures, note:

- The Hall effect requires both a current I passing through the sample and a magnetic field. The magnetic field is not shown in Fig. 3.2b, but the sample is oriented in the magnet to give the geometry shown in Fig. 3.1.
- Two reversing switches (RS1 and RS2) are present in the circuits, one to reverse the current in the magnet, the other to reverse the direction of current flow I in the Hall-effect device. The reason for RS2 is to compensate for thermal emfs, which can lead to small zero offsets.
- The reversing switch RS1 is included for a rather subtle reason: In an ideal Hall-effect device $\Delta V = 0$ if there is no magnetic field. But you will find that $\Delta V \neq 0$ even when $B = 0$. This occurs because the sample has a finite resistance and therefore a voltage drop occurs along the direction of current flow; the potential leads are soldered onto the sample at points which are not quite on the same equipotential line (in zero magnetic field). This potential difference between the two leads, which we'll call the IR effect, adds onto the Hall potential ΔV . In order to eliminate the IR effect, the magnetic field direction is reversed by using RS1. This changes the sign of ΔV , but since I is still flowing in the same direction, the IR effect will not change sign. The results for both magnetic field directions are averaged to get ΔV .



Ramp the current through the electromagnet down to zero before reversing RS1.

Gaussmeter You will measure the magnetic field using the LakeShore gaussmeter. It may be used in either SI mode (in milliteslas) or Gaussian mode (in gauss). Note that $1 \text{ T} = 10^4 \text{ G}$.

Measurements Establish a field of some 1000 to 1700 gauss and determine its polarity by means of a compass. **The current in the magnet circuit must not exceed 1.9 A!!** Estimate the power dissipation in the Hall-effect device for currents in the range **20 mA to 300 mA**. Use currents in this range to measure the potential difference across the Hall-effect sample. Vary both the current and magnetic field to establish the constancy of R and obtain a best value from your data. Evaluate the sign and density of charge carriers in the sample of indium arsenide.

Note: Consider the physical significance of your value for n .

Thermal Radiation

1. General

Any surface at a temperature above absolute zero radiates energy in the form of electromagnetic radiation. When the surface is in a vacuum, as in this case for the thermistor illustrated in Fig. 4.1, radiation is the only mechanism for energy loss. One imagines that the surface of [A] is at some temperature T and asks at what rate P thermal energy is radiated from the surface. Since [C] is a vacuum, the rate can depend only upon the nature of the surface and its temperature. The surface of the thermistor has been blackened in this experiment

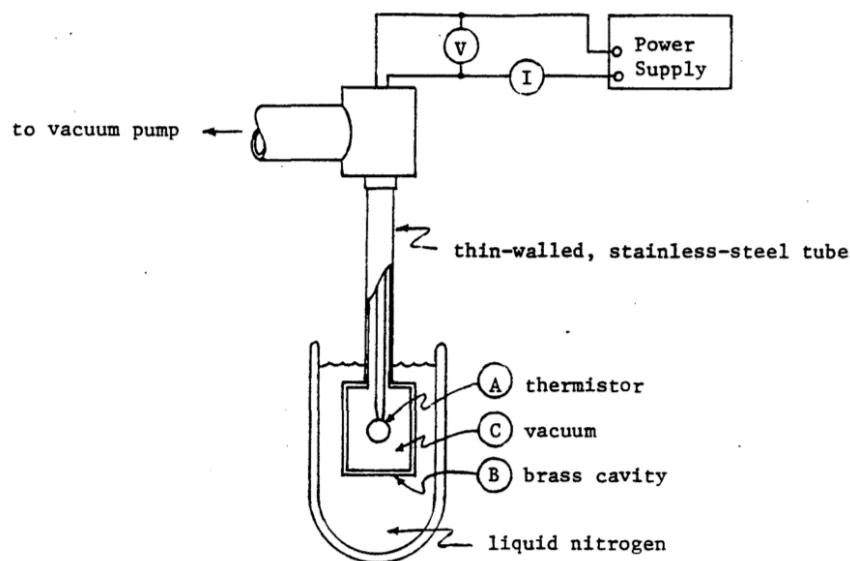


Figure 4.1: Thermal radiation apparatus.

to eliminate any effect due to the nature of the surface other than its area A . The rate of energy loss may be expressed as

$$P = \sigma AT^x \quad (4.1)$$

The universal constant σ is called the Stefan-Boltzmann constant. You are to determine σ and the exponent x relating P to the surface temperature.

The interior surface of the brass cavity [B] also radiates thermal energy to [A]. This surface, however, is maintained at a sufficiently low temperature (77 K) by contact with liquid nitrogen that the energy received by [A] from [B] is negligible. Further, [A] is suspended by wires of very small cross-section to reduce conduction of energy to [A] from the room. In equilibrium, then, the rate at which energy is radiated through the vacuum from [A] must equal the rate at which it is generated electrically within [A].

The radiating element [A] in this experiment is a thermistor. Thermal energy (in watts) is generated within it at a rate given by

$$P = VI \quad (4.2)$$

where V (in volts) and I (in amperes) are measured by the meters “V” and “I” shown. The resistance R of a thermistor varies as a function of its temperature. Since

$$R = V/I \quad (4.3)$$

measurement of V and I determines not only P but R as well.

2. Thermistor calibration

The conductivity of a thermistor (an undoped semiconductor) is proportional to the Boltzmann factor, $\exp(-E/kT)$, where T is the kelvin temperature, k the Boltzmann constant ($8.616 \times 10^{-5} \text{ eV K}^{-1}$), and E the band-gap energy of the semiconductor—the energy which must be acquired by an electron for it to participate in electrical conduction. The resistivity is the reciprocal of the conductivity so that the thermistor resistance may be expressed as

$$R = ae^{b/T} \quad (b = E/k) \quad (4.4)$$

The two constants, a and b , are determined by measuring R at two or more different temperatures. Thus, if R_1 and R_2 are the resistances of the thermistor at temperatures T_1 and T_2 , then

$$\ln(R_1/R_2) = b \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \quad a = R_1 e^{-b/T_1} = R_2 e^{-b/T_2} \quad (4.5)$$

Perform the calibration with atmospheric pressure in the apparatus to hasten equilibration. First measure R at room temperature as indicated by a thermometer. The circuit shown in Fig. 4.2 is used with a large (10-k Ω) series decade resistor (the black one) to limit the power dissipation in the thermistor to the order of microwatts (confirm this). The Lakeshore power supply is operated in program mode, in which an external resistor (the green one) is used to adjust its output current. The greater the control resistance, the smaller the current.

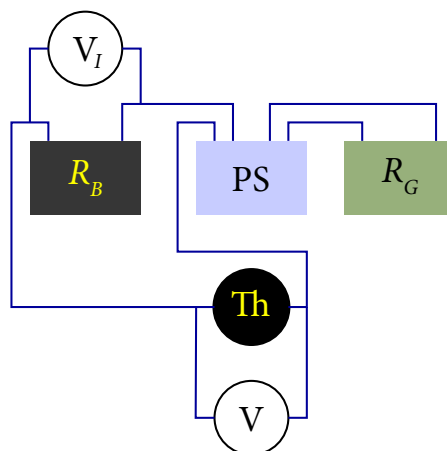


Figure 4.2: Thermistor circuit. The black resistor box (R_B) is used to measure the current through the thermistor (Th) and to limit that current. The output current of the power supply (PS) is adjusted using the green resistor box (R_G).

The leads of one voltmeter are connected across the thermistor. The second voltmeter reads the voltage drop across the decade resistor, from which you can determine the current in the thermistor. Use the potential and current to determine the thermistor resistance, R , at room temperature. Then heat some water and calibrate at a water temperature near 60°C . You may wish to take a few points at intermediate temperatures on the way. Then surround the thermistor with an ice-water bath to obtain another data point near the freezing temperature of water. You will probably find it helpful to use a stirring bar on the hot plate to ensure uniform bath temperature. (Record the potential across the $10\text{-k}\Omega$ resistor at one- or two-minute intervals to determine when equilibrium at ice temperature is reached.) Determine a and b (and E) from these data and a fit to Eq. (4.1) or equations Eq. (4.2). Equation (4.1) is then used to determine T in the experiment as written.

As soon as these data are obtained, have the instructor start the rough and molecular drag vacuum pumps. Approximately 30 minutes are required to reach the operating pressure of a few times 10^{-5} Torr. (Note that $1\text{ Torr} = 1/760\text{ atm} = 133.2\text{ Pa}$.) Use this time to analyze your calibration measurements to determine the constants a and b , and to set up a spreadsheet or Igor experiment to help you analyze data as it comes in. *You will need a fast way of converting between the current and voltage measurements and the temperature of the thermistor, so set one up in Excel, Igor, Python, or some other environment.*

3. Experimental procedure

The same circuit used to calibrate the thermistor is used to measure V and I (and thus R and P) in the remainder of the experiment **except that** the series resistor is initially re-

duced from $10\text{ k}\Omega$ to $1\text{ k}\Omega$. **You will have to reduce it further once the bath of liquid nitrogen cools the brass cylinder.** Otherwise, you can't supply enough power to the thermistor to keep it from cooling to the temperature of the walls. You need to be able to determine the thermistor temperature quickly using the resistance of the black box and the two voltage readings to make sure that the thermistor doesn't cool so far that you cannot revive it without removing the liquid nitrogen bath and warming up the brass canister. Do not immerse the canister in liquid nitrogen until you have spreadsheet or Igor panel set up to facilitate a rapid response.

When a sufficient vacuum is obtained, establish a current of $10 - 20\text{ mA}$ just before immersing the brass cavity in liquid nitrogen. The Lakeshore power supply can furnish **a maximum of 11 V and 50 mA in program mode**. Depending on how warm the thermistor is before you begin cooling, you may need to reduce the black resistance as low as $250\ \Omega$ to provide adequate current. Monitor its temperature carefully as you cool. Slowly raise the dewar of liquid nitrogen to cover the brass cavity. The current through the thermistor will drop a bit as its resistance increases. Wait several minutes for the temperature of the thermistor to stabilize. Record several voltage readings during this equilibration period to document the approach to equilibrium. Determine the thermistor temperature and the power radiated by the thermistor at equilibrium.

Vary the current slightly and again wait for equilibrium. Calculate T and P . Proceed in this way to generate data for P vs. T over a temperature range of approximately 0°C to $50-60^\circ\text{C}$.

The exponent x is most easily found by using a log-log plot of Eq. (4.1):

$$\log P = \log(\sigma A) + x \log T \quad (4.6)$$

The slope of the plot $\log P$ vs. $\log T$ yields the exponent in the "Stefan-Boltzmann radiation law." Of course, you can manage the log-log plot simply by double-clicking the axes and setting them to display logarithmically. Determine the values of σ and x from your data, given $A = 0.52\text{ cm}^2$. Think carefully about the best way to get a value for the Stefan-Boltzmann constant, σ .

4. A Note on Thermal Conduction in Gases

The brass canister must be evacuated to eliminate thermal conduction via diffusion through the gas. It is a curious fact that this conduction is *independent* of the gas pressure over a range of pressures that spans many orders of magnitude. It would seem that the greater the density of the gas, the more rapidly heat would be conducted away from the thermistor. However, the conducted power flux depends both on the number of particles striking the thermistor (and equilibrating at its temperature) per unit time, and the typical distance that they travel before colliding with another gas particle. This distance d is called the **mean free path**. Because d is inversely proportional to the gas density (for a reason I will outline below), while the number of gas particles is proportional to the density, the rate of heat conduction *by thermal diffusion* in the gas is independent of density until it becomes more probable for a gas particle leaving

the thermistor to collide with the walls of the canister instead of another gas particle. That is, until the mean free path is large compared to the dimensions of the canister.

We can estimate the mean free path of a gas molecule by pretending it is a sphere moving at a typical speed in the gas. By the equipartition theorem, the mean square velocity of a nitrogen molecule is given by

$$\langle mv^2/2 \rangle = \frac{3}{2} kT \quad \implies \quad v \approx \sqrt{\frac{3kT}{m}} \approx 500 \text{ m/s}$$

at room temperature. If it does not collide with another molecule or the wall, this molecule will trace out a cylinder in space whose volume is given by $V = avt$, where a is the cross-sectional area of the molecule. Roughly speaking, when this volume is equal to the average volume per particle in the gas, then we can expect that the molecule will collide. At room temperature, the average volume per particle is

$$\frac{V}{N} = \frac{RT}{p} = \frac{(8.314 \text{ J mol}^{-1} \text{ K}^{-1})(293 \text{ K})}{10^5 \text{ Pa}} \frac{10^{27} \text{ nm}^3}{6.02 \times 10^{23} \text{ mol m}^3} \approx 40 \text{ nm}^3$$

The “radius” of a nitrogen atom is 75 pm, but the bond length of the N_2 triple bond is 110 pm. If we then take the diameter of the molecule to be 200 pm, and imagine it is spinning rapidly enough to present a circular cross section, then its effective area is about $\pi D^2 = 0.13 \text{ nm}^2$ and the room-temperature mean free path is about $d = 300 \text{ nm}$ at atmospheric pressure.¹ The pressure and density must be reduced until d is significantly larger than the distance from the thermistor to the walls of the canister. At that point, conduction is proportional to the gas density (pressure). What is the maximum pressure, therefore, that you can tolerate to be confident that radiation dominates conduction by the gas?

¹Two molecules collide when their centers are one *diameter* apart, so the effective area of a molecule is πD^2 .

Photoelectric Effect

1. Background

Around the turn of the 20th century, Philipp von Lenard, studying a phenomenon originally observed by Heinrich Hertz, showed that ultraviolet light falling on a metal can result in the ejection of electrons from the surface.¹ This light-induced ejection of electrons is now known as the photoelectric effect. Einstein's explanation of this effect in 1905 (the year he also developed special relativity) is one of the cornerstones of quantum physics.

According to the classical theory of electromagnetic fields, the intensity of a light wave is directly proportional to the square of the electric field of the wave. An electron in some material exposed to this light wave should feel a force proportional to this electric field. For an intense enough illuminating light, the electron should be able to gain sufficient kinetic energy to escape the material. The energy gained by the electron depends only on the intensity of the light (and the nature of the material), not on the wavelength.

That, however, is *not* what is observed experimentally. In a series of very careful experiments in the 1910s, Robert Millikan showed that the maximum kinetic energy $K_{\max}$ of the ejected electron is independent of the intensity but linearly dependent on the frequency ν of the incident light:

$$K_{\max} = h\nu - W_0 \quad (5.1)$$

where h is a constant and W_0 is the “work function” characteristic of the material. Millikan found experimentally that h is numerically equal to the constant Max Planck introduced in his explanation of blackbody radiation.

In fact, Einstein's theory of the photoelectric effect in 1905 (hypothesized before Millikan's experiments) predicted just such a relationship, with h being identical to Planck's constant. In this theory, light exists in individual quanta, or photons. The energy of a photon is given by its frequency, $E = h\nu$. In the photoelectric effect a photon is absorbed by an electron,

¹See, e.g., Townsend, pp. 9–12.

which then acquires the energy lost by the photon. If the electron is right at the surface (so it doesn't lose any energy in inelastic collisions on the way to the surface), then the electron can escape, provided its kinetic energy is greater than the work function W_0 . Increasing the intensity of the incident light of a given frequency would simply mean that *more* electrons are produced with sufficient kinetic energy to escape; the maximum kinetic energy of the escaping electrons would remain constant. However, if the frequency of the incident light is so low that the photon energy is less than the work function then no electrons will have sufficient energy to escape the material.² The simple linear relationship between photon frequency and energy thus predicts Millikan's results. Two Nobel Prizes were awarded for work done on the photoelectric effect—one in 1921 to Einstein for his theoretical explanation, and one in 1923 to Millikan for his experimental work on this effect and for his more famous experiments establishing the charge of the electron.

2. Experiment

In this experiment you will determine the maximum kinetic energy of electrons photoejected from a metallic cathode in a vacuum tube under various illuminations. The maximum kinetic energy is determined by measuring the "stopping potential," the minimum reverse potential V between the cathode and the anode which reduces the photoelectric current in the tube to zero. In this case,

$$K_{\max} = eV \quad (5.2)$$

where e is the magnitude of the electron charge. Substituting this expression for K into Eq. (5.1) and solving for the stopping potential V gives

$$V = \left(\frac{h}{e} \right) \nu - \frac{W_0}{e} \quad (5.3)$$

Thus a plot of V vs. ν should give a straight line with a slope of h/e and an intercept of $-W_0/e$.

The experiment consists of two parts. In the first you will study the effect of light intensity on the stopping potential and test the predictions of the classical theory of electromagnetic radiation. In the second you will look carefully at the effect of light frequency on the stopping potential as a test of the quantum theory.

The experimental apparatus, made by PASCO Scientific, consists basically of a mercury vapor light source, diffraction grating, and a photodiode tube and associated electronics. The light source/diffraction grating setup allows you to study five spectral lines, from the near ultraviolet through yellow. Read quickly through the PASCO lab manual to get familiar with the equipment and procedures. You should assume that the basic alignment of the apparatus has already been accomplished, so that you will only need to properly locate the grating and

²For sufficiently intense illumination, it is, in fact, possible for two "sub-threshold" photons to be absorbed by a given electron, allowing it to escape the material, even though the individual incident photon energies are less than the work function. Such nonlinear effects require very intense laser beams.

photodiode detector for optimal performance. *Consult with your instructor before you make any other alterations.*



The mercury vapor lamp is a strong source of UV light. Never look directly into the beam, and always use UV-absorbing safety glasses when the lamp is on.

Using the PASCO manual as a rough guide, study the dependence of the stopping potential on both the intensity and frequency of the illuminating light. Your final analysis should include a determination of Planck's constant and also the work function of the photocathode.³

³Note from Eq. (5.1) that h/e has the dimensions of volt-sec (V s) and W_0/e has the dimensions of volts (V). From these results you can directly express h in terms of eV s and W_0 in terms of electron volts, where $1 \text{ eV} \equiv (\text{charge of electron}) \times (1 \text{ volt})$. If you had some independent determination of electron charge, you could then give these results in terms of, say, joules rather than electron volts, but that's not necessary here.

Rutherford Scattering

By 1911 general agreement existed that atoms contain a small number of electrons (< 100) with most of the atomic mass associated with positive charge. The problem was to determine how the positive charge and mass are distributed. Two extreme views were proposed by J. J. Thomson and Ernest Rutherford. Thomson considered the atom to be made of a space filling sphere of positive charge in which the electrons were embedded—the “plum pudding” model. Rutherford considered the positive charge and mass to be contained within a central, very dense nucleus—the “nuclear atom” model.

The test of these views was suggested by Rutherford and carried out by H. Geiger and E. Marsden in 1913. The experiment is the prototype for a great many contemporary “particle experiments” of the so-called “scattering” type. Experiments by Hofstadter, *et al.*, on the special distribution of charge within the nucleus itself are of this type. The experimental procedure is to send known particles (known mass, charge, etc.) with a given momentum into a thin target of the material under investigation and to observe the scattering (the change of momentum) of the emergent beam. Given any model of the target such that the forces arising between the particle and the target are known, the expected scattering can be calculated. The observed scattering then serves to eliminate those models for which the predictions disagree with experiment. Rutherford’s particles were alpha particles of relatively low energy arising in natural radioactive decay. Since only electromagnetic forces are significant in this case, the experiments served to eliminate models of the positive charge distribution in an atom. The plum pudding model was definitely crossed off. The nuclear atom model, on the other hand, predicted results in very good agreement with the data.

1. The Rutherford model with which the results of this experiment are compared is that of a positive charge distribution which is represented as a point charge of magnitude Ze , where Z is the atomic number of the target material. The mass distribution was considered to be the same as that of the charge or, at any rate, the center of mass was assumed rigidly attached to the point charge. The predicted angular distribution of particles of

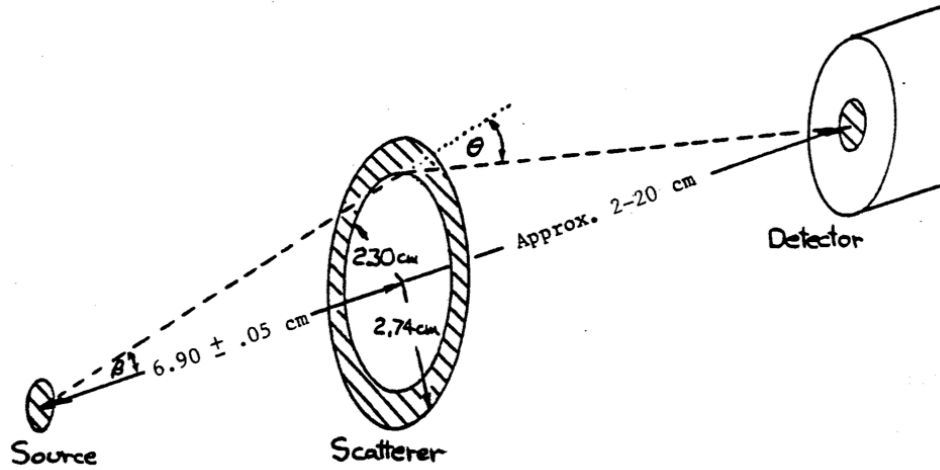


Figure 6.1: Scattering geometry.

mass m and charge $Z'e$ scattered from an incident beam of particles with velocity v by atoms of atomic number Z and mass M initially at rest is

$$\sigma(\theta) = \left(\frac{ZZ'e^2}{8\pi\epsilon_0\mu v^2} \right)^2 \frac{1}{\sin^4(\theta/2)} \quad \mu = \frac{mM}{m+M} \quad (6.1)$$

The “cross section,” $\sigma(\theta)$, is a measure of the probability that an incident particle will be scattered in a single collision into the angle θ to $\theta + d\theta$ measured with respect to the direction of the initial velocity.¹

A sketch of the relation of the source, scatterer, and detector of the alpha particles in the laboratory apparatus is shown in Fig. 6.1. The apparatus may be disassembled at the flanged end by removing four knurled nuts. **First**, however, read the following description. The source and scatterer are mounted together in a movable cage such that the angle β is fixed. The source is radioactive americium 241, which decays primarily by emitting a 5.29-MeV alpha particle.² The scatterer is an annulus of gold foil about $3.5\text{-}\mu\text{m}$ thick. Neither the americium source nor the gold foil may be touched, for obvious reasons.

The detector is a solid-state device consisting of a silicon wafer with a thin ($0.02\text{ }\mu\text{m}$) gold surface covering on one side and an aluminum surface on the other side. A po-

¹Why the quotation marks around “differential cross section”? Cross sections have dimensions of area. Imagine a uniform beam of intensity I (power per unit area or number per second per unit area) impinging on a single scattering particle, whose total cross section is σ . Then the total power removed from the forward-propagating beam is σI . The power scattered into solid angle $d\Omega = \sin\theta d\theta$ is $\frac{d\sigma}{d\Omega} d\Omega$. The derivative in this expression is abbreviated here by $\sigma(\theta)$.

²A thin cover over the radioactive source reduces the energy of the alpha particle somewhat.

tential difference of 30 V is placed across this “sandwich.” When an ionizing particle passes through the silicon, electrons are ejected by collision with the particle from the filled band to the empty conduction band of the silicon semiconductor. Both the electrons in the conduction band and the “holes” left in the valence band move under the applied field: the electron to the gold surface, the holes to the aluminum. Hence a pulse of charge is collected, with size proportional to the number of electrons injected into the conduction band, and thus to the energy loss of the ionizing particle in the silicon. You will count this pulse of charge with a scaler after it is amplified. Further details are given in the appendix to this experiment.



Do not touch the detector! The gold coating is fragile, the silicon can be ruined by contamination, and static electricity could damage the detector irreversibly.

The distance from the scattering foil to the detector may be varied from about 1 to 20 cm by means of the vacuum sealed plunger attached to the source cage and extending outside the apparatus. The scattering angle θ may thus be varied from about 27° to 90° .

Since the range of the alpha particles in air at normal pressure is only a few centimeters, it is necessary to evacuate the entire apparatus. The brass vacuum chamber is closed at one end by the sliding plunger and flange. The other end is closed by the mounting bracket of the detector seated against an O-ring seal.

Current pulses from the silicon detector generate voltage pulses in the amplifier circuit. These pulses are counted by a scaler. The experiment consists in determining the number of counts registered by the scaler in a measured time interval as the source cage plunger is moved in or out to vary the scattering angle θ .

2. Carefully study the apparatus prior to its evacuation. You will be given the minimum value of d (i.e., when the plunger is in as far as possible) for the apparatus. You will need this value together with your measurements of the external position of the plunger to compute the scattering angle θ and to correct for changes in the detector solid angle (see below). Begin collecting data with the plunger withdrawn as far as possible, to measure the counting rate for the smallest scattering angle. Record the time necessary to accumulate at least 100 counts at this and all other scattering angles.³ The standard deviation for N counts is $\sqrt{N}$ so that 10% statistics are obtained with 100 counts. The counting rate at minimum scattering angle will probably be of the order of 30 counts per minute, falling to some 4 counts per minute at the largest angles. This means that it will take something like 25 minutes to take a single data point when the plunger is pushed all the way in, which implies two things: (1) you had better plan your time carefully, and (2) you have plenty of time to plot the data as you take it.

³In taking data, choose intermediate plunger positions in light of the plot you will be making. (See below.)

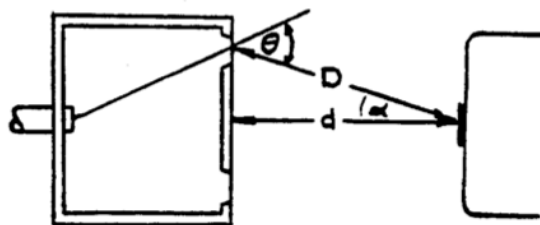


Figure 6.2: Scattering geometry. When the stick is pushed all the way in, the minimum value of $d = 1.80$ cm is obtained.

The counting rate must be corrected for the change in the solid angle subtended by the detector at the gold annulus. The apparent size of the detector as seen from the annulus is a function of their separation, d . This correction consists of two factors (if we ignore the finite size of the detector and the annulus width). First, the detector size would vary as $1/d^2$ were it viewed “head on” from the annulus. This is very nearly the case when d is much greater than the radius of the annulus. For small separations, however, the projection of the detector into the line of sight from the annulus must be taken into account. The projected area goes as $\cos \alpha$, or as d/D . Combining these two factors, the apparent size of the detector varies as $\frac{d}{D^3}$.

The counting rate is multiplied by the reciprocal of this factor to obtain a counting rate proportional to that which would have been measured with a detector whose size appeared always the same to the scattering annulus. The counting rate corrected for solid angle is proportional to the cross section $\sigma(\theta)$.

To compare your results with the predictions of the Rutherford model, plot the logarithm of the corrected counting rate vs. the logarithm of $\sin(\theta/2)$. (What should this plot look like according to the Rutherford model?) Enter your data in this plot with bars to indicate the standard deviations resulting from counting statistics.

1. The EGG Ortec Silicon Charged Particle Detector

Silicon is a semiconductor with a gap of 1.1 eV between the top of the filled band and the bottom of the (nearly empty) conduction band. At any temperature above absolute zero, some electrons will have enough thermal energy to reach the conduction band; for the detector you use, with 30 volts potential difference across the silicon wafer, this gives rise to a “dark current” of about 300 nA or 1×10^{12} electrons/second. (Incidentally, since the silicon wafer is about 150 μm thick, the electric field is $30 \text{ V} / 1.5 \times 10^{-4} \text{ m} = 200,000 \text{ V/m}$.)

When there is no voltage across the silicon wafer, the Fermi energies (see Townsend, Chapter 7, pp. 221–225 and Chapter 8, pp. 264–272) of the electrons in the gold, alu-

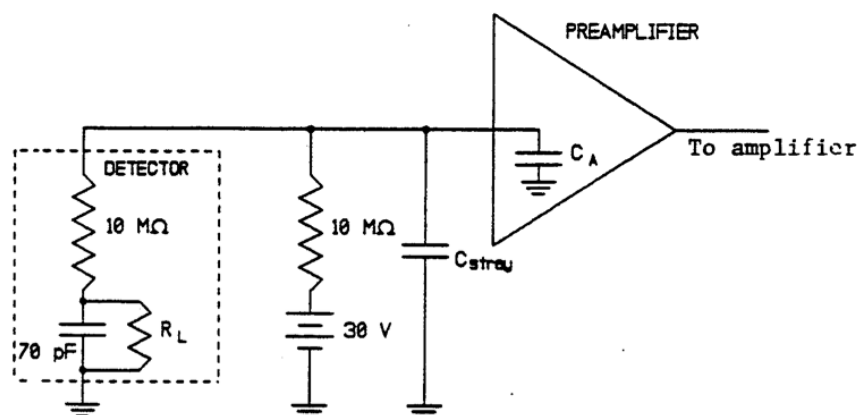


Figure 6.3: Circuit

minum, and silicon are equal; electrons move between these layers to change the potential of these layers until this equality is reached. The particular silicon wafer we use has donor impurities (see Townsend, Chapter 8, 268–272), so the Fermi energy in the silicon lies 0.16 eV below the bottom of the conduction band. There are, accordingly, thermally injected electrons in the conduction band. Once the 30 V power supply is turned on, these electrons are swept away, giving rise to the “dark current.”

When an α particle enters the silicon, it collides with electrons in the silicon lattice, giving many of them enough energy to reach the conduction band. The average energy lost by the α -particle to create an electron-hole pair is measured to be 3.6 eV. Thus a 5 MeV α -particle, completely stopped in the silicon, gives rise to $5 \times 10^6 / 3.6 = 1.4 \times 10^6$ electron-hole pairs, or 2.2×10^{-13} C. The capacitance of the detector is 70 picofarads (7×10^{-11} F), so collecting this charge causes a voltage change $\Delta V = (2 \times 10^{-13} \text{ C}) / (7 \times 10^{-11} \text{ F}) = 3 \text{ mV}$. The detector voltage is supplied through a 20-M Ω resistor, so the recovery time is $RC = (7 \times 10^{-11} \text{ F}) (2 \times 10^7 \Omega) = 1.4 \text{ ms}$.

Figure 6.3 shows a circuit diagram of the detector, its power supply, and the first (preamplifier) stage of amplification. Here R_L is the “equivalent resistance” of the silicon wafer; since the “dark current” is about 3×10^{-7} A for a potential of 30 V, $R_L = 100 \text{ M}\Omega$. The Model 109A preamplifier set at $10\times$ gain gives a pulse of 150 mV/MeV for a Si detector. This preamp also reduces the pulse width to approximately 50 μs . The amplifier following the preamp further reduces the pulse width and increases the peak voltage.

2. Addendum

Since the laboratory notes were written, the brass cylinder has been replaced by a plastic (Lexan) cylinder, which is semitransparent and slightly shorter. Accordingly, the angle of scattering of the alpha particles must be calculated with the new dimensions. The dimension

you need to know is d . **The shortest d available to you is 1.80 cm.**

Why the Lexan cylinder, and these changes?

For reasons I did not understand, this experiment usually produced an exponent in the range of -4.3 to -4.5 instead of the -4 which Professor Rutherford had in mind. Mark Chalice, '94, asked me two years ago if any of the alpha particles which go through the foil undeflected might then strike the brass cylinder wall and be scattered there. Indeed, most of the alpha particles that strike the foil do go through essentially undeflected, having lost some energy by many collisions with electrons, thus ionizing gold atoms. These alpha particles enter the brass. Most spend out their range losing energy in more electron collisions, but a few of them may indeed be scattered by the copper and zinc nuclei of atoms which make up brass. From the cross-section equation on Page 3-1 of the notes, we see that the scattering cross section depends on Z^2 . For gold, $Z = 79$; for copper, $Z = 29$, and for zinc, $Z = 30$. Accordingly, these brass nuclei are only about 14% as effective as gold in Rutherford scattering, but the path length in the brass can be considerable. From the same equation on Page 3-1, we also learn that as the alpha particle slows down, the probability of scattering increases. Thus the cylinder walls in front of the gold foil may constitute a significant second scatterer. The angle of scattering at which the alpha particle is detected is greater for these brass-scattered alphas, and hence they are no longer much detected as the foil nears the detector. Accordingly, we are led to believe that the power law is greater than 4.

The solution to the problem is not to use brass, but a plastic, for which the atoms in the wall are predominately carbon, hydrogen and oxygen. These have at most 1% of the scattering cross section of gold. Essentially all the alpha particles striking the wall lose their kinetic energy through electron collisions and are not scattered.

Joseph B. Platt, January 1994

Gamma Radiation Interactions

1. Introduction

Electromagnetic radiation of wavelength greater than 1 pm interacts with matter in just two ways: the photoelectric effect and Compton scattering. Both are nonclassical and most simply described as the interaction of a particle—the photon, or gamma ray—with an atom.

Both interactions remove an electron from the atom, and each is observed by detecting this electron. The identifying feature of the photoelectric interaction is that the electron emerges with a single energy $E = hc/\lambda$, since the photon is destroyed in the interaction then, for energy conservation, E is the photon energy. Compton scattering, on the other hand, is interpreted as elastic scattering of the photon of energy E and momentum $p = h/\lambda = E/c$ from an atomic electron. If the photon is scattered by an angle θ from its original direction, then its new wavelength λ' is related to its original wavelength λ by the Compton formula

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos \theta) \quad (7.1)$$

You may readily show, then, that the electron acquires the energy

$$E_e = \frac{E}{1 + \frac{mc^2}{E(1 - \cos \theta)}} \quad (7.2)$$

where mc^2 for an electron is 511 keV.

The experiment is performed with radiation from a radioactive source such as Cs^{137} incident upon the atoms in a 2-inch-diameter by 2-inch-thick crystal of (thallium-doped) sodium iodide. The radiation from Cs^{137} consists of gamma rays of energy 662 keV. Compton and photoelectrons freed from atoms of the crystal by this radiation rapidly come to rest within the crystal, producing a great number of electron-hole pairs, which lose energy to lattice vibrations until they recombine at a thallium ion, emitting a blue photon near 420 nm, which is readily detected by the photomultiplier tube. The photomultiplier converts the blue photons

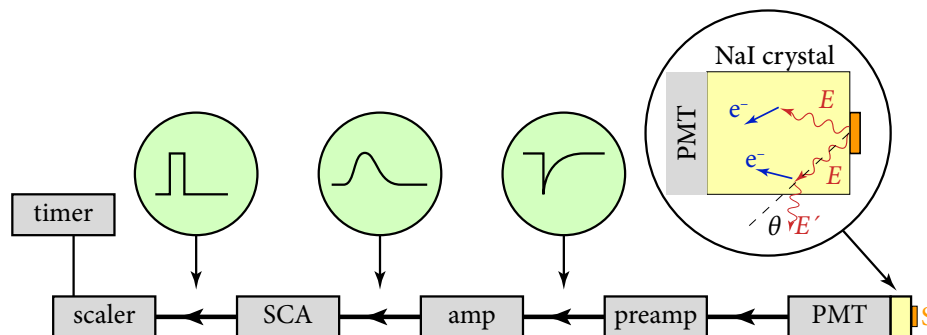


Figure 7.1: Schematic representation of the apparatus.

into an electrical pulse of amplitude proportional to the total energy of the secondary photons—hence also proportional to the initial electron energy. By observing the amplitude distribution of these pulses, we determine the energy distribution of electrons freed from atoms of the crystal by interactions with the incident radiation.

2. Experiment

The apparatus is shown schematically in Fig. 7.1. The source S is taped to the front of the sodium iodide crystal. The enlargement illustrates photoelectric and Compton interactions within it. Secondary photons are generated as the electrons e^- come to rest. These photons are detected by the photomultiplier tube PMT mounted in a common housing with the crystal. The preamplifier generates a negative output pulse as shown in the inset with amplitude proportional to the energy of the Compton or photoelectron feed within the crystal. The amplifier then inverts, shapes and amplifies the pulse as shown.

The amplifier output is directed to a single channel analyzer (SCA). This is the instrument with which the pulse amplitude distribution is determined. You will control two settings of the SCA—its lower level discriminator (LLD) and its window. The function of these controls is shown in Fig. 7.2. A, B, and C are three pulses from the amplifier to the SCA. The controls set the height of the line LLD and the width of the gap labeled “window.” As set, only pulse B will trigger an output window pulse from the SCA. The maximum amplitudes of A and C lie outside the window and neither pulse is sent on to the scaler. Were the LLD lowered so that the window embraced only pulse A, then only pulses of this maximum amplitude would be registered by the scaler. Were the window opened to include the maxima of both B and C, then both of these pulses would

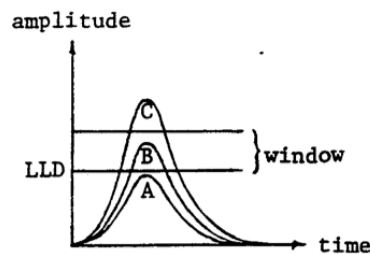


Figure 7.2: Discriminator levels.

be registered. The scaler is started and stopped by a timer and counts the number of pulses it receives from the SCA within this time which satisfy the condition:

$$\text{LLD} \leq \text{maximum pulse amplitude} \leq \text{LLD} + \text{window}$$

We wish to observe the amplitude distribution of pulses generated by electrons from gamma interactions within the sodium iodide crystal. First, connect an oscilloscope to the output of the amplifier. You will see a broad amplitude distribution, with a bright band of pulses of nearly the same maximum amplitude. These are generated by photoelectric interactions in the crystal. Set the amplifier gain so that these pulses have $\sim 5\text{--}8$ volts maximum amplitude. Remove the scope and reconnect the amplifier to the SCA. Set the SCA window¹ to 0.2 volts and the LLD somewhat below the maximum voltage of the bright band observed on the scope. Set the timer for 10 s or so and count the number of pulses received by the scaler at this setting of the SCA. Advance the LLD by 0.2 V, leaving the window fixed, and count again. Proceed in this way through the “photopeak.” You now know how to use the apparatus.

For Cs^{137} , the photopeak observed above occurs at 662 keV. This assumes that all the energy of the incident γ ray shows up as electron-hole pairs and then blue photons, as occurs in a photoelectric event. Since each component of the apparatus is linear (we assume), this number calibrates the entire voltage range of the LLD in electron-volts, although it is probably a good idea to add source to the mix (such as Na^{22} or Co^{60}) to have an independent check on linearity. You are now prepared to study the entire energy spectrum of electrons resulting from photoelectric and Compton interactions of gamma radiation from Cs^{137} with atoms of the sodium iodide crystal. Start with the LLD at zero volts and make three or four measurements at gradually increasing LLD. When you have completed this task and understand how the apparatus works, you have a choice: you may either proceed through the photopeak, one channel at a time. Or, you may get checked out by your instructor and switch to using the Maestro 32 multichannel data acquisition system, which will allow you to record the entire spectrum simultaneously. Instructions on using Maestro 32 are available at the apparatus.

If you choose the manual approach, be sure to plot as you go! Where the distribution is quite flat, you should increase the increments by which the LLD is advanced. Where it changes rapidly, you may wish to decrease them. Set the timer to obtain good statistics. Remember that the uncertainty in the number N of pulses counted in a given time interval is $\pm\sqrt{N}$.

2.1 Things to observe

Classically, one would expect no structure in the pulse amplitude distribution at all. The prominent peak you observe attests to the nonclassical photoelectric interaction first accounted for by Einstein.

The Compton interaction produces electrons of energy from zero, $\theta = 0$, to a maximum for $\theta = \pi$ (Eq. (7.1)). The maximum defines the “Compton edge.” Calculate where you expect

¹In “window” mode, the Window pot goes from 0–1 V, while the Lower Level pot goes from 0–10 V.

the edge to appear and locate it in your experimental distribution. You will probably also observe a rather broad peak at the energy $E - E_e$. This is the energy of a gamma ray which has scattered at $\theta \cong \pi$. This occurs for gammas emitted by the source away from the crystal, which are then scattered from the table, floor, walls, etc. back into the crystal. The peak may be enhanced by placing an aluminum plate immediately behind the source. You should also observe a narrow peak at an energy of ~ 30 keV. The radiation responsible is an x-ray from barium, the decay product of cesium within the source. Look up the known energy of this x-ray and compare it to your result. You may wish to place a lead brick behind the source and observe x-rays from lead (~ 70 keV) which arise with each photoelectric interaction in the lead.

Chaotic Motion

Newton's equations and the extensions supplied by Euler, d'Alembert, and Lagrange, built a mighty edifice able to explain the motions of planets and apples, pendulums and piano wires. The great French mathematician and astronomer, Pierre-Simon, Marquis de Laplace (1749–1827), summed up mechanical philosophy in *A Philosophical Essay on Probabilities* of 1825:

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

Towards the end of the nineteenth century, Henri Poincaré (1854–1912) came to appreciate that seemingly simple systems, such as planets orbiting the Sun, could exhibit quite complicated motions. We now know that the behavior of many (most?) mechanical systems shows very strong dependence on initial conditions: systems starting from nearly the same state evolve into later states that differ widely. Imperfect knowledge of the initial conditions prevents us from making valid predictions beyond a limited interval of time. Therefore, Laplace's classical determinism is a delusion.

To be chaotic, a system must be nonlinear and have more than one degree of freedom. In this experiment you will investigate the motion of a damped, driven, nonlinear pendulum. One degree of freedom is the pendulum's rotation θ . The second is the position of the drive arm, which will be forced to rotate at a constant rate. This system thus fulfills the requirements for exhibiting chaos.

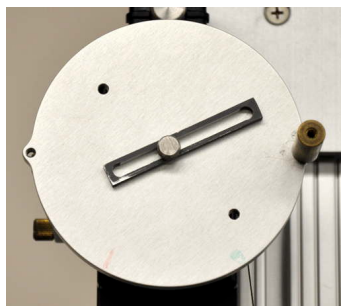


Figure 8.1: The disk of the chaotic motion apparatus is not perfectly symmetric, leading to an unwanted gravitational torque in the absence of the eccentric mass (shown here in the 3-o'clock position). The dark aluminum fixture (counterweight) pointing roughly at 2:30 is adjusted to move the center of mass of the disk onto the rotation axis.

1. Overview

You will study a pendulum consisting of an aluminum disk with an eccentric mass mounted near the edge. **(Please do not unmount the disk; just trust that I have properly measured its mass, since removing the disk will ruin the adjustment of the counterweight. See Fig. 8.1 and Table 8.1.)** A pulley is attached to the disk and a string attached to two springs is wound around the pulley. The string is adjusted so that the tension in each spring is equal when the eccentric mass is rotated to its highest position (the position of unstable equilibrium). The combination of the eccentric mass and the pair of springs creates two potential wells separated by the unstable equilibrium position with the eccentric mass at the top. A small magnet can be positioned near the disk to provide eddy-current damping, which is proportional to the angular velocity of the disk. By changing the gap between the magnet and the disk, you can adjust the strength of the damping torque.

If the pendulum starts at rest in one of the wells, and is driven at a modest amplitude and frequency, it oscillates steadily in that well after an initial transient. With increasing amplitude or frequency, however, it can surmount the barrier and explore both wells. Under some conditions, it executes a regular oscillation between both wells; under others, its motion seems utterly capricious—it bounces between wells in an apparently unpredictable way. Between these extremes lies period doubling and other strange behavior.

1.1 Visualizing the Motion

A straightforward way to portray the behavior of the system is to plot its angular position as a function of time, as shown in Fig. 8.3(a). The plot clearly reveals an initial transient period lasting about 50 seconds, followed by an apparently steady-state regime in which the pendulum oscillates solely in one of the two potential wells.

A second way to show the system's evolution is on a **phase plot**, which plots the pendulum's angular velocity $\omega = d\theta/dt$ as a function of its angular position. The phase plot for the data of Fig. 8.3(a) is shown in Fig. 8.3(b), and the phase plot for the portion of the data after $t = 1$ min is shown in Fig. 8.3(c), which clearly shows the



Figure 8.2: Chaos apparatus.

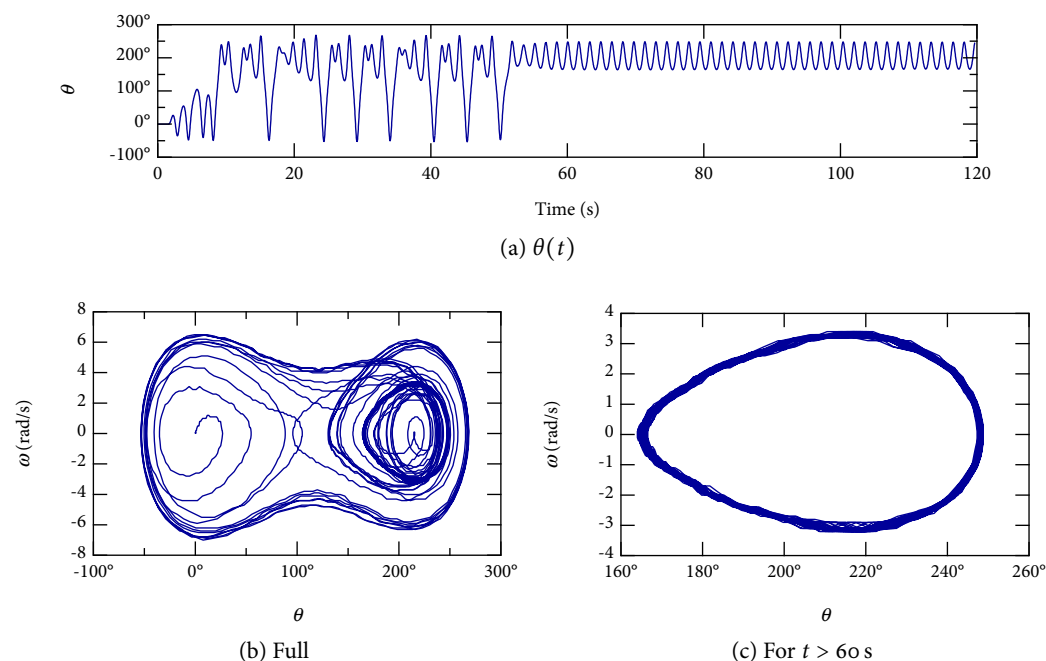


Figure 8.3: (a) Angular position vs. time in a non-chaotic regime. After transient behavior in which the pendulum samples both wells, it settles at $t \approx 52$ s into one well, where it undergoes periodic motion. (b) Phase plot (ω vs θ) for this run, beginning at $t = 0$. (c) Phase plot of the portion of the run after the system has stabilized in the right well.

periodicity of the motion after the initial transient has died out.

For driven motion such as this, there is a third common way of representing the response of the system, proposed initially by Poincaré. It is based on the phase plot, but involves recording the position of the system on the phase plane once per cycle of the drive. Essentially, the system is strobed at the driving frequency and its “phase” recorded. Figure 8.4 shows the Poincaré plot for the run we have been considering, superimposed on the phase plot. During the portion of the run in which the system executes a periodic motion, the Poincaré plot collapses to a single (blue) point, which is a tidy way of indicating the system’s stability.

2. Procedure

The steps outlined below will guide you through an exploration of the simple, periodic behavior of this system when driven in the non-chaotic regime, and then have you explore the conditions that give rise to chaotic behavior. So that you can attempt to model the behavior you observe, the initial explorations (along with data in the black binder next to the apparatus) should yield the necessary system parameters. You will be able to control several of these parameters:

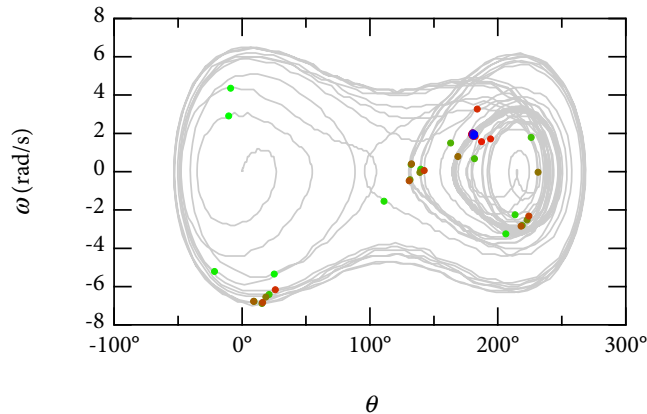


Figure 8.4: Poincaré plot superimposed on the phase plot of Fig. 8.3(a). The color of the dots of the Poincaré plot go from green at early times through red at intermediate times to blue towards the end of the run.

1. the drive frequency $0 \leq f \leq 1000$ mHz and acceleration of the motor as it comes up to speed
2. the drive amplitude $0 \leq a \leq 1.5$ in
3. the initial phase of the drive arm and the initial well the system occupies
4. the eddy-current damping rate, as controlled by the gap between the magnet and aluminum disk

Since this list implies a vast parameter space, while your time with the apparatus is quite limited, following the steps outlined below should allow you to get a sense of where the system behaves non-chaotically and where it might be interesting to explore for chaotic behavior.

2.1 Linear Behavior

When the eccentric mass is removed, the system is linear and well behaved.¹ You can use this behavior to measure the combined spring constant of the two springs by measuring the period of oscillation of the disk when released from rest.

1. Remove the eccentric mass from the aluminum disk and lock the drive arm in the horizontal position by holding it in place and then energizing the stepper motor.
2. Back out the damping magnet all the way.
3. Rotate the pendulum disk about half a turn from equilibrium, release it, and immediately start recording $\theta(t)$ in DataStudio.
4. Copy the trace to the clipboard.
5. Launch Igor Pro.

¹ When the eccentric mass is removed, the equation of motion of the undriven rotary pendulum is

$$I\ddot{\theta} + \beta\dot{\theta} + \kappa\theta = 0 \quad (8.1)$$

where I is the moment of inertia, β is the torsional damping coefficient, and κ is the effective torsional spring constant which depends linearly on θ and its time derivatives. When the eccentric mass is added, a term proportional to $\sin \theta$ must be added. This term has nonlinear dependence on θ .

6. Select **Auto DataStudio Paste** from the Data Studio submenu of the HMC menu. If you do not have these installed, see <http://www.physics.hmc.edu/igor/> for instructions.
7. You should see a trace of $\theta(t)$ in Igor. Fit a damped sinusoid to it to determine the period.
8. Repeat a few times to get a sense for the uncertainty in the determination of the period.
9. Using the period and the values in Table 8.1, determine the spring constant of the springs. Assume that the springs are identical.
10. Screw the magnetic damper in until the gap between it and the aluminum disk is about 4 turns of the screw (which has 20 threads per inch). Then take a few trials as before, determining the period. Is it different from the value when magnetic damping is negligible?

2.2 Using the Stepper Motor

You will drive the system using a stepper motor. Stepper motors advance one “tick” each time a current pulse is delivered to them. Between pulses, the shaft is effectively clamped in position, when the motor is energized. The Compumotor stepper currently installed in the apparatus rotates once for each 1000 pulses it receives, which means that if the controller sends it 1000 pulses per second, it will rotate at $f = 1000 \text{ mHz} = 1 \text{ Hz}$.



Do not attempt to rotate the stepper motor when the motor is energized. Use the switch on the power strip to turn off the stepper motor amplifier before rotating the motor.

- The direction of rotation is set by the leading $\pm$ sign on the position displayed on the Compumotor controller. Only use the ‘ $-$ ’ sign. (If you use the ‘ $+$ ’ sign, you will likely loosen the screw holding the drive arm in place. *This is a bad thing!*)
- Do not change the velocity setting while the motor is spinning. Press the Stop button,

Parameter	value	uncertainty	Parameter	value	uncertainty
<i>Aluminum disk</i>					
mass	120.40 g	0.02 g	diameter	9.50 cm	0.1 cm
off-axis mass	1.81 g	0.02 g	screw mass	1.47 g	0.02 g
<i>Three-radius pulley</i>					
mass	6.41 g	0.02 g	radii	0.5, 1.45, 2.4 cm	0.05 cm
moment of inertia	$< 18 \text{ g cm}^2$		radius to use	2.4 cm	
eccentric mass	14.53 g	0.02 g	damper thread	20 inch ⁻¹	
spring 1	10.16 g	0.02 g	spring 2	10.31 g	0.02 g

Table 8.1: System parameters.

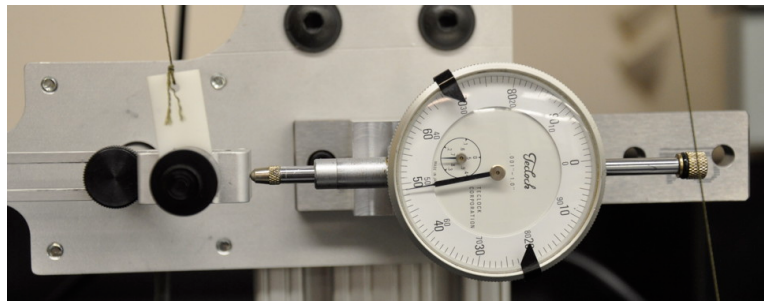


Figure 8.5: Using the dial indicator.

then change the setting.

- Do not use a pulse rate faster than **1000 pulses per second**, which corresponds to a frequency $f = 1$ Hz.
- To provide uniform initial conditions, I recommend orienting the drive arm horizontally, as shown in Fig. 8.5. *Make sure the motor is not energized while you rotate the arm.* Orient the pendulum to put the eccentric mass in the desired potential well and let everything come to rest. Start Data Studio (see Section 3. for detailed instructions on using the software) and let it run a second or two before pressing the start button on the Compumotor controller.

2.3 Stability of the Drive Motor

1. Set the drive amplitude to something between $3/4$ and 1 inch. You can measure the drive amplitude quite precisely using the dial indicator, which measures in thousandths of an inch (mils).
 - (a) Make sure the stepper motor power is off. Rotate the motor until the left spring is maximally stretched (the adjustable arm will be vertical and out of the way of the indicator).
 - (b) Install the dial indicator in the appropriate pair of holes.
 - (c) Pull the indicator pin to the right, out of the way of the adjustable arm, then rotate the arm to the orientation shown in Fig. 8.5. Carefully bring the tip of the pin into contact with the adjustable arm. Then energize the motor, making sure that the arm remains horizontal.
 - (d) Loosen the thumbscrew on the adjustable arm and adjust the length to the desired value.
 - (e) Tighten the thumbscrew firmly, squeezing on the lock washer. You don't want the screw coming loose in the middle of a long run, so be sure that it is quite snug.
 - (f) Turn off the power to the motor, rotate it back to the vertical, then carefully remove the dial indicator.

2. With the eccentric mass removed, start the motor at a frequency of about 500 mHz and record $\theta(t)$ for many cycles of the motor. How stable is the response? It may be useful to transfer the data to Igor, fit the steady-state portion to a sinusoid, and examine the residuals. Is the response of the disk-pendulum sinusoidal?
3. Change the damping and repeat this procedure. Does the damping affect the noise in the oscillation in the disk-pendulum?

2.4 Adjusting the Potential

1. Investigate the symmetry of the potential by releasing the rotor from rest near its unstable equilibrium position when the motor arm is in a horizontal position, once on each side of vertical. If it is asymmetric, carefully adjust the string on the clamp holding the string on the right-hand spring to symmetrize.
2. Measure the natural frequency in each potential well with the damping magnet far from the disk. Are they the same? Why or why not?
3. Conduct a series of trials, all starting from rest with the arm horizontal and the eccentric mass always in the same well, gradually increase the drive frequency. Do not change the angular velocity while the motor is running. It is not necessary to record these trials; just observe the behavior of the initial transient and see whether the system settles into a periodic motion. If it hasn't after a minute or two, stop it, increase the drive frequency, and try again. If it hasn't left the starting well by the time the frequency approaches the natural frequency, then either the damping is too great or the drive amplitude is too small to generate interesting behavior. Reduce the damping and try again.
4. Set the frequency just barely above the chaos threshold and record a long run (several minutes), observing the Poincaré plot. Alter the frequency a small amount and record again. Compare the Poincaré plots.
5. Explore how small changes in the amplitude, frequency, or damping near the chaos threshold affect the system's behavior.
6. See if you can find conditions that produce period doubling, such as illustrated in Fig. 8.6.

3. Data Studio Software

The Pasco software that operates the ScienceWorkshop 750 interface is called Data Studio. You will set it up to record angular position, θ , and angular velocity, ω , as functions of time, as well as the period of the drive. You can then create live plots to show θ vs. t , ω vs. t , a phase plot (ω vs. θ), and a Poincaré plot. You can also export data in text format to Igor Pro.

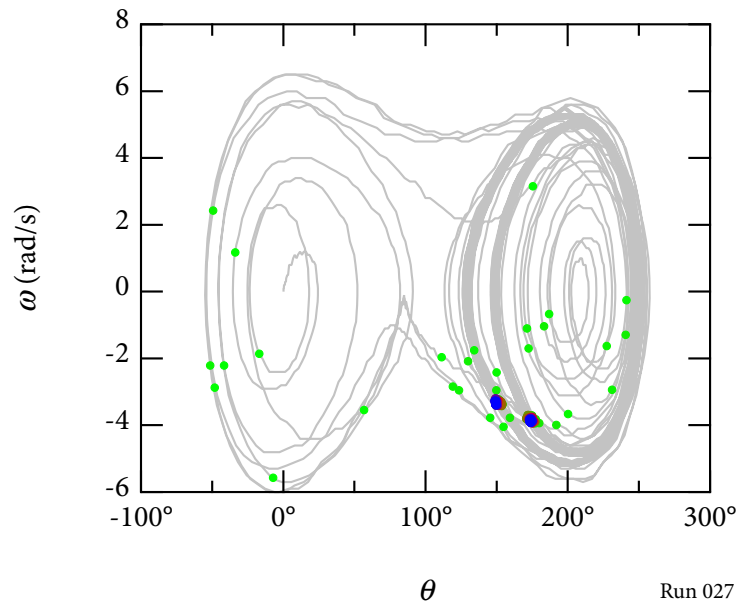


Figure 8.6: Period doubling. After an initial transient, the system settles into a motion which repeats itself after two cycles of the drive.



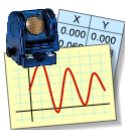
Data Studio does not handle large amounts of data nearly as gracefully as Igor does. I recommend transferring data you wish to keep from Data Studio to Igor soon after taking it, and then removing it from your Data Studio Activity file.

3.1 Setup

Hardware configuration

1. Plug the rotary motion sensor into channels one (yellow) and two (black) of the ScienceWorkshop 750 interface. (Reversing the plugs changes the sign of the data from the rotation sensor, which may be convenient, depending on which potential well you decide to start the eccentric mass in.)
2. Plug the photogate into channel 3.
3. Ensure that the interface's USB cable is connected to the computer. Then power on the ScienceWorkshop 750 interface.
4. Adjust the rotating arm on the drive motor to be between 0.75 and 1 inch away from the motor axis. **Be sure that the motor is not energized when you perform this rotation.**

Software configuration



1. Open Data Studio DataStudio.app and create a new activity. You should see a picture of the ScienceWorkshop 750 interface with its ports outlined in yellow. [You will only need to go through the following setup procedure once. Once you have created an activity with all the appropriate measurements and calculations, you can use it for subsequent data files. Just Save As a different file, then remove all the data using the Experiment menu.]
2. Click on Port 1 and add a **Rotary Motion Sensor** from the Digital Sensors tab.
3. Set it up to measure angular position (θ) in degrees, angular velocity (ω) in radians/s.
4. Set the sample rate to 50 Hz.
5. Set the Resolution to High (on the Rotary Motion Sensor tab). This corresponds to an angular resolution of 0.25° .
6. Add a photogate to Port 3, and have it measure “Velocity in Gate” (even though this makes no sense). We will use it instead to measure the period and to strobe θ and ω for the Poincaré plot.
7. Click the **Setup Timers...** button and label the timer Period. Using the Timing Sequence Choices area create a sequence of blocked–unblocked–blocked. This will record the period each time the drive motor completes a revolution.
8. Minimize the Experiment Setup window. You may need to come back to it later if you decide to change the sample rate or the resolution of the rotary motion sensor.

Calculations

1. You now need to set up a few calculations to be made from the data you will acquire. Click the **Calculate** button.
2. To calculate the frequency, click **new** and enter in the Definition box: $f = 1/T$. Click **Accept**. On the Variables popup, select “Period” in the **Data source** menu.
3. You now need to set up two additional calculations for the Poincaré plot, and one for the potential. The values of θ and ω are stored at the sample rate. In addition, we would like to record their values once each cycle, using the photogate measurements as the strobe. To manage this, we use a trick. We multiply the measurement of Channel 3 by zero and add the value of either θ or ω . Channel 3 is recorded only once per revolution of the drive, so including it in the calculation produces the desired strobing of θ and ω . Thus, create three more calculations:

$$\begin{aligned} \text{px} &= \text{strobe} * 0 + x \\ \text{py} &= \text{strobe} * 0 + y \\ \text{pot} &= -w * w \end{aligned}$$

where strobe is set to the value of Channel 3, x to Theta, and y and w to Omega. (Note

that I had written the equation for pot in a strange way because Data Studio binds the unary negative sign more tightly than exponentiation, so that $-w^2$ produces a positive number.) Close the Calculate window.

3.2 Potential Well

To explore the shape of the potential well, back the damping magnet all the way out, rotate the eccentric mass to the top, and release it from rest. If damping is negligible, then

$$E = U(\theta) + \frac{1}{2}I\omega^2 \quad (8.2)$$

where E is constant. Therefore, by plotting $-\omega^2$ vs. θ , you can explore the shape of the potential well. Prepare a graph of $-\omega^2$ vs. θ :

1. Drag the Data entry for pot onto the Graph entry in the Displays panel.
2. Mouse over the label for the x axis and popup the menu to select θ .
3. Raise the eccentric mass to the top, click the **Start** button, and release the disk. Stop the measurement after one complete oscillation.
4. Raise the eccentric mass to the top, but make sure it will fall on the opposite side, and repeat the procedure. Are the two potential wells symmetric? If not, adjust the string to symmetrize.

3.3 Measurements

Prepare graphs for θ vs. t , ω vs. θ (phase plot), and py vs. px (Poincaré plot). By double clicking on the name of the graph in the list at the left you can change its title to something suitable, which will make it easier to navigate to the graph you want to look at. You may also wish to create a frequency display by dragging the Data entry for frequency onto the Digits entry in the Displays panel. This display will allow you to check whether the Poincaré strobe agrees with the Compumotor frequency setting. Now, save the activity file.

A few last tips about graphs in Data Studio:

1. Graphs have an option called *Full Color*. When selected, each visible trace gets colored; when unselected, all traces are grayed except the selected trace.
2. You can use the “Smart Tool” to make measurements directly on a graph.
3. You can copy the selected trace to the clipboard. Both graphical and textual representations of the data are copied, so you can paste directly into a word processor document or into a data analysis program. A script is available to permit Igor Pro to load the data on the clipboard, preserving the run number from the Data Studio activity.

3.4 Transferring Data to Igor

As mentioned above, Igor handles large data sets much more gracefully than Data Studio does. I recommend transferring frequently and removing old data from the Data Studio ac-

tivity. You may view a short screencast illustrating the process at <http://www.physics.hmc.edu/igor/DSPaste.mov>. To set up Igor for easy data transfer,

1. Start a new Igor experiment, if necessary.
2. From the HMC menu select *Auto Data Studio Paste*.
3. Switch back to Data Studio (command-tab), bring forward a graph of θ vs. t whose data you wish to copy, and press Copy (command-C). This will copy the currently selected trace to the clipboard as text.
4. Switch back to Igor (command-tab). You should see a new graph with the data you just copied. If not, click on the small Data Studio Panel to bring it to the front.
5. Switch back to Data Studio and bring to the front the graph of ω vs. θ for the same run. Copy it and switch back to Igor.
6. If the phase plot isn't made automatically, click once again on the Data Studio Panel. Finally, go back and grab the data for the same run from the Poincaré plot.
7. I recommend keeping a log in the Igor experiment of the runs and their corresponding parameters. The Notes command (Command-4) is particularly convenient for this.

Potential Gotchas

Ideally, I could script Data Studio to manage the transfers automatically. Unfortunately, the program is not scriptable, nor will Pasco share the data file format they use. The code in the Igor Procedure file DS.ipf tries to manage as much of the drudgery on the Igor end as possible, so that you merely have to copy the data in Data Studio and switch the Igor, which should manage an automatic pasting operation, which sets the units of the data and prepares a plot. How does it manage this?

It is possible to attach a “hook” function to a window in Igor Pro. This function is called whenever window activity takes place, including when the window is activated (brought to the front). When you switch to Data Studio, the front Igor window is deactivated; when you switch back, it is activated again. If that window has the appropriate hook function installed, it can run the data pasting operation. The Data Studio Panel gets the hook, as does any of the graphs made by the pasting operation. So, if a paste doesn't take place automatically, bring one of these windows to the front.

1. Data Studio numbers runs sequentially from 1. If you delete a run, however, Data Studio reuses the number. Since it is a good strategy to keep the amount of data in Data Studio to a minimum, you will frequently be removing the last run from your Data Studio file, leading to run-number conflicts when you paste the data into Igor. To overcome this problem, when the Auto number box is checked, Igor will automatically assign to the new data the next available run number. If it is not checked, you will see a dialog box warning about the conflict and asking you to resolve it.

4. Modeling

A sample *Mathematica* notebook illustrating how to integrate the equations of motion for a system such as the chaotic pendulum is available from the course web page. No prior experience with *Mathematica* is required, in principle, but it won't hurt! Another possibility is to use Igor Pro to integrate the differential equations. Once again, head to the course web site for further information on this topic.

The Franck-Hertz Experiment

This experiment was performed by Franck and Hertz in 1914, following by one year Bohr's publication of the theory of the hydrogen spectrum. The Bohr theory, using Rutherford's nuclear atom, is based upon a mechanical model—an electron circling about a proton in a manner described by a new law of mechanics. The observations supporting the theory, and which necessitated a new description of atomic systems, were electromagnetic. Light is emitted and absorbed by atoms. The Bohr theory of hydrogen was a success because the energy difference between the various mechanical states of the electron-proton system corresponded, through the Einstein frequency condition $E = h\nu$, to observed frequencies of emitted and absorbed radiation. The Franck-Hertz experiment, on the other hand, was a direct mechanical confirmation of an essentially mechanical theory.

The optical spectrum of mercury vapor shows distinct emission and absorption lines corresponding to transitions between discrete energy levels of the mercury atom. Franck and Hertz found that discrete transitions of the mercury atom could also be produced by the inelastic scattering of electrons from the atom. Consider the system of an electron with some initial kinetic energy incident upon a mercury atom at rest in the ground state. If the electron energy is less than the energy required to excite the atom to its first excited state, the collision must be elastic. The kinetic energy of the electron-atom system cannot change. Due to the disparity of masses, the kinetic energy of the electron itself is essentially unchanged in the collision. If the electron energy equals or exceeds the energy for exciting the first level, however, the collision may in some cases be inelastic. The kinetic energy of the system is, in these cases, different after the collision than before. In an inelastic collision some of the initial kinetic energy is converted to potential or "excitation" energy of the atom. In due course, this energy is radiated from the excited atoms as light, but the primary interaction is one described in mechanical terms. Franck and Hertz observed such inelastic collisions by monitoring the current of electrons passing through a mercury vapor.

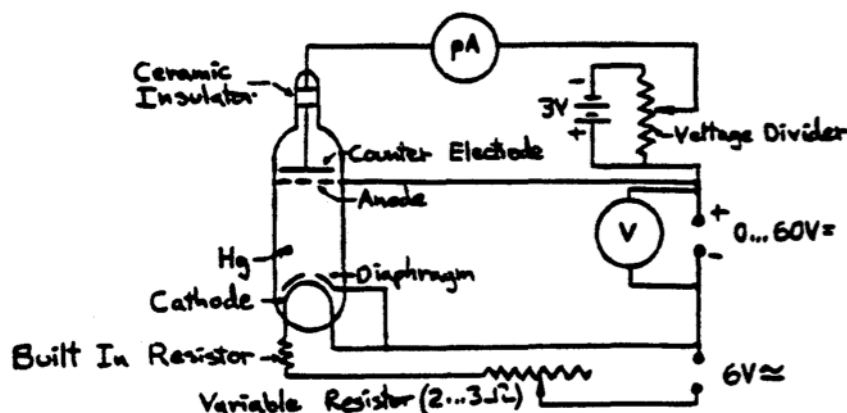


Figure 9.1: Franck-Hertz Electron Tube and Circuit.

1. Apparatus

The apparatus consists of a special electron tube containing a small quantity of mercury. The vapor pressure of mercury in the tube is adjusted by placing the tube in a furnace whose temperature may be varied. Electrons emitted from the cathode must, then, traverse a controlled mercury atmosphere in reaching the anode of the tube (see Fig. 9.1).

The anode is perforated so that many of the electrons will pass through it and collect on the counter-electrode.

Emission current from the cathode is controlled by the temperature of the cathode and by the potential applied to the anode. A diaphragm connected to the cathode limits the current and eliminates secondary and reflected electrons, making the electric field more uniform.

Electrons which pass through the hole in the diaphragm are accelerated through the mercury atmosphere by the positive potential applied to the anode. The counter-electrode is maintained at a potential of approximately -1.5 V with respect to the anode. Thus no electrons which pass through the perforated anode with energy less than 1.5 eV can reach the counter-electrode.

If the electrode and the cathode were of the same material and if all electrons were released from the cathode with zero kinetic energy, then the current of electrons collected by the counter-electrode would vary with the anode potential in the following way. No current would be observed until the potential of the anode exceeded 1.5 V . As the potential is increased, all electrons passing through the anode would reach the counter-electrode and the current would show a continuous increase with rising potential until a potential corresponding to the energy transition from the ground to the first excited state of mercury is reached. At this point the current would drop abruptly with increasing potential, since many electrons would make inelastic collisions with mercury atoms and have insufficient kinetic energy to reach the counter-electrode. If the potential were sufficiently increased, however, the elec-

trons would again reach the counter-electrode even after making an inelastic collision. This second increase in current would continue until the electrons gained enough energy to make two inelastic collisions, again not being left with enough kinetic energy to reach the counter-electrode. This would result in a second sharp drop in current. If the above conditions were satisfied, a succession of current maxima with sharp breaks would be observed with increasing potential.

The fact that the counter-electrode and the cathode may not be of the same material makes the first maximum an unreliable measure of the excitation potential of mercury. A so-called “contact” potential must be added to or subtracted from the observed potential. Evaluation of the contact potential is avoided by measuring potential differences between succeeding maxima. The fact that electrons are not emitted from the cathode with zero kinetic energy means that the actual energy distribution is superimposed upon that established by the anode potential. Sharp breaks are thus washed out of the current-voltage curve.

1.1 Electrical Circuit

The circuit employed in this experiment is shown in Fig. ?? All voltage sources indicated in the figure are located in a power supply which is connected to the furnace by means of various power cables. The measuring amplifier is a separate unit, but the microammeter which reads the counter-electrode current is mounted in the front of the power supply. A voltmeter (0–50 V) on the front of the power supply reads the anode potential, and a third small voltmeter is used for the bias voltage and the voltage across the filament. Filament current is supplied by a 6.3 V transformer and bridge rectifier. The anode and bias potentials are derived from dry cells mounted inside the power supply. All potentials are controlled by knobs on the front of the power supply. There are also two switches on the front of the power supply, the “POWER” switch and the “ANODE” switch. Filament potential alone is supplied with only the POWER switch on. The ANODE switch must be on for both anode and bias potentials.

2. Procedure

The oven in which the tube is mounted should be turned on immediately upon entering the laboratory. The temperature inside the oven is controlled by an external regulator. A thermometer protruding from the top of the cabinet provides a measurement of the temperature near the mercury tube. Close attention should be given to the reading of the thermometer so that a temperature of 180°C is at no time exceeded. By means of the thermostat control knob, the temperature of the oven should be adjusted initially to 170°C $\pm$ 5°C. Check the oven temperature every few minutes.

The Leybold measuring amplifier should also be turned on immediately so that it will have ample time to stabilize. As the temperature of the tube approaches its operating value, the power supply POWER switch should be turned on.

Set the sensitivity range of the measuring amplifier at 30×10^{-10} , turn the sensitivity control knob to its central position, and zero the microammeter by means of the “zero” control knob

on the amplifier. The input signal to the amplifier may be grounded at the time these settings are made, but the grounding connection on the front of the amplifier must be turned off, θ , before any current measurements can be made.

When the operating temperature is reached, **first make sure that the Anode voltage control knob is turned to the full counterclockwise (zero) position**; then turn the ANODE switch on.

Increase the filament voltage to the value suggested at your station. Slowly increase the anode potential to about 35 V. Watch the microammeter carefully. **If the current increases suddenly, an electrical breakdown has occurred in the tube, and the potential must be reduced to zero immediately.** If such a breakdown has occurred, reduce the filament voltage very slightly and again try to raise the anode potential to 35 V. (If the tube still breaks down, see your instructor.) Now very slowly increase the anode potential to 40 V and adjust the sensitivity knob to give a full scale deflection of the microammeter for the maximum current observed in the 30–40 V range. Allow time for the current to stabilize after each small adjustment and watch out for the onset of electrical breakdown.

The apparatus is now ready for taking measurements. Slowly decrease the anode potential and observe the microammeter deflection. Dips and peaks in the current should be obvious in the 0–40 V range. The sensitivity range of the amplifier may have to be changed in order to define the maxima clearly at the lower anode voltages. Record counter-electrode currents vs. anode potential. Note that small temperature changes will affect the current at a given voltage (why?), so monitor the oven temperature as you make your measurements. Plot your data and determine the excitation energy of mercury. Repeat your measurements while slowly increasing the anode voltage from 0 to 40 V. Repeat with different oven temperatures. (Do not exceed 180°C.) For each temperature you should reset the sensitivity knob following the original procedure. From all your data obtain your best estimate of the excitation energy of mercury.