

## Conservation of Angular Momentum

## Experimental Setup and Procedure:

In this experiment we tested the law of conservation of angular momentum,  $P_{\text{initial}} = P_{\text{final}}$ , in a system of rotating disks. We used two identical steel disks suspended on a central axis by air cushions (to reduce friction). A digital sensor was positioned next to the spinning disks to determine the angular speed  $\omega$  in bars/second. The disks spun independently until the pin was removed, at which point they collided, began rotating at the same speed.

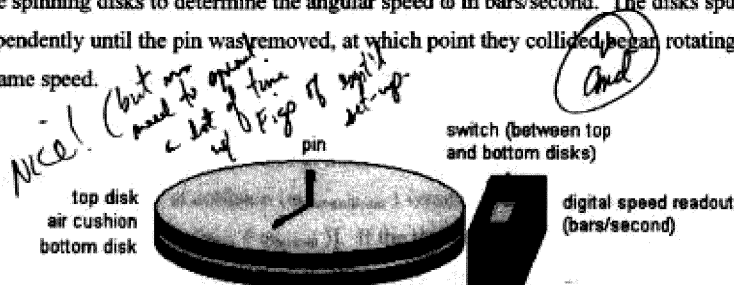


Figure 1. Experimental apparatus

We designed our experiment so that we could determine the speed of both the top and bottom disks at the instant of collision. There were two difficulties in this requirement: the digital sensor only reported the speed every two seconds, and it could only sense the speed of one disk at a time. In order to determine the speeds at the collision instant, we employed an extrapolation technique. For each collision, we took readings of the speed over a period of 40 seconds. At  $t = 0, 5, 10$  we recorded the velocity of the top disk (we arbitrarily assigned clockwise the positive direction). At  $t = 15, 20, 25$  we recorded the velocity of the bottom disk. Immediately after the 25 second reading, we pulled out the pin and the disks collided (at  $t = 25$  sec). We then read the velocity of the combined disks at  $t = 30, 35, 40$ . We fit a line to the velocities in order to

extrapolate to  $t = 25$  for both the top disk and the combined post-collision disks. See Figure 2.

### Interpretation of Results:

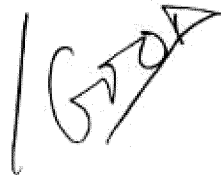
The law of conservation of angular momentum states that  $P_{\text{initial}} = P_{\text{final}}$ . For rotating objects,  $P = I \omega$ , where  $I$  is the object's rotational inertia and  $\omega$  is its angular velocity.  $I = \int r^2 dm$ , where  $r$  is the radius and  $m$  is the mass of the object. For this experiment,  $I_{\text{top}} = I_{\text{bottom}} = I$ , and  $I_{\text{post-collision}} = 2I$  (because the mass is twice as much, but the radii of all the  $dm$ 's are the same). From the conservation equation, we predict

$$P_{\text{initial}} = P_{\text{final}}$$

$$I \omega_{\text{top}} + I \omega_{\text{bottom}} = 2I \omega_{\text{post-collision}}$$

$$\omega_{\text{post-collision}} = \frac{1}{2} (\omega_{\text{top}} + \omega_{\text{bottom}})$$

typically  $\omega_{\text{L}} \gg \omega_{\text{top}}$  and  $\omega_{\text{B}} \approx \omega_{\text{bottom}}$



where all the velocities are measured at the instant of collision. To test this prediction, we plotted the velocity at collision ( $\omega_{\text{post-collision}}$ ) versus the average of the top and bottom velocities at collision [ $\frac{1}{2} (\omega_{\text{top}} + \omega_{\text{bottom}})$ ]. If the theory holds, our data points should fall on the line  $y = x$ . In fact, they fall on the line  $y = .994x + 2.00$  with a chi-squared (per degree of freedom) of about 2 (see Figure 3).  $0.544 \pm 0.001$  ~~2.00~~ 0.3

In order to calculate the uncertainties of our data, we took into consideration the fact that the digital speedometer only reports the speed at two-second intervals. The velocities we recorded, therefore, could be as much as two seconds off the time we assumed we were taking them. To calculate the uncertainty in our reported velocities, we used the extrapolation lines (see Figure 2) from our original data to calculate the velocity at  $t = 27$  (2 seconds after the collision). The difference between this number and the velocity calculated at  $t = 25$  (at the time of the collision) is a good estimate of our error due to the 2-second sampling rate of the speedometer. The error in collision velocity estimated in this way gives a relatively good chi-squared value of 2 (per degree of freedom).



The equation  $y = .994x + 2.00$  fits our data remarkably well, when account is taken of the uncertainty due to sampling the speed in 2-second intervals. This equation,

however, is not quite the  $y = x$  that is predicted by the law of conservation of momentum. The slope is very close to one, but the y-intercept is slightly higher than might be expected. The source of this systematic error may lie in the fact that the collision actually happened at a slightly later time than  $t = 25$  seconds. First, we took a reading of the top velocity at  $t = 25$ , so we would have had to pull the pin out a little after that time. Also, the disks slid a small amount on top of each other before actually "colliding" and spinning at the same speed. If the collision did in fact take place at  $t > 25$  sec, the extrapolated "collision speed" of the combined disks would indeed have been systematically too high (because the slope of the collision line is negative) (see Figure 2). Since our calculated "velocity at collision" was systematically high by a small amount, the y-intercept of our line is slightly larger than 0 (see Figure 3).

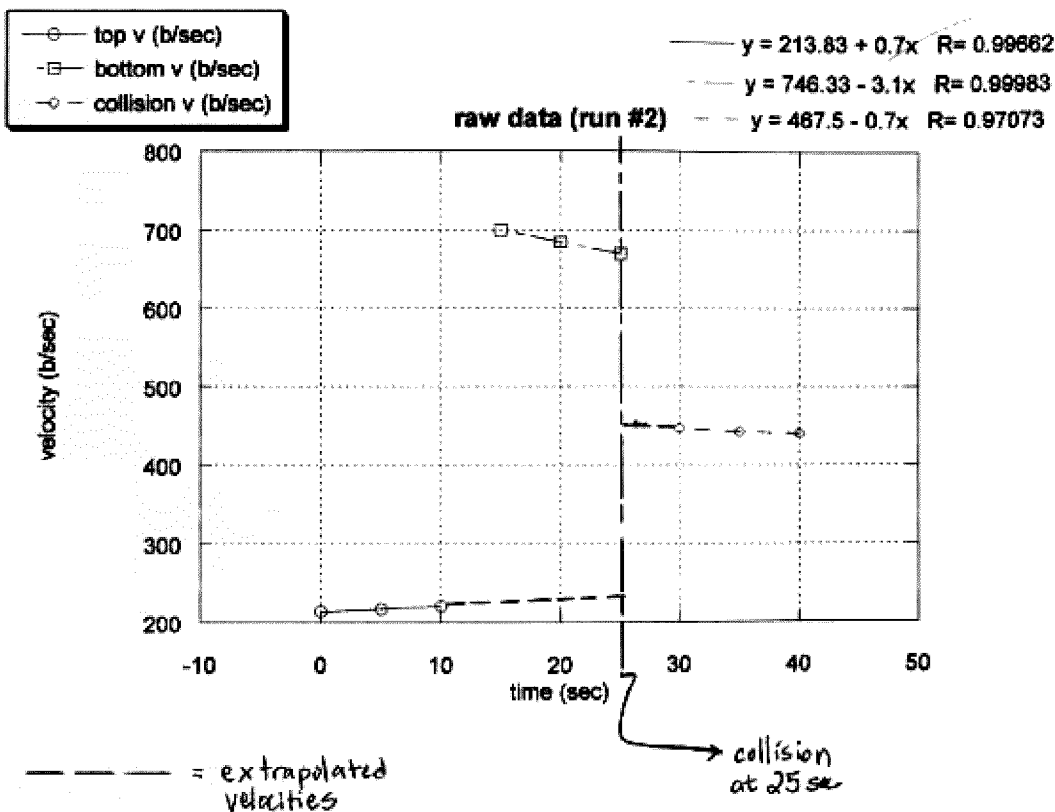
Overall, our data is consistent with the law of conservation of angular momentum. Our two uncertainties, due to 2-second intervals of speed sampling and delayed time of actual collision, account for all discrepancies.

but you need to sum the #5  
to see if it  
holds water.

Nice work!! Please  
see me if any questions.  
Tom

4.5/5

Figure 2. Example of data extrapolation



VERY NICE!

Figure 3. Correlation of data with theory

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