

Physics 54
Modern Physics Laboratory

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1. Schedule

You will conduct four two-week experiments and extend one of the experiments for your technical report. The experiments will be done in pairs and in a rotation to be established after the Orientation meeting in the first week, in which you will be introduced to each experiment. The manual describes more experiments than are currently available. Some of those unavailable at the time this manual was prepared may become serviceable during the term. If so, we can use a lottery system to decide which teams may conduct them, if more than one team is interested.

Currently, the following experiments are working and available:

1. Rutherford scattering (3)
2. The Hall effect (4) + the photoelectric effect (7) (each lasts only one week)
3. Gamma radiation interactions (5)
4. Barrier penetration (6)
5. The Cavendish experiment (8)
6. Chaotic motion (9)

A vacuum system has been ordered for the Thermal Radiation experiment (Expt. 2), but it may not arrive before spring break. The Franck-Hertz experiment (Expt. 1) may replace the photoelectric experiment at some point during the semester, if the latter gets sick.

Week	Section 1 (Thurs.)	Section 2 (Fri.)
January 21–22	<i>Orientation</i>	<i>Orientation</i>
January 28–29	1A	1A
February 4–5	1B	1B
February 11–12	2A	2A
February 18–19	2B	2B
February 25–26	3A	3A
March 4–5	3B	3B
March 11–12	4A	Tech Report Data
March 18–19	<i>Spring Break</i>	<i>Spring Break</i>
March 25–26	4B	<i>Cesar Chavez Day</i>
April 1–2	Tech Report Data	Tech Report Work
April 5		Tech Report Due
April 8–9	Tech Report Work	4A
April 15–16	Tech Report Due	4B
April 22–23	<i>Free</i>	Revised Tech Report Due
April 29–30	Revised Tech Report Due	<i>Free</i>

2. General Instructions

In preparing for the first laboratory meeting for each experiment, you should carefully read the instructions for a scheduled experiment before reporting to the laboratory meeting. In many cases, you will also find it useful to do some background reading on the subject, in your Physics 52 textbook, in various sources found in the library, or on the internet. (Beware of material on the internet. Some of it is very good, but there is no guarantee that the information is relevant, useful, or correct!) You are encouraged to take notes on your reading and attach these notes in your lab book. Be sure to include references to your sources. While you may find some details of the instructions will have been changed in the lab (as we are constantly tweaking the equipment), complete familiarity with the objectives and general procedures of the experiment before the laboratory period will help you in working efficiently in lab.

Please observe the precautions emphasized in the laboratory instructions and appendices and accord the research-type equipment the respect it deserves. Note that much of the equipment in this laboratory is one-of-a-kind, delicate, difficult to repair, and expensive! Much of it can be damaged if used incorrectly. If you have any questions about how to use the equipment, be sure to ask the instructor before turning it on or starting a new procedure. Report any damaged equipment to your instructor immediately.

- Always bring your lab manual, a calculator, a (non-erasable) pen, and a laboratory notebook (brown-cover National 43-648 “Computation Notebook”) with you to lab.

Laboratory Notebook

You will follow the same general rules for your laboratory notebook that were used in Physics 53. The notebook will be an essential part of your laboratory work this year, and it should contain a running account of the work you do. *Entries should be made while the experiment is in progress*, and you should use a standard format. Your notebook should:

1. provide the reader with a table of contents at the beginning, page 1, listing the number and title of the experiment, the date or dates when it was done, the page numbers in the notebook, and the name of your partner (see below);
2. contain all pertinent information, schematic diagrams, observations, data, rough calculations, results, and conclusions. Think of your entries as being those in an informal diary or journal relating daily experiences.

In the laboratory, each experiment will be performed by a team of two investigators. Each person is responsible for the complete documentation of the work performed and its analysis. That is, while you may discuss the experimental results with your lab partner, *your analysis of the experiment should be done individually*. Remember, you will write a technical report based on one of the experiments, and therefore a complete record of your observations and conclusions is essential.

Computers are available for data gathering, plotting, and analysis. Do not alter the computer operating system or programs in any way. Any such tampering, even if intended to be harmless, is considered a serious offense.

There are some general rules for making entries in your laboratory notebook:

1. Use permanent ink, not pencil or erasable ink.
2. Do not use scratch paper—all records must be made directly in the notebook. (Left-hand pages may be used for scratch work.)
3. Do not erase or use “white out”—draw a single line through an incorrect entry and write the correct value nearby. Apparent errors sometimes later prove to be important.
4. Record data in tabular form when possible with uncertainties and give units in the heading of each column.
5. Define all symbols used in diagrams, graphs, and equations.
6. Determine the uncertainties in your data and results as you go, and let the calculations determine the number of measurements needed.
7. Record qualitative observations as well as numbers and diagrams.
8. Take care in drawing graphs—a sloppy graph or one drawn to an inadequate scale is next to useless. Graphs often dictate what you should do next. Try to determine the probable range of your data before starting your graph. Draw graphs in your notebook as the experiment proceeds to help record and understand the experiment in real time. You will often find it useful to use the computer to generate graphs while you are taking data as well as when you are doing final analysis or fitting.

Append *clearly and fully labelled* computer-generated or hand-drawn graphs securely to the notebook pages. Describe these graphs in your narrative. Place the graphs in your lab books as close as possible to the relevant data, preferably on left-hand pages facing the corresponding data tables. Include error bars and properly weighted fits to the data wherever possible. Also include plots of residuals (with appropriate error bars) where applicable.

9. Do not fall into the habit of recording only your data in lab, leaving blank pages or spaces for description and calculation to be finished later. Entries should be made in order corresponding to the work you are doing, much like a diary report, although complicated computations and analyses are usually undertaken after the data taking procedures have been completed.
10. While not everyone can produce a showcase-type notebook, your work should be as neat and orderly as possible. Sloppiness and carelessness cannot be overlooked even when the results are good.
11. Your notebook will be a success if you or a colleague could use it as a guide in repeating or expanding upon the particular experiment at a much later date.

Because your success in the laboratory will depend to a large extent upon your notebook and the write-up you produce from it, the following specific instructions for your notebook are provided.

Notebook The physics laboratory books (National 43-648 “Computation Notebook”) are bound notebooks ruled horizontally and vertically into squares. Leave two pages at the beginning of the notebook for a table of contents. Pages are numbered in the upper right-hand corner, beginning with the first page in the book. Put your table of contents on the last page of the notebook, but start your lab entries on the first page. When the table of contents meets the actual contents, somewhere in the middle, the notebook is complete. The table of contents should contain the following column headings:

Expt. #	Title	Partner	Dates	Pages	Grade
Expt. 1	Delta Radiation	Marge Innovera	1/19–1/26/10	3–13	

You should date each page of your notebook. Also date any entry added after lab.

Description of Intent and Proposed Procedure Head each experiment with the material used in the Table of Contents. Begin with a brief statement of the purpose of the experiment and a brief outline of the procedure your intended procedure. Two or three sentences should be enough. There is no need to copy that contained in the laboratory manual.

Please note that you do not always have at the beginning all the information you need to prepare a full description of purpose or procedure. The objectives may be poorly defined at the start and become crystallized only in the final stages of the experiment. Your notion about how to proceed may change after you have proved something else. For these reasons we are reluctant to prescribe very definite rules about laboratory work and laboratory records

Sketch of Apparatus Whenever practical, include a large, clear drawing, sketch, and/or block diagram of your experimental arrangement, to scale when necessary. Indicate clearly on the sketch critical quantities such as dimensions, volumes, masses, etc. Avoid excessive detail; include only essential features. Record the manufacturer, model name or number, and HMC identification number and, if appropriate, the accuracy of all apparatus you use, as it may be essential that you get the *same* apparatus later, or someone else may wish to compare their results with yours.

Preliminary Data Frequently there are certain preliminary data that must be measured or looked up in the tables and recorded, such as switch settings on equipment. Collect such information in a single table in your notebook and devote a separate line to each quantity. Include units and uncertainty where relevant.

Tabulation of Data Put data in tabular form where practical. The easiest and most frequently used procedure is to organize the data in clear tabular form, leaving empty columns if necessary for calculated results that will be made later. This requires advanced planning. At the head of each column should appear a symbol or notation for the quantity that is to be recorded in that column, with the units in which this quantity is measured in parentheses, thus T (K) or E (mV). This avoids the necessity of writing the unit after each entry. Always use one self-consistent system of units. Uncertainties must be included for all measurements (unless you’re told otherwise). If the uncertainties vary from datum to datum, each should be followed by its own uncertainty. Uncertainties should also be listed at the top of each column if possible. Thus, T ($^{\circ}\text{C}$) ($\pm 0.1^{\circ}\text{C}$), E (mV) (± 0.2 mV), wt (g) (± 0.2 mg)

Computation and Results After the data are recorded, there will generally be some calculations to be made. Make calculations as you go along to verify that your data is giving reasonable results; do not postpone all calculations until the final week of the experiment. If the data are all treated by some standard procedure, describe the procedure briefly for each calculation, giving any formulas that are to be used. (Define any quantities appearing in the formula that have not previously been defined.) Always give a sample calculation, starting with the formula, substituting experimental numbers, and carry the numerical work down to the result. DO NOT SUBMIT a computer program in lieu of a sample calculation. Your results should be compared with accepted values if possible. Information from a handbook or any reference source must be identified by book and page.

Summary of the Experiment At the end of an experiment you should type a summary of one to two pages containing a concise discussion of important points of the experiment, including the purpose, theoretical predictions you are testing (if appropriate), a brief outline of experimental methods, results, analysis, and conclusions. There is no need to put in detailed procedural discussions unless they bear directly on understanding some aspect of the data. In discussing results, you can make references (with page numbers) to specific entries—such as tables, figures, and calculations—in your lab notebook. Be sure to refer to or include in your summary well-documented graphs of important findings. Discuss the major sources of random and systematic errors, including possible methods of reducing them. If you have a hunch about the source of a discrepancy, make some order of magnitude estimates, make some approximations, and check quantitatively to see if hunch could reasonably explain the discrepancy. If relevant, compare your results with theoretical predictions. Your results should be as quantitative and precise as possible. Your conclusions should also be as specific as possible, given your experimental results and analysis. This summary should be attached to the end of your lab notebook write-up; it is a significant part of the record of your experiment.

Honor System All work that is handed in for credit in this course, including laboratory reports, is regulated by the Harvey Mudd Honor Code, which is described in general terms in the student handbook. In application, this Code means simply that all work submitted for credit shall be your own. You should not hesitate to consult texts, the instructor, or other students for general aid in the preparation of laboratory reports. However, you must not transcribe another student's work without direct credit to him or her, and you must give proper credit for any substantial aid from outside your partnership. Again remember that while you may discuss the experiment with your lab partner, your analysis of the experiment should be done individually.

The Franck-Hertz Experiment

This experiment was performed by Franck and Hertz in 1914, following by one year Bohr's publication of the theory of the hydrogen spectrum. The Bohr theory, using Rutherford's nuclear atom, is based upon a mechanical model—an electron circling about a proton in a manner described by a new law of mechanics. The observations supporting the theory, and which necessitated a new description of atomic systems, were electromagnetic. Light is emitted and absorbed by atoms. The Bohr theory of hydrogen was a success because the energy difference between the various mechanical states of the electron-proton system corresponded, through the Einstein frequency condition $E = h\nu$, to observed frequencies of emitted and absorbed radiation. The Franck-Hertz experiment, on the other hand, was a direct mechanical confirmation of an essentially mechanical theory.

The optical spectrum of mercury vapor shows distinct emission and absorption lines corresponding to transitions between discrete energy levels of the mercury atom. Franck and Hertz found that discrete transitions of the mercury atom could also be produced by the inelastic scattering of electrons from the atom. Consider the system of an electron with some initial kinetic energy incident upon a mercury atom at rest in the ground state. If the electron energy is less than the energy required to excite the atom to its first excited state, the collision must be elastic. The kinetic energy of the electron-atom system cannot change. Due to the disparity of masses, the kinetic energy of the electron itself is essentially unchanged in the collision. If the electron energy equals or exceeds the energy for exciting the first level, however, the collision may in some cases be inelastic. The kinetic energy of the system is, in these cases, different after the collision than before. In an inelastic collision some of the initial kinetic energy is converted to potential or “excitation” energy of the atom. In due course, this energy is radiated from the excited atoms as light, but the primary interaction is one described in mechanical terms. Franck and Hertz observed such inelastic collisions by monitoring the current of electrons passing through a mercury vapor.

1. Apparatus

The apparatus consists of a special electron tube containing a small quantity of mercury. The vapor pressure of mercury in the tube is adjusted by placing the tube in a furnace whose temperature may be varied. Electrons emitted from the cathode must, then, traverse a controlled mercury atmosphere in reaching the anode of the tube (see Fig. 1.).

The anode is perforated so that many of the electrons will pass through it and collect on the

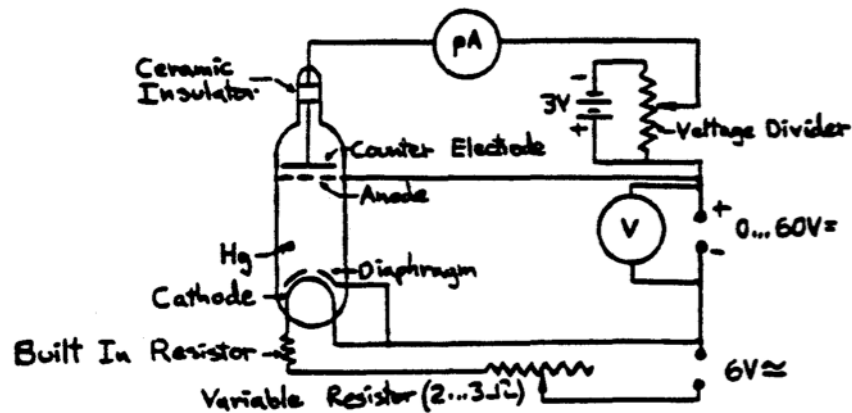


Figure 1.1: Franck-Hertz Electron Tube and Circuit.

counter-electrode.

Emission current from the cathode is controlled by the temperature of the cathode and by the potential applied to the anode. A diaphragm connected to the cathode limits the current and eliminates secondary and reflected electrons, making the electric field more uniform.

Electrons which pass through the hole in the diaphragm are accelerated through the mercury atmosphere by the positive potential applied to the anode. The counter-electrode is maintained at a potential of approximately -1.5 V with respect to the anode. Thus no electrons which pass through the perforated anode with energy less than 1.5 eV can reach the counter-electrode.

If the electrode and the cathode were of the same material and if all electrons were released from the cathode with zero kinetic energy, then the current of electrons collected by the counter-electrode would vary with the anode potential in the following way. No current would be observed until the potential of the anode exceeded 1.5 V. As the potential is increased, all electrons passing through the anode would reach the counter-electrode and the current would show a continuous increase with rising potential until a potential corresponding to the energy transition from the ground to the first excited state of mercury is reached. At this point the current would drop abruptly with increasing potential, since many electrons would make inelastic collisions with mercury atoms and have insufficient kinetic energy to reach the counter-electrode. If the potential were sufficiently increased, however, the electrons would again reach the counter-electrode even after making an inelastic collision. This second increase in current would continue until the electrons gained enough energy to make two inelastic collisions, again not being left with enough kinetic energy to reach the counter-electrode. This would result in a second sharp drop in current. If the above conditions were satisfied, a succession of current maxima with sharp breaks would be observed with increasing potential.

The fact that the counter-electrode and the cathode may not be of the same material makes the first maximum an unreliable measure of the excitation potential of mercury. A so-called “contact” potential must be added to or subtracted from the observed potential. Evaluation of the contact potential is avoided by measuring potential differences between succeeding maxima. The fact that electrons are not emitted from the cathode with zero kinetic energy means that the actual energy distribution is superimposed upon that established by the anode potential. Sharp breaks are thus washed out of the current-voltage curve.

1.1 Electrical Circuit

The circuit employed in this experiment is shown in Fig. 1. All voltage sources indicated in the figure are located in a power supply which is connected to the furnace by means of various power cables. The measuring amplifier is a separate unit, but the microammeter which reads the counter-electrode current is mounted in the front of the power supply. A voltmeter (0–50 V) on the front of the power supply reads the anode potential, and a third small voltmeter is used for the bias voltage and the voltage across the filament. Filament current is supplied by a 6.3 V transformer and bridge rectifier. The anode and bias potentials are derived from dry cells mounted inside the power supply. All potentials are controlled by knobs on the front of the power supply. There are also two switches on the front of the power supply, the “POWER” switch and the “ANODE” switch. Filament potential alone is supplied with only the POWER switch on. The ANODE switch must be on for both anode and bias potentials.

2. Procedure

The oven in which the tube is mounted should be turned on immediately upon entering the laboratory. The temperature inside the oven is controlled by an external regulator. A thermometer protruding from the top of the cabinet provides a measurement of the temperature near the mercury tube. Close attention should be given to the reading of the thermometer so that a temperature of 180°C is at no time exceeded. By means of the thermostat control knob, the temperature of the oven should be adjusted initially to 170°C $\pm$ 5°C. Check the oven temperature every few minutes.

The Leybold measuring amplifier should also be turned on immediately so that it will have ample time to stabilize. As the temperature of the tube approaches its operating value, the power supply POWER switch should be turned on.

Set the sensitivity range of the measuring amplifier at 30×10^{-10} , turn the sensitivity control knob to its central position, and zero the microammeter by means of the “zero” control knob on the amplifier. The input signal to the amplifier may be grounded at the time these settings are made, but the grounding connection on the front of the amplifier must be turned off, θ , before any current measurements can be made.

When the operating temperature is reached, **first make sure that the Anode voltage control knob is turned to the full counterclockwise (zero) position**; then turn the ANODE switch on.

Increase the filament voltage to the value suggested at your station. Slowly increase the anode potential to about 35 V. Watch the microammeter carefully. **If the current increases suddenly, an electrical breakdown has occurred in the tube, and the potential must be reduced to zero immediately.** If such a breakdown has occurred, reduce the filament voltage very slightly and again try to raise the anode potential to 35 V. (If the tube still breaks down, see your instructor.) Now very slowly increase the anode potential to 40 V and adjust the sensitivity knob to give a full scale deflection of the microammeter for the maximum current observed in the 30–40 V range. Allow time for the current to stabilize after each small adjustment and watch out for the onset of electrical breakdown.

The apparatus is now ready for taking measurements. Slowly decrease the anode potential and observe the microammeter deflection. Dips and peaks in the current should be obvious in the 0–40 V range. The sensitivity range of the amplifier may have to be changed in order to define the maxima clearly at the lower anode voltages. Record counter-electrode currents vs. anode potential. Note

that small temperature changes will affect the current at a given voltage (why?), so monitor the oven temperature as you make your measurements. Plot your data and determine the excitation energy of mercury. Repeat your measurements while slowly increasing the anode voltage from 0 to 40 V. Repeat with different oven temperatures. (Do not exceed 180°C.) For each temperature you should reset the sensitivity knob following the original procedure. From all your data obtain your best estimate of the excitation energy of mercury.

Thermal Radiation

1. General

The apparatus for study of the propagation of thermal energy by radiation is shown schematically in Fig. 2.1

Any surface at a temperature above absolute zero radiates energy in the form of electromagnetic radiation. When the surface is in a vacuum, as in this case, radiation is the only mechanism for energy loss. One imagines that the surface of [A] is at some temperature T and asks at what rate W thermal energy is radiated from the surface. Since [C] is a vacuum, the rate can depend only upon the nature of the surface and its temperature. The surface has been blackened in this experiment to eliminate any effect due to the nature of the surface other than its area A . The rate of energy loss may be expressed as

$$P = \sigma AT^x \quad (2.1)$$

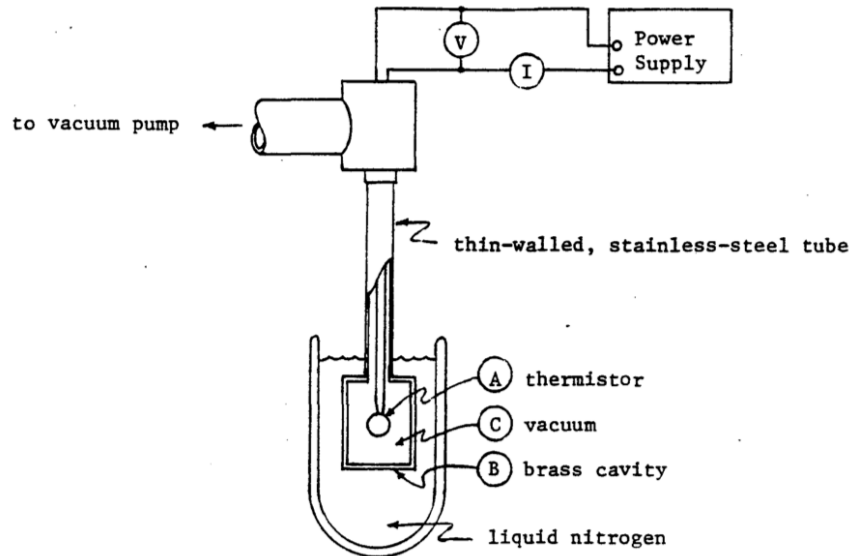


Figure 2.1: Thermal radiation apparatus.

The universal constant σ is called the Stefan-Boltzmann constant. We are to determine σ and the exponent x relating W to the surface temperature.

The interior surface of the brass cavity [B] also radiates thermal energy to [A]. This surface, however, is maintained at a sufficiently low temperature (77 K) by contact with liquid nitrogen that the energy received by [A] from [B] is negligible. Further, [A] is suspended by wires of very small cross-section to reduce conduction of energy to [A] from the room. In equilibrium, then, the rate at which energy is radiated through the vacuum from [A] must equal the rate at which it is generated electrically within [A].

The radiating element [A] in this experiment is a thermistor. Thermal energy (in watts) is generated within it at a rate given by

$$P = VI \quad (2.2)$$

where V (in volts) and I (in amperes) are measured by the meters “V” and “I” shown. The resistance R of a thermistor varies as a function of its temperature. Since

$$R = V/I \quad (2.3)$$

measurement of V and I determines not only P but R as well.

2. Thermistor calibration

The conductivity of a thermistor (semiconductor) is proportional to the Boltzmann factor, $\exp(-E/kT)$, where T is the temperature (in kelvins), k the Boltzmann constant (8.616×10^{-5} eV K⁻¹) and E the “band-gap” energy of the semiconductor—the energy which must be acquired by an electron for it to participate in electrical conduction. The resistivity is the reciprocal of the conductivity so that the thermistor resistance may be expressed as

$$R = ae^{b/T} \quad (b = E/k) \quad (2.4)$$

The two constants, a and b , are determined by measurement of R at two different temperatures. Thus, if R_1 and R_2 are the resistances of the thermistor at temperatures T_1 and T_2 , then

$$\ln(R_1/R_2) = b \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \quad a = R_1 e^{-b/T_1} = R_2 e^{-b/T_2} \quad (2.5)$$

Calibration is performed with atmospheric pressure in the apparatus to hasten equilibrium. First measure R at room temperature as indicated by a thermometer. The following circuit is used with a large (10 k Ω) series decade resistor to limit the power dissipation in the thermistor to the order of microwatts: The leads of one voltmeter are connected to points [1] and [3] to give the potential across the thermistor. The second voltmeter reads the voltage drop across the decade resistor (points [2] and [3]), from which you can determine the current in the thermistor.¹ Use the potential and current to determine R_1 at room temperature. Then surround the thermistor bath to obtain R_2 at $T_2 = 273$ K. (Record the potential across the 10 k Ω resistor at one- or two-minute intervals to determine when equilibrium at ice temperature is reached.) Determine a and b (and E) from these data and equations Eq. (2.2). Equation (2.1) is then used to determine T in the experiment as written.

¹The 1 k Ω resistor between the power supply and the thermistor is included to protect the thermistor from damage when the decade resistance is changed.

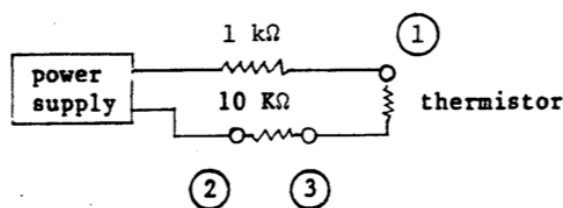


Figure 2.2: Thermistor circuit.

As soon as these data are obtained, have the instructor start the rough and molecular drag vacuum pumps. Approximately 30 minutes are required to reach the operating pressure of a few times 10^{-5} Torr. Use this time to analyze your calibration measurements and determine the constants a and b .

3. Experimental procedure

The same circuit used to calibrate the thermistor is used to measure V and I (and thus R and P) in the remainder of the experiment **except that** the series resistor is reduced from $10\text{ k}\Omega$ to $1\text{ k}\Omega$.

When a sufficient vacuum is obtained, establish a current of $\sim 10\text{ nA}$ just before immersing the brass cavity in liquid nitrogen. (If power is not applied to the thermistor before cooling, it will rapidly cool after immersion to a resistance so high that sufficient power cannot be applied to control its temperature.) Slowly raise the dewar of liquid nitrogen to cover the brass cavity. The current through the thermistor will drop a bit as its resistance increases. Wait several minutes for the temperature of the thermistor to stabilize. Record several voltage readings during this equilibration period to document the approach to equilibrium. Determine the thermistor temperature and the power radiated by the thermistor at equilibrium.

Vary the current slightly and again wait for equilibrium. Calculate T and P . Proceed in this way to generate data for P vs. T over a temperature range of approximately 0°C to $50\text{--}60^\circ\text{C}$.

The exponent x is most easily found by using a log-log plot of equation (1):

$$\log P = \log(\sigma A) + x \log T \quad (2.6)$$

The slope of the plot $\log P$ vs. $\log T$ yields the exponent in the “Stefan-Boltzmann radiation law.” Determine this law from your data and evaluate the Stefan-Boltzmann constant σ , given $A = 0.52\text{ cm}^2$. Think carefully about the best way to get a value for σ .

Rutherford Scattering

By 1911 general agreement existed that atoms contain a small number of electrons with most of the atomic mass associated with positive charge. The problem was to determine how the positive charge and mass are distributed. Two extreme views were proposed by J. J. Thomson and Ernest Rutherford. Thomson considered the atom to be made of a space filling sphere of positive charge in which the electrons were embedded—the “plum pudding” model. Rutherford considered the positive charge and mass to be contained within a central, very dense nucleus—the “nuclear atom” model.

The test of these views was suggested by Rutherford and carried out by H. Geiger and E. Marsden in 1913. The experiment is the prototype for a great many contemporary “particle experiments” of the so-called “scattering” type. Experiments by Hofstadter, *et al.*, on the special distribution of charge within the nucleus itself are of this type. The experimental procedure is to send known particles (known mass, charge, etc.) with a given momentum into a thin target of the material under investigation and to observe the scattering (the change of momentum) of the emergent beam. Given any model of the target such that the forces arising between the particle and the target are known, the expected scattering can be calculated. The observed scattering then serves to eliminate those models for which the predictions disagree with experiment. Rutherford’s particles were alpha particles of relatively low energy arising in natural radioactive decay. Since only electromagnetic forces are significant in this case, the experiments served to eliminate models of the positive charge distribution in an atom. The plum pudding model was definitely crossed off. The nuclear atom model, on the other hand, predicted results in very good agreement with the data.

1. The Rutherford model with which the results of this experiment are compared is that of a positive charge distribution which is represented as a point charge of magnitude Ze , where Z is the atomic number of the target material. The mass distribution was considered to be the same as that of the charge or, at any rate, the center of mass was assumed rigidly attached to the point charge. The predicted angular distribution of particles of mass m and charge $Z'e$ scattered from an incident beam of particles with velocity v by atoms of atomic number Z and mass M initially at rest is

$$\sigma(\theta) = \left(\frac{ZZ'e^2}{8\pi\epsilon_0\mu v_0^2} \right)^2 \frac{1}{\sin^4(\theta/2)} \quad \mu = \frac{mM}{m+M} \quad (3.1)$$

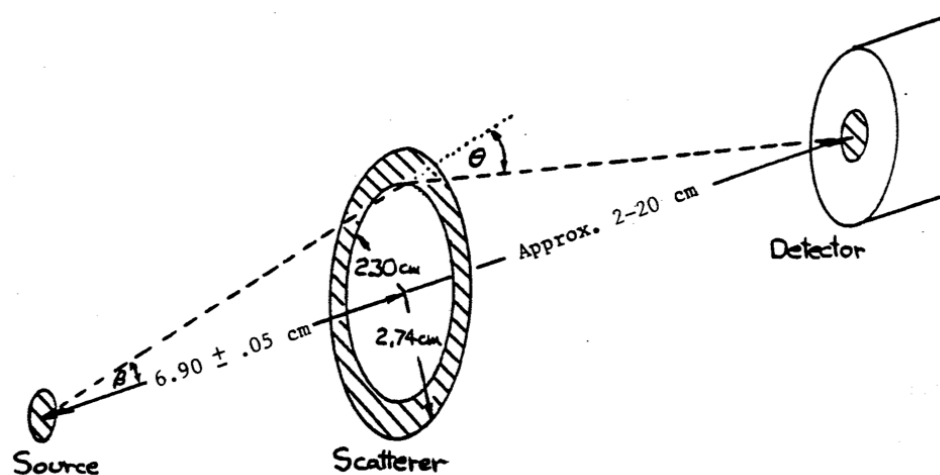


Figure 3.1: Scattering geometry.

The “cross section,” $\sigma(\theta)$, is a measure of the probability that an incident particle will be scattered in a single collision into the angle θ to $\theta + d\theta$ measured with respect to the direction of the initial velocity.

A sketch of the relation of the source, scatterer, and detector of the alpha particles in the laboratory apparatus is shown in Fig. 3.1. The apparatus may be disassembled at the flanged end by removing the four knurled nuts. **First**, however, read the following description. The source and scatterer are mounted together in a movable cage such that the angle β is fixed. The source is radioactive americium $^{241}_{87}\text{Am}$, which decays primarily by emitting a 5.29-MeV alpha particle.¹ The scatterer is an annulus of gold foil about $3.5 \mu\text{m}$ thick. Neither the americium source nor the gold foil may be touched, for obvious reasons.

The detector is a solid-state device consisting of a silicon wafer with a thin ($0.02 \mu\text{m}$) gold surface covering on one side and an aluminum surface on the other side. A potential difference of 30 V is placed across this “sandwich.” When an ionizing particle passes through the silicon, electrons are ejected by collision with the particle from the filled band to the empty conduction band of the silicon semiconductor. Both the electrons in the conduction band and the “holes” left in the valence band move under the applied field: the electron to the gold surface, the holes to the aluminum. Hence a pulse of charge is collected, with size proportional to the number of electrons injected into the conduction band, and thus to the energy loss of the ionizing particle in the silicon. You will count this pulse of charge with a scaler after it is amplified. Further details are given in the appendix to this experiment.



Do not touch the detector! The gold coating is fragile, the silicon can be ruined by contamination, and static electricity could damage the detector irreversibly.

The distance from the scattering foil to the detector may be varied from about 1 to 20 cm by means of the vacuum sealed plunger attached to the source cage and extending outside the

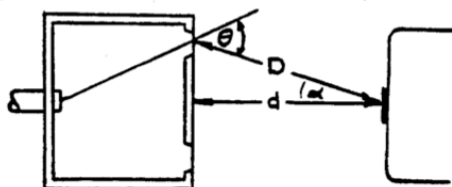
¹A thin cover over the radioactive source reduces the energy of the alpha particle somewhat.

apparatus. The scattering angle θ may thus be varied from about 27° to 90° .

Since the range of the alpha particles in air at normal pressure is only a few centimeters, it is necessary to evacuate the entire apparatus. The brass vacuum chamber is closed at one end by the sliding plunger and flange. The other end is closed by the mounting bracket of the detector seated against an O-ring seal.

Current pulses from the silicon detector generate voltage pulses in the amplifier circuit. These pulses are counted by a scaler. The experiment consists in determining the number of counts registered by the scaler in a measured time interval as the source cage plunger is moved in or out to vary the scattering angle θ .

2. Carefully study the apparatus prior to its evacuation. You will be given the minimum value of d (i.e., when the plunger is in as far as possible) for your apparatus. You will need this value together with your measurements of the external position of the plunger to compute the scattering angle θ and to correct for changes in the detector solid angle (see below). Begin collection of data with the plunger withdrawn as far as possible to measure the counting rate for the smallest scattering angle. Record the time necessary to accumulate at least 100 counts at all scattering angles.² The standard deviation for N counts is $\sqrt{N}$ so that 10% statistics are obtained with 100 counts. The counting rate at minimum scattering angle will probably be of the order of 30 counts per minute, falling to some 4 counts per minute at the largest angles.



The counting rate must be corrected for the change in the solid angle subtended by the detector at the gold annulus. The apparent size of the detector as seen from the annulus is a function of their separation, d . Ignoring the finite size of the detector and the annulus width, this correction consists of two factors. First, the detector size would vary as $1/d^2$ were it viewed “head on” from the annulus. This is very nearly the case when d is much greater than the radius of the annulus. For small separations, however, the projection of the detector into the line of sight from the annulus must be taken into account. The projected area goes as $\cos \alpha$, or as d/D . Combining these two factors, the apparent size of the detector varies as $\frac{d}{D} \frac{1}{D^2}$.

The counting rate is multiplied by the reciprocal of this factor to obtain a counting rate proportional to that which would have been measured with a detector whose size appeared always the same to the scattering annulus. The counting rate corrected for solid angle is proportional to the cross section $\sigma(\theta)$.

To compare your results with the predictions of the Rutherford model, plot the logarithm of the corrected counting rate vs. the logarithm of $\sin(\theta/2)$. (What should this plot look like

²In taking data, choose intermediate plunger positions in light of the plot you will be making. (See below)

according to the Rutherford model?) Enter your data in this plot with bars to indicate the standard deviations resulting from counting statistics.

1. The EGG Ortec Silicon Charged Particle Detector

Silicon is a semiconductor with a gap of 1.1 electron volts between the top of the filled band and the bottom of the (nearly empty) conduction band. At any temperature above absolute zero, some electrons will have enough thermal energy to reach the conduction band; for the detector you use, with 30 volts potential difference across the silicon wafer, this gives rise to a “dark current” of about 300 nA or 1×10^{12} electrons/second. (Incidentally, since the silicon wafer is about $150 \mu\text{m}$ thick, the electric field is $30 \text{ V} / 1.5 \times 10^{-4} \text{ m} = 200,000 \text{ V/m}$.)

When there is no voltage across the silicon wafer, the Fermi energies (see Eisberg and Resnick, Chapter 13, pp. 507 *et seq.*) of the electrons in the gold, aluminum, and silicon are equal; electrons move between these layers to change the potential of these layers until this equality is reached. The particular silicon wafer we use has donor impurities (see E & R, p. 507), so the Fermi energy in the silicon lies 0.16 eV below the bottom of the conduction band. There are, accordingly, thermally injected electrons in the conduction band. Once the 30 V power supply is turned on, these electrons are swept away, giving rise to the “dark current.”

When an α particle enters the silicon, it collides with electrons in the silicon lattice, giving many of them enough energy to reach the conduction band. The average energy lost by the α -particle to create an electron-hole pair is measured to be 3.6 eV. Thus a 5 MeV α -particle, completely stopped in the silicon, gives rise to $5 \times 10^6 / 3.6 = 1.4 \times 10^6$ electron-hole pairs, or 2.2×10^{-13} coulombs. The capacitance of the detector is 70 picofarads ($7 \times 10^{-11} \text{ F}$), so collecting this charge causes a voltage change $\Delta V = (2 \times 10^{-13} \text{ C}) / (7 \times 10^{-11} \text{ F}) = 3 \text{ mV}$. The detector voltage is supplied through a 20-M Ω resistor, so the recovery time is $RC = (7 \times 10^{-11} \text{ F}) 2 \times 10^7 \Omega = 1.4 \text{ ms}$.

Figure 3.2 shows a circuit diagram of the detector, its power supply, and the first (preamplifier) stage of amplification. Here R_L is the “equivalent resistance” of the silicon wafer; since the “dark current” is about $3 \times 10^{-7} \text{ A}$ for a potential of 30 V, $R_L = 100 \text{ M}\Omega$. The Model 109A preamplifier set at $10\times$ gain gives a pulse of 150 mV/MeV for a Si detector. This preamp also reduces the pulse width to approximately $50 \mu\text{s}$. The amplifier following the preamp further reduces the pulse width

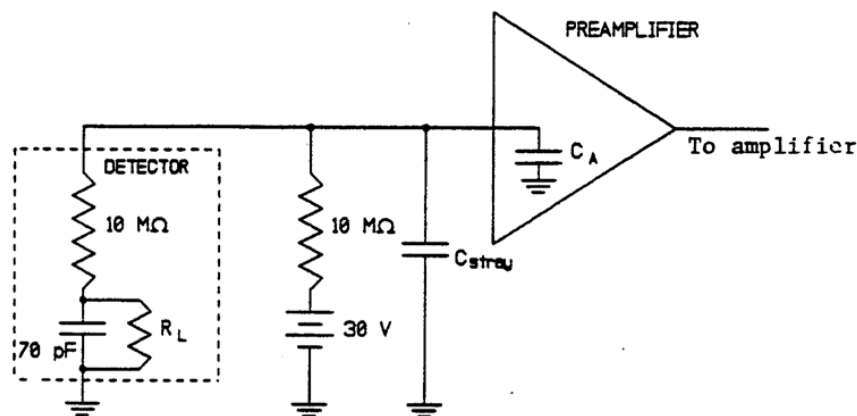


Figure 3.2: Circuit

and increases the peak voltage.

2. Addendum

Since the laboratory notes were written, the brass cylinder has been replaced by a plastic (Lexan) cylinder, which is semitransparent and slightly shorter. Accordingly, the angle of scattering of the alpha particles must be calculated with the new dimensions. The dimension you need to know is d Page 3-3 of the notes. d the distance from the plane of the gold scattering foil to the surface of the detector. It is also the distance between the two knurled knobs, one the handle on the plunger and the other the one the rod slides through, less 0.08 cm. We have placed a sleeve of length 2.00 cm on the plunger rod (to keep a bump on the can carrying the americium source and the foil from striking the detector), and therefore the shortest d available to you is 1.92 cm.

Why the Lexan cylinder, and these changes?

For reasons I did not understand, this experiment usually produced an exponent in the range of -4.3 to -4.5 instead of the -4 which Professor Rutherford had in mind. Mark Chalice, '94, asked me two years ago if any of the alpha particles which go through the foil undeflected might then strike the brass cylinder wall and be scattered there. Indeed, most of the alpha particles that strike the foil do go through essentially undeflected, having lost some energy by many collisions with electrons, thus ionizing gold atoms. These alpha particles enter the brass. Most spend out their range losing energy in more electron collisions, but a few of them may indeed be scattered by the copper and zinc nuclei of atoms which make up brass. From the cross-section equation on Page 3-1 of the notes, we see that the scattering cross section depends on Z^2 . For gold, $Z = 79$; for copper, $Z = 29$, and for zinc, $Z = 30$. Accordingly, these brass nuclei are only about 14% as effective as gold in Rutherford scattering, but the path length in the brass can be considerable. From the same equation on Page 3-1, we also learn that as the alpha particle slows down, the probability of scattering increases. Thus the cylinder walls in front of the gold foil may constitute a significant second scatterer. The angle of scattering at which the alpha particle is detected is greater for these brass-scattered alphas, and hence they are no longer much detected as the foil nears the detector. Accordingly, we are led to believe that the power law is greater than 4.

The solution to the problem is not to use brass, but a plastic, for which the atoms in the wall are predominately carbon, hydrogen and oxygen. These have at most 1% of the scattering cross section of gold. Essentially all the alpha particles striking the wall lose their kinetic energy through electron collisions and are not scattered.

J. B. Platt, January 1994

The Hall Effect

In 1879, American physicist Edwin Herbert Hall (1855–1938) observed a small potential difference across a conducting sample through which a current flowed perpendicular to an applied magnetic field. In this experiment you will study this phenomenon, called the Hall effect, in a semiconductor. Measurements of the Hall potential will yield the sign and density of the charge carriers in the semiconductor.

1. Theoretical Background

The charge carriers that conduct electricity in metals are electrons. If a metal strip is placed in a magnetic field and a current is established in the strip, then a small transverse electric field is set up across the strip. The resulting difference of potential is the Hall potential. Note that this potential is perpendicular to both the current flow and the magnetic field (see Fig. 4.1). When semiconducting materials are used in place of the metal, the Hall potential is generally much larger and may be of opposite sign. The change in sign implies that, in such cases, the charge carriers are positive and that a different conduction process is occurring than in metals.

Two quantities are of interest here. First, the sign of the Hall potential, which depends on the sign of the charge carriers. Second, the magnitude of the potential, from which may be deduced the density of charge carriers. This deduction is briefly the following: Let E_H be the transverse electric field generated in the strip carrying a current of density j in a field B in the geometry shown above. The quantity

$$R = \frac{E_H}{jB}$$

is called the Hall coefficient. In equilibrium, the transverse force of qE_H acting on the charge carriers of charge magnitude q must just compensate the Lorentz force of qvB acting upon these charges

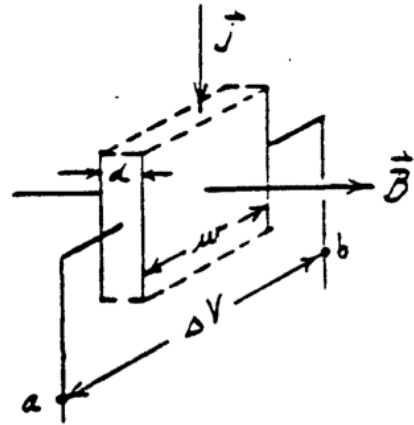


Figure 4.1: The Hall effect.

moving with drift velocity v . Since also $j = qnv$, where n is the density of charge carriers, we have

$$R = \frac{E_H}{jB} = \frac{vB}{qnvB} = \frac{1}{qn}$$

For both metals and semiconductors, it turns out that $|q| = e$, the magnitude of the charge of an electron. Thus n may be calculated from measured values of R .

For a strip of width w and thickness d , carrying a uniform current density j , the conduction current is $I = jwd$.

The Hall potential is $\Delta V = E_H w$. Thus,

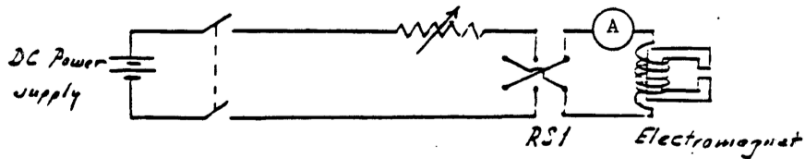
$$R = \frac{\Delta V}{I} \frac{wd}{B} = \frac{\Delta V}{IB} d \quad (4.1)$$

2. Measurement of the Hall Potential

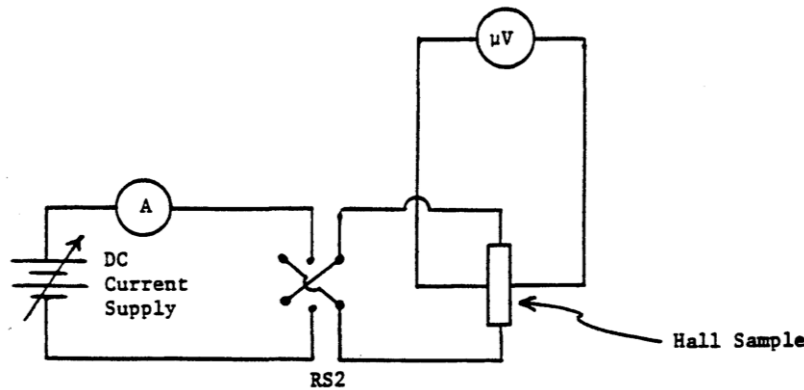
Circuits Figs. 4.2(a) and 4.2(b) show the circuits used in obtaining the Hall potential. Fig. 4.2(a) shows the circuit used to produce the required magnetic field and Fig. 4.2(b) shows the Hall effect circuit itself.

Concerning these figures, note:

- The Hall effect requires both a current I passing through the sample and a magnetic field. The magnetic field is not shown in Fig. 2b, but the sample is oriented in the magnet to give the geometry shown in Fig. 4.1.



(a) Electromagnet circuit



(b)

Figure 4.2: Circuits

- Two reversing switches (RS_1 and RS_2) are present in the circuits, one to reverse the current in the magnet, the other to reverse the direction of current flow I in the Hall effect device. The reason for RS_2 is to compensate for thermal emf's which can lead to small zero offsets.
- The reversing switch RS_1 is included for a rather subtle reason: In an ideal Hall effect device $\Delta V = 0$ if there is no magnetic field. But you will find that $\Delta V \neq 0$ even when $B = 0$. This occurs because the sample has a finite resistance and therefore a voltage drop occurs along the direction of current flow; the potential leads are soldered onto the sample at points which are not quite on the same equipotential line (in zero magnetic field). This potential difference between the two leads, which we'll call the IR effect, adds onto the Hall potential ΔV . In order to eliminate the IR effect, the magnetic field direction is reversed by using RS_1 . This changes the sign of ΔV , but since I is still flowing in the same direction, the IR effect will not change sign. The results for both magnetic field directions are averaged to get ΔV .



Ramp the current through the electromagnet down to zero before reversing RS_1 .

Gaussmeter You will measure the magnetic field using the LakeShore gaussmeter. It may be used in either SI mode (in milliteslas) or Gaussian mode (in gauss). Note that $1 \text{ T} = 10^4 \text{ G}$.

Measurements Establish a field of some 1000 to 1700 gauss and determine its polarity by means of a compass. **The current in the magnet circuit must not exceed 1.9 amps!!** With a current of some 20 mA to 300 mA in the Hall probe, measure the potential difference across the Hall-effect sample. Vary both the current and magnetic field to establish the constancy of R and obtain a best value from your data. Evaluate the sign and density of charge carriers in the sample of indium arsenide.

Note: Consider the physical significance of your value for n .

Gamma Radiation Interactions

1. Introduction

Electromagnetic radiation of wavelength greater than 1 pm interacts with matter in just two ways: the photoelectric effect and Compton scattering. Both are nonclassical and most simply described as the interaction of a particle—the photon, or gamma ray—with an atom.

Both interactions remove an electron from the atom, and each is observed by detecting this electron. The identifying feature of the photoelectric interaction is that the electron emerges with a single energy $E = hc/\lambda$, since the photon is destroyed in the interaction then, for energy conservation, E is the photon energy. Compton scattering, on the other hand, is interpreted as elastic scattering of the photon of energy E and momentum $p = h/\lambda = E/c$ from an atomic electron. If the photon is scattered by an angle θ from its original direction, then its new wavelength λ' is related to its original wavelength λ by the Compton formula

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos \theta) \quad (5.1)$$

You may readily show, then, that the electron acquires the energy

$$E_e = \frac{E}{1 + \frac{mc^2}{E(1 - \cos \theta)}} \quad (5.2)$$

where mc^2 for an electron is 511 keV.

The experiment is performed with radiation from a radioactive source such as cesium 137 incident upon the atoms in a 2-inch-diameter by 2-inch-thick crystal of sodium iodide. The radiation from Cs^{137} consists of gamma rays of energy 662 keV. Compton and photoelectrons freed from atoms of the crystal by this radiation rapidly come to rest within the crystal, converting their energy into that of low energy photons. In sodium iodide, these secondary photons are in the visible spectrum and are detected by a photomultiplier tube. The photomultiplier converts them into an electrical pulse of amplitude proportional to the total energy of the secondary photons—hence also proportional to the initial electron energy. By observing the amplitude distribution of these pulses, we determine the energy distribution of electrons freed from atoms of the crystal by interactions with the incident radiation.

2. Experiment

The apparatus is shown schematically in Fig. 5.1.

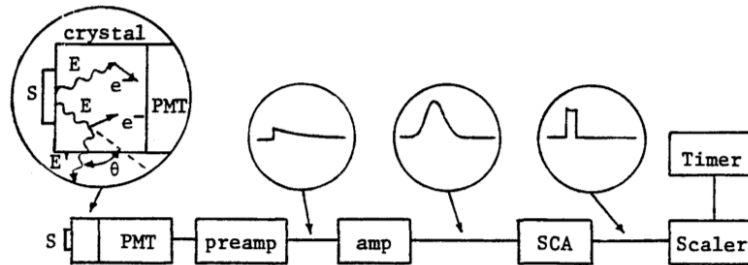


Figure 5.1: Schematic representation of the apparatus.

The source S is taped to the front of the sodium iodide crystal. The enlargement illustrates photoelectric and Compton interactions within it. Secondary photons are generated as the electrons e^- come to rest. These photons are detected by the photomultiplier tube PMT mounted in a common housing with the crystal. The preamplifier generates a negative output pulse as shown in the inset with amplitude proportional to the energy of the Compton or photoelectron feed within the crystal. The amplifier then inverts, shapes and amplifies the pulse as shown.

The amplifier output is directed to a single channel analyzer SCA. This is the instrument with which the pulse amplitude distribution is determined. You will control two settings of the SCA—its lower level discriminator LLD and its window. The function of these controls is shown in Fig. 5.2. A, B, and C are amplitude three pulses from the amplifier to the SCA. The controls set the height of the line LLD and the width of the gap labeled “window.” As set, only pulse B will trigger an output window pulse from the SCA. The maximum amplitudes of A and C lie outside the window and neither pulse is sent on to the scaler. Were the LLD lowered so that the window embraced only pulse A, then only pulses of this maximum amplitude would be registered by the scaler. Were the window opened to include the maxima of both B and C, then both of these pulses would be registered. The scaler is started and stopped by a timer and counts the number of pulses it receives from the SCA within this time which satisfy the condition:

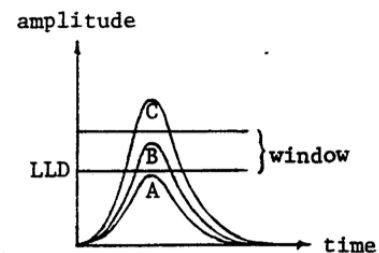


Figure 5.2: Discriminator levels.

$$\text{LLD} \leq \text{maximum pulse amplitude} \leq \text{LLD} + \text{window}$$

We wish to observe the amplitude distribution of pulses generated by electrons from gamma interactions within the sodium iodide crystal. First, connect an oscilloscope to the output of the amplifier. You will see a broad amplitude distribution, with a bright band of pulses of nearly the same maximum amplitude. These are generated by photoelectric interactions in the crystal. Set the amplifier gain so that these pulses have $\sim 5\text{--}8$ volts maximum amplitude. Remove the scope and reconnect the amplifier to the SCA. Set the SCA window¹ to 0.2 volts and the LLD somewhat below the maximum voltage of the bright band observed on the scope. Set the timer for 10 s or so and count the

¹In “window” mode, the Window pot goes from 0–1 V, while the Lower Level pot goes from 0–10 V.

number of pulses received by the scaler at this setting of the SCA. Advance the LLD by 0.2 V, leaving the window fixed, and count again. Proceed in this way through the “photopeak.” You now know how to use the apparatus.

For Cs^{137} , the photopeak observed above occurs at 662 keV. Since each component of the apparatus is linear, this number calibrates the entire voltage range of the LLD in electron-volts. You are now prepared to study the entire energy spectrum of electrons resulting from photoelectric and Compton interactions of gamma radiation from Cs^{137} with atoms of the sodium iodide crystal. Start with the LLD at zero volts and make three or four measurements at gradually increasing LLD. When you have completed this task and understand how the apparatus works, you have a choice: you may either proceed through the photopeak, one channel at a time. Or, you may get checked out by your instructor and switch to using the Maestro 32 multichannel data acquisition system, which will allow you to record the entire spectrum simultaneously. Instructions on using Maestro 32 are available at the apparatus.

If you choose the manual approach, be sure to plot as you go! Where the distribution is quite flat, you should increase the increments by which the LLD is advanced. Where it changes rapidly, you may wish to decrease them. Set the timer to obtain good statistics. Remember that the uncertainty in the number N of pulses counted in a given time interval is $\pm\sqrt{N}$.

2.1 Things to observe

Classically, one would expect no structure in the pulse amplitude distribution at all. The prominent peak you observe attests to the nonclassical photoelectric interaction first accounted for by Einstein.

The Compton interaction produces electrons of energy from zero, $\theta = 0$, to a maximum for $\theta = \pi$ (Eq. (5.1)). The maximum defines the “Compton edge.” Calculate where you expect the edge to appear and locate it in your experimental distribution. You will probably also observe a rather broad peak at the energy $E - E_e$. This is the energy of a gamma ray which has scattered at $\theta \cong \pi$. This occurs for gammas emitted by the source away from the crystal, which are then scattered from the table, floor, walls, etc. back into the crystal. The peak may be enhanced by placing an aluminum plate immediately behind the source. You should also observe a narrow peak at an energy of ~ 30 keV. The radiation responsible is an x-ray from barium, the decay product of cesium within the source. Look up the known energy of this x-ray and compare it to your result. You may wish to place a lead brick behind the source and observe x-rays from lead (~ 70 keV) which arise with each photoelectric interaction in the lead.

Barrier Penetration

1. Introduction

A striking consequence of quantum mechanics is the prediction that a particle of total energy E located in a potential well of depth $V_0 > E$ has a finite probability of escaping if the walls of the potential well have a finite thickness. This phenomenon, known as *barrier penetration* or tunneling, is not uncommon at the atomic or subatomic scale; for example, α decay occurs via tunneling of the α particle through the Coulomb barrier of the radioactive nucleus (see Section 9.3 of Townsend or Section 16-2 of Eisberg and Resnick). While barrier penetration is hardly commonplace on the macroscopic scale, it can be seen; in fact, barrier penetration is a property of both classical and quantum mechanical wave motion.

An optical analog of barrier penetration, known as “frustrated total internal reflection,” is described formally by the same equations that describe quantum mechanical tunneling. In this phenomenon, a light beam traveling through glass (or any other transparent medium with an index of refraction $n > 1$) is incident on the glass-air interface. For sufficiently small angles of incidence, the light is partly reflected and partly transmitted into the air. But for angles of incidence greater than the “critical angle” $\sin^{-1}(1/n)$, the beam is totally reflected back into the glass; no light is transmitted into the air. The oscillating electromagnetic field of the light does not stop precisely at the interface, however; it extends some distance into the air. If another piece of glass is brought close enough to the interface, this electromagnetic field can then propagate away from the interface (thus the total internal reflection is “frustrated”). The trick is getting the second piece of glass close enough, to within about a wavelength of the interface. Unless the interface is very flat, the effect won’t occur; in any case, the gap is so small as to be invisible. One can change the scale of electromagnetic radiation to the microwave region, where wavelengths are on the order of centimeters. Then this phenomenon can be easily observed. For radiation with wavelengths of a few centimeters, polyethylene becomes a good substitute for glass; it is almost transparent to microwaves and has an index of refraction very similar to that of glass for optical frequencies. A microwave beam traveling through a polyethylene block and incident on the polyethylene-air interface at an angle of 45° undergoes total internal reflection, provided the interface is isolated. Again, there is an oscillating electromagnetic field extending into the air beyond the interface, as you will see. If another polyethylene block is brought close enough to the interface, it should allow a transmitted wave to propagate away from the interface. You will study this phenomenon.

In the experiment there are two 45° - 45° - 90° polyethylene prisms arranged so that the two hypotenuse faces can be brought close together. A microwave beam is incident on the first prism perpendicular to one base, travels through the prism and strikes the hypotenuse at 45° . If the perpendicular separation of the two prisms is d , then the fraction of the microwave radiation intensity that can penetrate the gap between the prisms (T) is given by

$$T = (1 + \alpha \sinh^2 \beta d)^{-1} \quad (6.1)$$

The form of this equation is identical to that seen for quantum mechanical barrier penetration (see Section 4.7 of Townsend). The coefficients α and β can be obtained from classical electromagnetic theory. For the geometry of this experiment one obtains¹

$$\alpha = \frac{(n^2 - 1)^2}{n^2(n^2 - 2)} \quad (6.2)$$

$$\beta = \frac{2\pi}{\lambda} \sqrt{(n^2 - 2)/2} \quad (6.3)$$

where n is the index of refraction of the polyethylene and λ is the wavelength in air.

2. Preliminary Measurements

To predict the intensity of the microwave radiation transmitted across the gap, you need to know the microwave wavelength and the index of refraction of polyethylene. And there is one more subtle question, namely, does the detector measure the intensity of the microwave radiation? That is, does the detector respond linearly to the square of the microwave electric field strength? You will perform some preliminary measurements to determine these three parameters.

Transmitter and receiver The microwave transmitter operates at 10.5 GHz. The microwaves emitted from the horn are polarized parallel to the long axis of the Gunn diode (the slender shiny cylinder located at the base of the horn). While the transmitter can be rotated to change the polarization axis, for this experiment the Gunn diode should be kept vertical (0 on the scale). The microwave receiver consists of a detector diode mounted similarly at the base of the detector horn. The diode responds only to the component of the microwave signal that is parallel to the diode axis. There are four amplification ranges and a variable gain control on the receiver. **Always start at the least sensitive range ($30\times$) to avoid damaging the electronics.**

Wavelength A good way to measure the wavelength is to use a Fabry-Perot interferometer. Follow the procedures in the PASCO Instructions and Experiments Manual, Experiment 9, p. 26. Compare your result with the expected value for 10.5-GHz radiation.

Receiver Response You can check to what extent the meter reading on the microwave receiver is proportional to the intensity by seeing how the meter reading changes as the diode axis of the receiver is rotated relative to the polarization direction of the transmitted electric field (transmitter diode axis). If the meter responds directly to the electric field strength, then the meter reading should be proportional to $\cos \theta$, where θ is the angle of the detector diode relative to

¹See, e.g., J. Strong, *Concepts of Classical Optics* (1958), Section 6-9.

the E field. If the meter responds to intensity, then the meter reading should be proportional E^2 , hence to $\cos^2 \theta$. In fact, the detector diode is a nonlinear device, so that the way the meter responds can vary with the strength of the field. To test the meter in the relevant range, you should separate the transmitter and receiver by about 2.5 m, roughly the same distance you will be using in the barrier penetration experiment. (The goniometer arm on which the transmitter and receiver were mounted for the wavelength determination should be removed from the table to minimize spurious reflections.) Adjust the detector position to get a maximum signal when both transmitter and receiver diodes are vertical. Set the meter reading to full scale. Then rotate the receiver in 15° increments up to 90° and record the meter readings. Compare with $\cos \theta$ and $\cos^2 \theta$. The meter response should be quite close to $\cos^2 \theta$ (intensity) at this separation. If it is not, see your instructor.

Index of refraction Light (or microwave radiation) incident on a prism is refracted on entering and leaving the prism. If the prism is oriented so that the angle with which the beam leaves the prism (where as usual the angle is measured relative to the normal to the surface it is exiting) is the same as the angle at which it enters (relative to the normal to the front surface)—see the figure—then the index of refraction is given by

$$n \equiv \frac{\sin \theta_i}{\sin \theta_r} = \frac{\sin(\psi + \alpha/2)}{\sin(\alpha/2)} \quad (6.4)$$

where θ_i is the angle of incidence, θ_r is the angle of refraction, α is the apex angle of the prism, and ψ is defined as shown in Fig. 6.1.

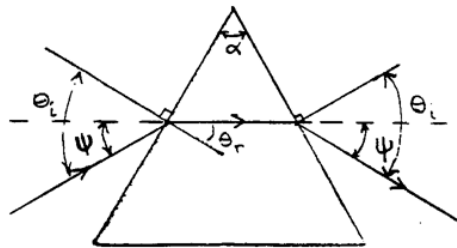


Figure 6.1: Symmetric propagation.

(**Note:** As part of your writeup for this experiment, you should derive this formula.) Two rotating goniometer arms are attached to the platform supporting the fixed polyethylene prism. (You may have to roll back the second polyethylene prism to see the second goniometer arm.) The angle scales marked for each arm correspond to the angle ψ in the figure above. Position the transmitter on one arm and the receiver on the other. Rotate the arms symmetrically relative to the 45° apex angle of the prism (i.e., both angles must be the same) and locate the angle ψ where the receiver signal is a maximum. Use this to determine the index of refraction of the prism.

3. Barrier penetration measurement

Remove the transmitter and receiver from the goniometer arms and rotate the arms so that they are parallel to the base of the prism. Use the pegs provided to fix these arms in position. Now roll

the second prism all the way up to the first prism. Place the receiver in the guides on the movable platform behind the second prism. (The separation of the prism and the receiver should be about 30 cm to the base of the receiver horn.) Place the source about 190 cm from the front face of the fixed prism and align it carefully to give a maximum reading on the receiver meter. Slide the receiver forward and backward a few centimeters in the guides to maximize the signal. Set the sensitivity for full-scale reading when the two prisms are touching. Now roll the prism back. You should see the meter reading drop quickly, with essentially 0 reading for a separation of several centimeters.

If the reading does not drop to at most 5-10% of the initial reading, you need to realign the source and detector and look for any causes of extraneous reflections. With the second prism still several centimeters away, adjust the position of the second receiver, located to detect the beam reflected from the hypotenuse of the fixed prism. This detector should be about 30 cm from the fixed prism. Adjust its position for a maximum meter reading and set this to 1 when the movable prism is “far” away. Now roll the second prism toward the fixed prism and note the meter readings on the two receivers. Describe qualitatively what you see.

Since the receiver monitoring the reflected beam can produce spurious reflections which affect the transmitted beam receiver, it should be removed for the rest of the experiment. **Be sure to turn off the receivers after you are finished with them, as their batteries run down quickly.** Recheck the transmitter receiver for the extreme positions of the movable prism, and if necessary readjust and reset before taking quantitative measurements. Vary the movable prism location, measure the perpendicular separation d of the prisms, and plot the resulting transmission coefficient T . On the same graph, plot the predicted transmission coefficient. Compare the two and comment.

Photoelectric Effect

1. Background

Around the turn of the century, Philipp von Lenard, studying a phenomenon originally observed by Heinrich Hertz, showed that ultraviolet light falling on a metal can result in the ejection of electrons from the surface.¹ This light-induced ejection of electrons is now known as the photoelectric effect. Einstein's explanation of this effect in 1905 (the year he also developed special relativity!) is one of the cornerstones of quantum physics.

According to the classical theory of electromagnetic fields, the intensity of a light wave is directly proportional to the square of the electric field of the wave. An electron in some material exposed to this light wave should feel a force proportional to this electric field. For an intense enough illuminating light, the electron should be able to gain sufficient kinetic energy to escape the material. The energy gained by the electron depends only on the intensity of the light (and the nature of the material), not on the wavelength.

That, however, is *not* what is observed experimentally. In a series of very careful experiments in the 1910s, Robert Millikan showed that the maximum kinetic energy $K_{\max}$ the ejected electron is independent of the intensity but linearly dependent on the frequency ν of the incident light:

$$K_{\max} = h\nu - W_0 \quad (7.1)$$

where h is a constant and W_0 is the “work function” characteristic of the material. Millikan found experimentally that h is numerically equal to the constant Max Planck introduced in his explanation of blackbody radiation.

In fact, Einstein's theory of the photoelectric effect in 1905 (hypothesized before Millikan's experiments) predicted just such a relationship, with h being identical to Planck's constant. In this theory, light exists in individual quanta, or photons. The energy of a photon is given by its frequency, $E = h\nu$. In the photoelectric effect a photon is absorbed by an electron, which then acquires the energy lost by the photon. If the electron is right at the surface (so it doesn't lose any energy in inelastic collisions on the way to the surface), then the electron can escape, provided its kinetic energy is greater than the work function W_0 . Increasing the intensity of the incident light of a given frequency would simply mean that *more* electrons are produced with sufficient kinetic energy to

¹See, e.g., Robert Eisberg, Robert Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*, 2nd ed. (Wiley, New York, 1985) Ch. 2.

escape; the maximum kinetic energy of the escaping electrons would remain constant. However, if the frequency of the incident light is so low that the photon energy is less than the work function then no electrons will have sufficient energy to escape the material.² The simple linear relationship between photon frequency and energy thus predicts Millikan's results. Two Nobel Prizes were awarded for work done on the photoelectric effect—one in 1921 to Einstein for his theoretical explanation, and one in 1923 to Millikan for his experimental work on this effect and for his more famous experiments establishing the charge of the electron.

2. Experiment

In this experiment you will determine the maximum kinetic energy of electrons photoejected from a metallic cathode in a vacuum tube under various illuminations. The maximum kinetic energy is determined by measuring the “stopping potential,” the minimum reverse potential V between the cathode and the anode which reduces the photoelectric current in the tube to zero. In this case,

$$K_{\max} = eV \quad (7.2)$$

where e is the magnitude of the electron charge. Substituting this expression for K into Eq. (7.1) and solving for the stopping potential V gives

$$V = \left(\frac{h}{e}\right)\nu - \frac{W_0}{e} \quad (7.3)$$

Thus a plot of V vs. ν should give a straight line with a slope of h/e and an intercept of $-W_0/e$.

The experiment consists of two parts. In the first you will study the effect of light intensity on the stopping potential and test the predictions of the classical theory of electromagnetic radiation. In the second you will look carefully at the effect of light frequency on the stopping potential as a test of the quantum theory.

The experimental apparatus, made by PASCO Scientific, consists basically of a mercury vapor light source, diffraction grating, and a photodiode tube and associated electronics. The light source/diffraction grating setup allows you to study five spectral lines, from the near ultraviolet through yellow. Read quickly through the PASCO lab manual to get familiar with the equipment and procedures. You should assume that the basic alignment of the apparatus has already been accomplished, so that you will only need to properly locate the grating and photodiode detector for optimal performance. *Consult with your instructor before you make any other alterations.*



The mercury vapor lamp is a strong source of UV light. Never look directly into the beam, and always use UV-absorbing safety glasses when the lamp is on.

Using the PASCO manual as a rough guide, study the dependence of the stopping potential on both the intensity and frequency of the illuminating light. Your final analysis should include a determination of Planck's constant and also the work function of the photocathode.³

²For sufficiently intense illumination, it is, in fact, possible for two “sub-threshold” photons to be absorbed by a given electron, allowing it to escape the material, even though the individual incident photon energies are less than the work function. Such nonlinear effects require very intense laser beams.

³Note from Eq. (7.1) that h/e has the dimensions of volt-sec (Vs) and W_0/e has the dimensions of volts (V). From these results you can directly express h in terms of eVs and W_0 in terms of electron volts, where $1 \text{ eV} \equiv (\text{charge of electron}) \times (1 \text{ volt})$. If you had some independent determination of electron charge, you could then give these results in terms of, say, joules rather than electron volts, but that's not necessary here.

The Cavendish Experiment

1. Background

Isaac Newton's (1642–1727) theory of gravitation explained the motion of terrestrial objects and celestial bodies by positing a mutual attraction between all pairs of massive objects proportional to the product of the two masses and inversely proportional to the square of the distance between them. In modern notation, the law of universal gravitation is expressed

$$F = \frac{GMm}{r^2} \quad (8.1)$$

where M and m are the masses of the two objects, r the distance separating them, and G is the universal constant of gravitation. Newton was not particularly concerned to evaluate the constant of proportionality, G , for two reasons. First, a consistent unit of mass was not in widespread use at the time. Second, he judged that since the gravitational attraction was so weak between any pair of objects whose mass he could sensibly measure, being so overwhelmed by the attraction each feels toward the center of the Earth, any measurement of G was impractical.

Notwithstanding Newton's pessimism, towards the latter half of the 18th century several scientists attempted to weigh the Earth by observing the gravitational force on a test mass from a nearby large mountain. These efforts were hampered, however, by very imperfect knowledge of the composition and average density of the rock composing the mountain. Spurred by his interest in the structure and composition of the interior of the Earth, Henry Cavendish in a 1783 letter to his friend Rev. John Michell discussed the possibility of devising an experiment to "weigh the Earth." Borrowing an idea from the French scientist Coulomb who had investigated the electrical force between small charged metal spheres, Michell suggested using a torsion balance to detect the tiny gravitational attraction between metal spheres and set about constructing an appropriate apparatus. He died in 1793, however, before conducting experiments with the apparatus.

The apparatus eventually made its way to Cavendish's home/laboratory, where he rebuilt most of it. His balance was constructed from a 6-foot wooden rod suspended by a metal fiber, with 2-inch-diameter lead spheres mounted on each end of the rod. These were attracted to 350-pound lead spheres brought close to the enclosure housing the rod, roughly as illustrated in the figure below. He began his experiments to "weigh the world" in 1797 at the age of 67, and published his result in 1798 that the average density of the Earth is 5.48 times that of water. His work was done with such care

that this value was not improved upon for over a century. The modern value for the mean density of the Earth is 5.52 times the density of water. Cavendish's extraordinary attention to detail and to the quantification of the errors in this experiment has led C.W.F. Everitt to describe this experiment as the first modern physics experiment. In this experiment you will use a torsional balance similar to Cavendish's to "weigh the Earth" by determining a value for G .¹

2. Theory

The Cavendish torsional balance is illustrated in Fig. 8.1. Two small metal balls of mass m are attached to opposite ends of a light, rigid, horizontal rod which is suspended from a torsion fiber. When the "dumbbell" formed by the rod and masses is twisted away from its equilibrium position (angle), the fiber generates a restoring torque proportional to the angle of twist, $\tau = -\kappa\theta$. In the absence of damping, the dumbbell executes an oscillatory motion whose period is given by $T = 2\pi\sqrt{\frac{I}{\kappa}}$, where I is the rotational inertia of the dumbbell, $I = 2m\left(d^2 + \frac{2}{5}r^2\right)$. In this expression, r is the radius of the small masses m , and d is the distance from the center of the rod to the center of one of the masses, and we have neglected the mass of the thin rod. Knowledge of m , d , and r , and a careful measurement of the period of oscillation T allows one to calibrate the torsion fiber, obtaining its spring constant κ . From κ and a measurement of the twist caused by the large masses M you can deduce the gravitational force between the masses, and hence G .

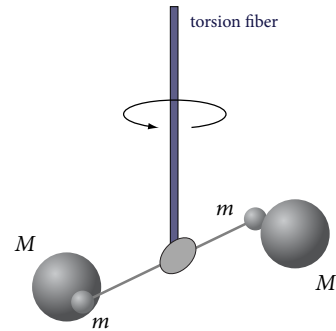


Figure 8.1: The Cavendish torsional balance.

2.1 Gravitational Torque

When the large metal spheres are positioned as shown in the figure, the gravitational attraction between the large and small spheres produces a torque that rotates the dumbbell clockwise. Only the component of the force on each mass that is perpendicular to the horizontal bar produces a torque about the center of the rod. The magnitude of the torque between the two adjacent masses is given by $\tau_g = 2F_{\perp}d$, where the factor of 2 comes from the fact that the torque is equal on the two masses m . This torque displaces the equilibrium angle of the dumbbell by an amount given by $\tau = -\kappa\theta_0$. Hence, if one can measure the equilibrium angle θ_0 very carefully, one can deduce the gravitational force that produces the torque and finally G .

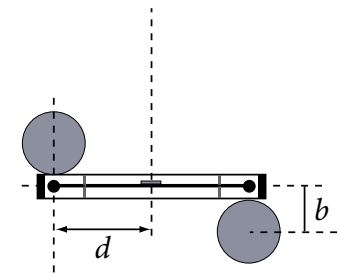


Figure 8.2: Top view.

2.2 Light Lever

Cavendish mounted a finely ruled scale near the end of the dumbbell, which he could read with a telescope to one-hundredth of an inch. The telescope allowed him to remain outside the experi-

¹In 1998 the accepted value for G was known to only 0.15% precision, a surprisingly crude number, reflecting the miniscule forces involved. In May 2000 in an experimental *tour de force*, Jens Gundlach and Stephen Merkowitz of University of Washington improved on that precision by a factor of 100, finding a value of $(6.67390 \pm 0.00009) \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$.

mental chamber, thus eliminating air currents and his gravitational influence on the oscillator.

We will take advantage of a light lever to magnify the dumbbell's tiny rotation into an easily observed displacement on a far screen. The light lever is produced by bouncing a laser beam off a mirror mounted to the dumbbell. When light bounces off a mirror, the angle the incoming beam makes with the normal (perpendicular) of the mirror is equal to the angle the outgoing beam makes with the normal. If the mirror rotates through a small angle α , the outgoing beam rotates through the angle 2α , since both the incoming and outgoing angles change by the same amount. By measuring the motion of the laser spot on a far screen, and knowing the distance between the mirror and the screen, you can determine the angle α , from which you can infer the rotation of the dumbbell, θ .

2.3 Damping

In the absence of damping, the motion of the dumbbell is a sinusoidal oscillation with the period given by

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \quad (8.2)$$

Viscous damping of the pendulum's motion caused by air resistance produces a drag torque proportional to the angular speed of the dumbbell. This causes the sinusoidal oscillation to decay exponentially, with a time constant τ that is long compared to the oscillation period. The motion is described by the function

$$\theta = \theta_o + Ae^{-t/\tau} \sin(2\pi t/T + \phi_o) \quad (8.3)$$

where ϕ_o is the initial phase of the motion, θ_o is the equilibrium position, A is the amplitude of the motion, and τ is the time for the oscillation to fall to $1/e$ of its original value. (Note that this time has nothing to do with the torques, which, unfortunately use the same symbol!) By measuring $\theta(t)$ and fitting your data to this equation, you can determine both T and θ_o , from which you can determine G .

2.4 Parameters

According to the Pasco manual, the parameters of the apparatus include the following:

Variable	Value
m	$(38.3 \pm 0.2) \text{ g}$
r	9.53 mm
d	50.0 mm
b	46.5 mm

No uncertainties are given for the last three values. Since $m \propto r^3$, one could infer an uncertainty in r of 0.02 mm resulting from the uncertainty in m given above. Based on the precision of the value given for d one might reasonably assume an uncertainty of 0.1 mm in that value. The uncertainty in b depends on the accuracy of the horizontal alignment of the pendulum (See the Pasco manual for the proper alignment procedure; your check of the alignment should give you an estimate of the uncertainty in b .)

2.5 Safety



The laser pointer that forms the light lever for this experiment is a class III laser capable of damaging retinas. **Do not look directly into the beam.** Please ensure that nobody looks into the beam. Note that it is safe to look at the diffuse spot the beam produces as it reflects from an object, such as a meter stick.

3. Procedure

1. See the instructions next to the apparatus for proper alignment of the oscillator. You should be sure that when the freely suspended dumbbell remains at rest, it is equidistant from the front and back faces of the enclosure. This alignment should be done with the large lead balls removed.
2. Carefully weigh the large lead balls. Place a styrofoam tray on the electronic scale and tare it. Then gently place the lead ball into the tray. If the lead balls are dropped, they will become misshapen, which will severely compromise the accuracy of the experiment.
3. Lock the dumbbell using the screws on the bottom of the enclosure, then gently place the large lead balls in the armature. Rotate the armature until the balls just touch the sides of the enclosure. Gently lower the support screws until the dumbbell rotates freely. If it rotates so much that the dumbbell bounces off the sides of the enclosure, use the support screws to settle the motion.
4. Once the dumbbell oscillates freely, begin recording the position of the reflected laser spot as a function of time. Record at least two full periods.
5. Gently rotate the armature until the large balls once again touch the sides of the enclosure in the opposite orientation and record the position of the reflected laser spot as a function of time for at least another two periods.
6. If you have time you should finally return the armature to its original position and again record the position of the laser spot as a function of time for at least another two periods.
7. From the distance between the mirror and the screen on which you measured the laser spot position, you can deduce the angle of rotation of the dumbbell. As described above, fit a damped sinusoid to each set of data and extract the centers of rotation. From their difference and the data given in the parameters table above, you can deduce the value of G .

Created 10/22/99 by Peter N. Saeta
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Chaotic Motion

Newton's equations and the extensions supplied by Euler, d'Alembert, and Lagrange, built a mighty edifice able to explain the motions of planets and apples, pendulums and piano wires. The great French mathematician and astronomer, Pierre-Simon, marquis de Laplace (1749–1827), summed up mechanical philosophy in *A Philosophical Essay on Probabilities* (Fifth edition, 1825):

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

Towards the end of the nineteenth century, Henri Poincaré (1854–1912) came to appreciate that seemingly simple systems, such as planets orbiting the Sun, could exhibit quite complicated motions. We now know that the behavior of many (most?) mechanical systems shows very strong dependence on initial conditions: systems starting from nearly the same state evolve into later states that differ widely. Imperfect knowledge of the initial conditions prevents us from making valid predictions beyond a limited time interval. Therefore, Laplace's classical determinism is a delusion.

To be chaotic, a system must be nonlinear and have more than one degree of freedom. In this experiment you will investigate the motion of a damped, driven, nonlinear pendulum.

1. Overview

The pendulum consists of an aluminum disk (9.5-cm diameter, 120 g) with an eccentric mass (15 g) mounted near the edge. A pulley is attached to the disk and a string attached to two springs winds around the pulley. The string is adjusted so that the tension in each spring is equal when the eccentric mass is rotated to its highest position (the position of unstable equilibrium). A small magnet can be positioned near the disk to provide eddy-current damping, which is proportional to the angular velocity.

For *small* oscillations about either stable equilibrium position, the natural frequency should be independent of amplitude and depend only on the rotational inertia of the rotor and the effective

spring constant of the restoring torque (provided by the combination of springs and gravity on the eccentric mass).

1.1 Procedure Overview

1. Investigate the symmetry of the potential by releasing the rotor from rest near its unstable equilibrium position when the motor arm is in a horizontal position, once on each side of vertical. If it is asymmetric, carefully adjust the string on the clamp holding the string on the right-hand spring to symmetrize.
2. Measure the natural frequency in each potential well with the damping magnet far from the disk. Are they the same? Why or why not?
3. Briefly investigate the influence of the damper by recording a sequence of releases as you bring the magnet closer to the rotor.
4. Set the drive amplitude to about 2.5 cm and the magnet about 3–4 mm from the rotor. Set the voltage on the DC power supply to about 3 V. You should observe stable oscillation of the rotor in one of the potential wells. Is the motion sinusoidal? You may wish to perform a fast Fourier transform (FFT) to calculate the spectrum.
5. Gradually increase the drive voltage, allowing plenty of time for the oscillator to settle after each small change. Eventually the oscillator may begin to explore both wells. If it hasn't left the well by the time the frequency approaches the natural frequency, then either the damping is too great or the drive amplitude is too small to generate interesting behavior. Reduce the damping and try again.
6. Set the voltage just barely above the chaos threshold and record a long run (several minutes), observing the Poincaré plot. Alter the frequency a small amount and record again.
7. Prepare a few phase plots and Poincaré plots for drive frequencies just below and just above the chaos threshold.
8. Play and explore!

2. Details

The Pasco software that operates the ScienceWorkshop 750 interface is called Data Studio. You will set it up to record angular position, θ , and angular velocity, ω , as functions of time, as well as the period of the drive. You can then create live plots to show θ vs. t , ω vs. t , a phase plot (ω vs. θ), and a Poincaré plot, which is a phase plot with discrete points plotted once per cycle of the drive. Of course, you can also export data in text format to Igor Pro or other analysis packages.

2.1 Setup

Hardware configuration

1. Plug the rotary motion sensor into channels one (yellow) and two (black) of the ScienceWorkshop 750 interface.
2. Plug the photogate into channel 3.

3. Ensure that the interface's USB cable is connected to the computer. Then power on the ScienceWorkshop 750 interface.
4. Connect the red and black banana jacks of the DC motor to the DC power supply. You may find it useful to connect a digital multimeter first, so you can monitor the supply voltage more precisely than is possible with the analog meter on the power supply.
5. Adjust the rotating arm on the drive motor to be about 2.5 cm off axis, and rotate the arm to a horizontal position (so that the bottom end of the drive spring is approximately in the middle of its range of travel).

Software configuration

1. Open Data Studio and create a new activity. You should see a picture of the ScienceWorkshop 750 interface with its ports outlined in yellow. [You will only need to go through the following setup procedure once. Once you have created an activity with all the appropriate measurements and calculations, you can use it for subsequent data files. Just Save As a different file, then remove all the data using the Experiment menu.]
2. Click on Port 1 and add a **Rotary Motion Sensor** from the Digital Sensors tab.
3. Set it up to measure angular position (θ) in degrees, angular velocity (ω) in radians/s.
4. Set the sample rate to 50 Hz.
5. Set the Resolution to High (on the Rotary Motion Sensor tab). This corresponds to an angular resolution of 0.25° .
6. Add a photogate to Port 3, and have it measure "Velocity in Gate" (even though this makes no sense). We will use it instead to measure the period and to strobe θ and ω for the Poincaré plot.
7. Click the **Setup Timers...** button and label the timer Period. Using the Timing Sequence Choices area create a sequence of blocked–unblocked–blocked. This will record the period each time the drive motor completes a revolution.
8. Minimize the Experiment Setup window. You may need to come back to it later if you decide to change the sample rate or the resolution of the rotary motion sensor.

Calculations

1. You now need to set up a few calculations to be made from the data you will acquire. Click the **Calculate** button.
2. To calculate the frequency, enter in the Definition box: $f = 1/T$. Click on the Variables popup, select undefined variable, which will still be referring to "x". It will now mention the undefined variation "T", which you should tie to the Period measurement (Data source).
3. You now need to set up two additional calculations for the Poincaré plot, and one for the potential. The values of θ and ω are stored at the sample rate. In addition, we would like to record their values once each cycle, using the photogate measurements. To manage this, we use a trick. We multiply the measurement of Channel 3 by zero and add the value of either θ or ω . Channel 3 is recorded only once per revolution of the drive, so including it in the calculation produces the desired strobing of θ and ω . Create these two calculated quantities, then close the Calculate window.

2.2 Potential Well

To explore the shape of the potential well, back the damping magnet all the way out, rotate the eccentric mass to the top, and release it from rest. If damping is negligible, then

$$E = U(\theta) + \frac{1}{2}I\omega^2 \quad (9.1)$$

where E is constant. Therefore, by plotting $-\omega^2$ vs. θ , you can explore the shape of the potential well. Set up a calculation for $-\omega^2$. Then prepare a graph of $-\omega^2$ vs. θ :

1. Drag the Data entry for $-\omega^2$ onto the Graph entry in the Displays panel.
2. Mouse over the label for the x axis and popup the menu to select θ .
3. Raise the eccentric mass to the top, click the **Start** button, and release the disk. Stop the measurement after one complete oscillation.
4. Raise the eccentric mass to the top, but make sure it will fall on the opposite side, and repeat the procedure. Are the two potential wells symmetric? If not, adjust the string to symmetrize.

2.3 Measurements

Set the damping magnet to be somewhere between 3 and 5 mm from the disk and determine the undriven period of the oscillator. Then turn on the power supply and set the voltage to about 3 V. After an initial transient, you should see the pendulum oscillate in a periodic fashion. Record data for a minute or so. Are the oscillations sinusoidal?

You'll probably want to set up a temporal plot (θ vs. t) and a phase plot (ω vs. θ) to observe the changing behavior of the oscillator as you *gradually* increase the voltage, allowing the behavior to settle down after each small change. (You'll probably also want to turn off individual data points in these two plots.) Make a rough determination of the voltage/frequency at which the pendulum begins to explore both potential wells. You should also set up the Poincaré plot at this point and make sure that your activity file has been saved.

A few last tips about graphs in Data Studio:

1. Graphs have an option called *Full Color*. When selected, each visible trace gets colored; when unselected, all traces are grayed except the selected trace.
2. You can use the "Smart Tool" to make measurements directly on a graph.
3. You can copy the selected trace to the clipboard. Both graphical and textual representations of the data are copied, so you can paste directly into a word processor document or into a data analysis program. A script is available to permit Igor Pro to load the data on the clipboard, preserving the run number from the Data Studio activity.