Physics 53
Electricity and Optics

Laboratory Course Manual

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"Nothing in the world can take the place of persistence. Talent will not; nothing is more common than unsuccessful men with talent. Genius will not; unrewarded genius is almost a proverb. Education will not; the world is full of educated derelicts. Persistence and determination alone are omnipotent."

Calvin Coolidge

Contents

Part I

Introductory Material

Quick Start Guide for First Meetings

The following is a brief summary of the essential information you'll need to prepare for the first meetings of E&M Lab. Further details will be found elsewhere in this manual and in the other resources provided to you by the instructors.

First Meeting The first meetings of each lab section will be during the week of Aug. 29 (except for Monday sections which will first meet on Sept. 5). The sections will then meet on succeeding weeks for two hours throughout the semester, following the schedule of meetings and breaks on the following page. Although it is not heavy, there is some preparation for the first meeting. In the first meeting there will be a brief orientation by the instructor to all the experiments, with related discussions of methods of data analysis. You will then perform Experiment #1, which is an introduction to simple DC electrical circuits. Prior to coming to lab, you must read the lab manual introduction and description for Experiment #1. Also come to lab prepared with a brown cover **National Brand #43-648 Computation Notebook** (available from the bookstore). Experiment #1 is shorter and simpler than the other experiments in the course, so you should be able to complete it in the abbreviated time available and experience a relatively easy "warm-up" for the later lab activities.

Plotting Exercise In this course, everyone will be responsible for being fluent in a data plotting/fitting software package. To help reinforce this, everyone is required to complete and turn in a data plotting exercise. As soon as possible (but before your #2a meeting) go to the web page at `http://www.physics.hmc.edu/analysis/exercise.php` and complete the exercise. Turn in the finished plot to your instructor. **Do we want this? Do we need another one?**

Subsequent Meetings After the first week, you will perform four of the five available experiments according to a schedule to be worked out with your instructor. With your instructor's consultation, you will sign up for the experiments and lab partners you will work with during the semester.

The experiments in this course are challenging, and require advance reading and preparation! It is very important that you come to the lab prepared. Before the first laboratory meeting for a particular experiment, read the appropriate section of this lab manual and watch the short movie introducing the experiment you will be doing. The movies are available at the course web site: `http://www.physics.hmc.edu/courses/p053/`. We will not require pre-lab homework or exercises; however, part of your grade will be based on your instructor's assessment of how prepared you are for lab.

The semester schedule has built-in days off from lab meetings, so please take advantage of this in managing your time, and budget time for lab preparation. The lab periods run for 2 hours every meeting, and each experiment (except #1) will take two lab periods. The first week (meeting "a") will typically be useful for familiarizing yourself with and calibrating the apparatus and making a first set of measurements; the second week (meeting "b") will normally be used for completion of data taking, recovery from any problems or mistakes in the first week, and consultation with your instructor about analysis and write-up.

Write-up of the Experiment A typical Experiment Summary is 2–3 typed pages, turned in along with your lab book for documentary support, at a time and place to be announced by each instructor. All write-ups should be typed/word processed, with all graphics neatly and professionally incorporated. The Experiment Summary will be what is primarily read and marked by your instructor. The lab book may not be graded in detail, however the degree to which it serves as a valid reproducible archival record of your work in lab will form a part of your grade. In the write-up, you may assume your reader is familiar with your experiment, the apparatus, and the basic procedures, so re-hashing of these is not needed. However, you should detail innovations, important observations, or unique insights that you have made during the experiment period. *The write-up should be entirely derived from the content of your laboratory notebook, and not from memory or other undocumented sources.* See the rest of this manual for more detailed discussion of writeups and lab books.

Schedule

	Week of	Section 1 **M 12:40** **Donnelly**	Section 3 **Tu 12:40** **Lynn**	Section 5 **W 12:40** **Saeta**	Section 6 **W 15:15** **Eckert**	Section 8 **Th 12:40** **Lyzenga**	Section 10 **F 12:40** **Hoard**
1	8/29		Expt. 1	Expt. 1	Expt. 1	Expt. 1	Expt. 1
2	9/5	Expt. 1	Mtg. 2a	Mtg. 2a	Mtg. 2a	Mtg. 2a	Mtg. 2a
3	9/12	Mtg. 2a	Mtg. 2b	Mtg. 2b	Mtg. 2b	Mtg. 2b	Mtg. 2b
4	9/19	Mtg. 2b	*open*	*open*	*open*	*open*	*open*
5	9/26	*open*	Mtg. 3a	Mtg. 3a	Mtg. 3a	Mtg. 3a	Mtg. 3a
6	10/3	Mtg. 3a	Mtg. 3b	Mtg. 3b	Mtg. 3b	Mtg. 3b	Mtg. 3b
7	10/10	Mtg. 3b	Mtg. 4a	Mtg. 4a	Mtg. 4a	Mtg. 4a	Mtg. 4a
8	10/17			Mtg. 4b	Mtg. 4b	Mtg. 4b	Mtg. 4b
9	10/24	Mtg. 4a	Mtg. 4b	*open*	*open*	*open*	*open*
10	10/31	Mtg. 4b	*open*	Mtg. 5a	Mtg. 5a	Mtg. 5a	Mtg. 5a
11	11/7	Mtg. 5a	Mtg. 5a	Mtg. 5b	Mtg. 5b	Mtg. 5b	Mtg. 5b
12	11/14	Mtg. 5b	Mtg. 5b	Report Prep	Report Prep	Report Prep	Report Prep
13	11/21	*open*	*open*	*open*	*open*		
14	11/28	Report Prep	Report Prep	*open*	*open*	*open*	*open*
15	12/5	*Makeup Week*					

	Week of	Section 2 **M 15:15** **Donnelly**	Section 4 **Tu 15:15** **Lynn**	Section 7 **W 18:00** **Eckert**	Section 9 **Th 15:15** **Lyzenga**	Section 11 **F 15:15** **Hoard**
1	8/29		Expt. 1	Expt. 1	Expt. 1	Expt. 1
2	9/5	Expt. 1	*open*	*open*	*open*	*open*
3	9/12	*open*	Mtg. 2a	Mtg. 2a	Mtg. 2a	Mtg. 2a
4	9/19	Mtg. 2a	Mtg. 2b	Mtg. 2b	Mtg. 2b	Mtg. 2b
5	9/26	Mtg. 2b	*open*	*open*	*open*	*open*
6	10/3	*open*	Mtg. 3a	Mtg. 3a	Mtg. 3a	Mtg. 3a
7	10/10	Mtg. 3a	Mtg. 3b	Mtg. 3b	Mtg. 3b	Mtg. 3b
8	10/17			Mtg. 4a	Mtg. 4a	Mtg. 4a
9	10/24	Mtg. 3b	Mtg. 4a	Mtg. 4b	Mtg. 4b	Mtg. 4b
10	10/31	Mtg. 4a	Mtg. 4b	*open*	*open*	*open*
11	11/7	Mtg. 4b	*open*	Mtg. 5a	Mtg. 5a	Mtg. 5a
12	11/14	Mtg. 5a	Mtg. 5a	Mtg. 5b	Mtg. 5b	Mtg. 5b
13	11/21	Mtg. 5b	Mtg. 5b	Report Prep		
14	11/28	Report Prep	Report Prep	*open*	Report Prep	Report Prep
15	12/5	*Makeup Week*				

Overview

Eppur si muove (but it does move...)

Galileo Galilei, 1564–1642

Our understanding of the physical universe derives from comparing honest, careful observations to theoretical models. The principal goal of this course is for you to learn how to conduct experiments:

1. to figure out just what you need to do to obtain the necessary data
2. to make careful observations
3. to record those observations accurately
4. to compare them to a theory
5. to communicate the observations, results, and conclusions clearly and succinctly

None of these is as easy as it may appear at first glance. Your success in the course will be determined by the care and thought you demonstrate in these four aspects of conducting experiments.

1. What do I need to do?

A great chef uses a recipe as a source of ideas, as a suggestion about how to accomplish a goal; she or he doesn't necessarily follow the recipe meticulously, the way a lesser cook might. It would be possible for us to spell out in excruciating detail each step you should follow in conducting the experiments of this course, but we choose not to. We are uninterested in educating lesser cooks who are bound merely to follow the recipes developed by others. Such an approach gives an impoverished view of experimental science. Rather, we want you to understand how to conduct original research, which involves experimental design in addition to careful observations and analysis. We won't be answering for you questions such as

- "How many data points do I need?"
- "Do I need to measure both doublet lines, or is one enough?"
- "Should I measure the resistance of the inductor?"
- "Can you set up this circuit for me?"

even though this may be frustrating at times. Learning how to answer such questions is an important part of understanding what it means to conduct an original experiment.

2. Fiddling

Never trust your equipment! Rather, assume that Murphy's law applies: Anything that can wrong will go wrong. Check out your equipment. Make sure you understand how it works, and how well it works. When you use a multimeter to measure resistance, for example, what value does it report when you simply connect the leads together? Does the scale read zero when nothing is on it? What does it read when a 50-g test mass sits in the pan? Does the detector signal change when you block the laser beam?

Before you begin to take data for any experiment, play with the equipment to make sure you understand how it behaves.

3. Document

The second principal task is to record what you do. Not what you should have done, or thought you might do, or what you think you did yesterday. What you *actually did*. Every scientist can tell you stories about efforts wasted because he or she failed to keep adequate notes on what went on in the lab, from hours, to days, to months or even more. Our memories fade annoyingly quickly, but it is critical to know whether *these* data were taken before or after *that* setting was changed. Document your work as you go.

- **Sketches and figures are good**. Use them to clarify (and sometimes avoid) discussion, define symbols, explain equations, etc. A picture speaks 10^3 words.

- **Tables are good**. Almost all measurements should be repeated, and the results collected in a table. Figure out what columns you'll need, head them and indicate units and uncertainties (or put uncertainties in their own column).

- **Lists are good**. If you have a multi-part procedure, a bulleted or numbered list is easier to figure out than such glowing phrases as "Then we turned the *xxx* on and …", "Then we fiddled with the alignment of the *xxx* until it was good …", "Then …" A list saves time and is easier to read.

- **Algebra is good**. Using numbers without associated algebraic explanation is **evil**.

- **Units are good**. Failing to put units on a value is **evil**.

- Set off key results and formulas with boxes.

4. Analyze

An experiment is about understanding. The more understanding you can develop *while* you are conducting the experiment, the better your experiment will be. Although it is tempting to lapse into "data-taking mode" — mindlessly writing down measurements — don't! Instead, ask yourself if your data are making sense as you take them.

For this, a spreadsheet and plot are invaluable. Use one of the lab computers or your own notebook computer to allow you to monitor data quality as you go, or plot by hand in your notebook. If you use a computer to record your data, print the data table out and tape it into your notebook. If you finish a section of the experiment and have produced a graph, print it out and tape it into the notebook, as well. Annotate the graph before moving on: what conclusion(s) can you draw from the graph?

Uncertainties

Most measurements are not infinitely precise, either because you use instruments of imperfect accuracy to make the measurement or because the precise position or configuration you are attempting to quantify is difficult to determine. For example, how tall are you?

One way to estimate the uncertainty of a measurement of a quantity y is to measure y several times and compute the mean ($\bar{y}$), standard deviation (σ), and standard deviation of the mean ($\sigma_{\bar{y}}$) of the measurements,

$$\bar{y} = \frac{1}{N}\sum_{n=1}^{N} y_n$$

$$\sigma_y = \left[\frac{1}{N-1}\sum_{n=1}^{N}(y_n - \bar{y})^2\right]^{1/2}$$

$$\sigma_{\bar{y}} = \frac{\sigma_y}{\sqrt{N}}$$

and then assume that your determined value is $\bar{y} \pm \sigma_{\bar{y}}$. These expressions are justified when the errors are random and normally distributed. Fortunately, this situation is quite common. See Chapter 4 of Taylor for further discussion.

Sometimes you will estimate the uncertainty based on the accuracy of a measuring device. Usually you do both and take the larger, or add them in quadrature. *Please record in your notebook how you arrived at your uncertainty estimate.*

Error Propagation

Suppose that you are measuring the speed of an object by measuring the time t it takes to travel a small distance d. The speed is then $v = d/t$, but what is the uncertainty in speed, δv? In this case, the desired quantity v is a function of two measured quantities, $d \pm \delta d$ and $t \pm \delta t$, namely $v(d,t) = d/t$.

Let's say that you measured the distance d with a ruler and you performed the time measurement with a stopwatch. First assume that the time measurement is much less certain than the distance measurement, so that we may neglect δd compared to δt. As illustrated in Fig. 1, the magnitude of the uncertainty in the deduced value of v for a given uncertainty δt depends on the value of t. For the comparatively small value t_1, δt produces a large uncertainty in the value of v_1, whereas for the larger value t_2, it produces a much smaller uncertainty in v_2.

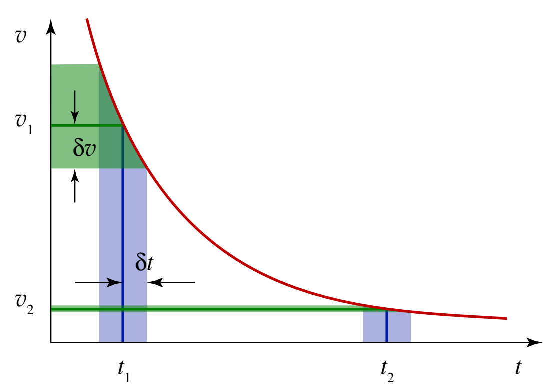


Figure 1 Effect of uncertainty in a time measurement on the uncertainty in a deduced speed.

In general, you may approximate the uncertainty in a derived quantity y from an uncertainty in the measured quantity x using a straight-line approximation to the function $y(x)$:

$$\delta y \approx \left|\frac{\partial y}{\partial x}\right| \delta x \tag{1}$$

Returning to our example of measuring speed, let us now assume that the uncertainty in d is not negligible and that the deduced value of v is uncertainty both because of δt and δd. There is no reason to suspect that if you measured the distance to be higher than its true value, you are more likely to have measured a time that is higher than its true value. These measurements are **uncorrelated**. When this is the case, add the two errors in quadrature using the formula

$$\boxed{\delta v = \left[\left(\frac{\partial v}{\partial d}\,\delta d\right)^2 + \left(\frac{\partial v}{\partial t}\,\delta t\right)^2\right]^{1/2}}$$

The extension to more uncorrelated (independent) variables is straightforward. *This is really all you need to know.*

We can specialize this formula to a common case. If $f = x^m y^n$, then

$$\frac{\delta f}{f} = \sqrt{\left(m\frac{\delta x}{x}\right)^2 + \left(n\frac{\delta y}{y}\right)^2}$$

This expresses the **fractional error** in the function $f(x, y)$ in terms of the fractional error of its arguments x and y.

Example: As an example, suppose you have measured the speed of an object to an error of 3% and its mass to within 1%. What is the error in its kinetic energy?

Solution: Since $K = \frac{1}{2}mv^2$, the relative error is

$$\begin{aligned}\frac{\delta K}{K} &= \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(2\frac{\delta v}{v}\right)^2} \\ &= \sqrt{(0.01)^2 + [2(0.03)]^2} = 0.0608\end{aligned}$$

which is 6%. Thus, the uncertainty in the speed produces essentially all the uncertainty in the kinetic energy, and this relative uncertainty is double the relative uncertainty in the speed, because kinetic energy depends on the second power of speed.

You should work out error expressions *in the lab* and evaluate them numerically for a sample data point. This will allow you to determine which terms in the quadrature sum are negligible. A spreadsheet can be a great time-saver for error propagation, showing you quickly which terms dominate and allowing you to replicate the sample calculation for all your data. If you have questions, please ask your instructor.

5. Report

Following the second week of each experiment, you will type a few-page summary of the lab containing a concise discussion of important points, including procedure, analysis, and results, sometimes with references to specific entries in your lab notebook, as appropriate. Include a discussion of the major sources, effects, and possible methods of diminishing experimental uncertainties. Include any conclusions you have reached on the basis of your experience in that lab or its results. This summary should be turned in with your lab notebook write-up; it is the primary end product of your experiment. The lab book, while secondary, must provide documentary support for the results and claims of the summary.

All work that is handed in for credit in this course, including tech reports, is regulated by the HMC Honor Code, which is described in general terms in the student handbook. In application, this Code means simply that *all* work submitted for credit shall be your own. You should not hesitate to consult texts, the instructor, or other students for general aid in the preparation of laboratory reports. However, you must *not* transcribe another student's work without direct credit to him or her, and you must *give proper credit* for any substantial aid from outside your partnership. Again remember that while you may discuss the experiment with your lab partner, your analysis of the experiment should be done individually.

General Instructions

The first week consists of an introduction to the laboratory and basic DC circuits. After that, four experiments will be done, each scheduled for two meetings (consult your section's schedule for details). The first week (meeting "a") of each experiment will typically be devoted to understanding and calibrating the apparatus and making a first round of measurements; the second week (meeting "b") will normally be for completion of data taking and analysis. Makeup work for excused absences must be arranged with your laboratory instructor, and is generally done during the last week of the semester.

Early in the semester, your instructor will work out with you the selection of four experiments and lab partners you will work with during the semester. Although every effort will be made to accommodate individual preferences as to experiments, scheduling restrictions may prevent some students from getting all their first choices; we will appreciate your flexibility here. The usual means of evaluating your lab work will be the Experiment Summary, as described in the Quick Start section of this manual. This, together with the documentary support of your lab book will form the bulk of your instructor's basis for grades. These lab write-ups are to be prepared and turned in individually by each student. Collaboration between lab partners follows rules similar to those for homework in classes; partners may share ideas and general methods (and data, obviously), but the final product turned in by each must be his/her own work.

About once during the semester (more or less; see your instructor), the written Experiment Summary will be replaced with an Oral Report presented by the lab partners as a team. The purpose is essentially the same as that of the written summaries, but presented in an oral presentation format, usually lasting about 15 minutes with graphical exhibits and a question and answer period. Again, your instructor will provide you with more detailed information as to the scheduling and format of these presentations.

What to bring to the laboratory

You may find it helpful to bring your Physics 51 textbook, *Physics* (Vol. 2), by Halliday, Resnick, and Krane "HRK" to each laboratory meeting. Also bring with you

1. A good (non-erasable) pen
2. A calculator (and a knowledge of such things as degree/radian mode, etc.)
3. A brown cover laboratory notebook of the approved type: National Brand #43-648 Computation Notebook
4. This laboratory experiment manual
5. The error analysis textbook, *An Introduction to Error Analysis*, by John R. Taylor. Part I of this text is assumed knowledge and must be mastered for satisfactory completion of the experiments.

If you have a notebook computer, you may find it very useful to bring to the laboratory to help in data recording and analysis.

Laboratory Notebook

The notebook will be an essential part of your laboratory work in this course, and it should contain a running account of the work you do. Entries should be made while the experiment is in progress, and you should use a standard format. Your notebook should:

1. provide the reader with a table of contents at the beginning, page 1, listing the number and title of the experiment, the date or dates when it was done, the page numbers in the notebook, and the name of your partner (see p. vii below);
2. contain all pertinent information, schematic diagrams, observations, data, rough calculations, results, and conclusions. Think of your entries as being those in an informal diary or journal relating daily experiences.

In the laboratory, each experiment will be performed by a team of two investigators. Each person is responsible for the complete documentation of the work performed and its analysis. That is, while you may discuss the experimental results with your lab partner, **your analysis of the experiment should be done individually**. Remember, you will write a semester-end Technical Report based on one of the experiments, and therefore a complete record of your observations and conclusions is essential when trying to recall the pertinent facts of an experiment that you may have completed weeks previously. Besides its use for the Tech Report, your lab notebook, along with the written summary and/or oral presentation, will be part of your grade on all four major experiments.

The exact form for any particular day's record will depend on the type of experiment and may vary considerably from one experiment to the next. Please observe the precautions emphasized in the laboratory instructions and appendices and accord the research-type equipment the respect it deserves. Report any damaged equipment to your instructor immediately. In two of the experiments, computers are used to speed data gathering and analysis.

> **Warning: Do not alter the data acquisition computers or their programs in any way.**

Do not install desktop backgrounds, utilities, or software on any laboratory computer. Tampering with these computers, even if intended to be harmless, violates the academic Honor Code.

There are some general rules for making entries in your **laboratory notebook**:

1. Use permanent ink, not pencil or erasable ink.

2. Do *not* use scratch paper—*all* records must be made directly in the notebook. Cultivate the skill of committing your thoughts to paper as you are working; this will prove valuable to you later. (Left-hand pages may be used for scratch work.)

3. Do not erase or use "white out"—draw a single line through an incorrect entry and write the correct value nearby. Apparent errors sometimes later prove to be important.

4. Record data in tabular form whenever possible *with uncertainties* and *give units* in the heading of each column.

5. Define all symbols used in diagrams, graphs, and equations whenever necessary to make the description or discussion understandable by another reader.

6. Take care in drawing graphs—a sloppy graph or one drawn to an inadequate scale is next to useless. Graphs often dictate what you should do next. Try to determine the probable range of your data before starting your graph. Draw graphs *in your notebook as the experiment proceeds* to help record and understand the experiment in real time. Computer generated graphs are usual when doing final analysis or fitting. Append computer generated or graph paper plots securely to the notebook pages, fully labelled and described in the narrative.

7. Determine the uncertainties in your data and results *as you go*, and let the calculations determine the number of measurements needed.

8. Record qualitative observations as well as numbers and diagrams. This is often very important to give meaning to otherwise unintelligible pages of numbers.

9. Do not fall into the habit of recording only your uncommented data in lab, leaving blank pages or spaces for description and calculation to be finished later. Entries should be made in order corresponding to the work you are doing, much like a diary report, although complicated computations and analyses are usually undertaken after the data taking procedures have been completed.

10. Not all students, or all professors, can produce a showcase-type notebook, but your work should be as neat and orderly as possible. Sloppiness and carelessness cannot be overlooked even when the results are good.

11. Your notebook will be a success if you *or a colleague* could use it as a guide in repeating or expanding upon the particular experiment at a much later date.

Because your success in the laboratory will depend to a large extent upon your notebook and the write-up you produce from it, the following additional requirements for notebooks are provided.

The physics lab books are bound notebooks of paper ruled horizontally and vertically into squares. Leave four pages at the beginning of the notebook for a table of contents. Pages are numbered in the upper right-hand corner, beginning with the first page in the book. Page 1 will be your table of contents. It should contain the column headings shown in Table 1.

Head each experiment with the same material provided in the Table of Contents. Begin with a *brief* statement of the purpose of the experiment and a brief outline of the procedure or intended procedure. Two or three sentences should be enough. There is no need to

Table 1: Table of contents of a laboratory notebook.

Expt. #	**Title**	**Partner**	**Grade**	**Date**	**Page #s**
Expt. 1	Sampling theory	Marge Inovera		10/03/06	3–11

copy that contained in the laboratory manual. Please note that you do not always have at the beginning all the information you need to prepare a full description of purpose or procedure. The objectives may be poorly defined at the start and become crystallized only in the final stages of the experiment.

Whenever practical and appropriate, include a large, clear drawing, sketch, block diagram, or photograph of your experimental arrangement, to scale when necessary. Indicate clearly on the sketch critical quantities such as dimensions, volumes, masses, etc. Avoid excessive detail; *include only essential features*. In some cases you will want to record the manufacturer, model name or number, and HMC identification number of apparatus you use, as it may be essential that you get the SAME apparatus later, or someone else may wish to compare his results with yours. In any case you will need to know and record the relevant accuracies and limitations of the instruments you use.

After the data are recorded, there will generally be some calculations to be made. **Make necessary calculations in the lab notebook as you go along to verify that your data is giving reasonable results; do not postpone all calculations after the experiment.** If the data are all treated by some standard procedure, describe the procedure *briefly* for each calculation, giving any formulas that are to be used. (Define *any* quantities appearing in the formula that have not previously been defined.) *Always give a sample calculation*, starting with the formula, substituting experimental numbers, and carry the numerical work down to the result.

Grading policies

Your laboratory grade will be based upon:

75% Lab write-up and notebook, oral report(s), prelab preparation and in-lab performance and progress.

25% Technical Report on one of the experiments.

Semester-end technical report

Each student is required to write one technical report for this course. This represents a significant fraction of your grade, so it is important that it is turned in on time and is well written. It will be based on an in-depth treatment of one of the four experiments you performed during the semester. After consultation with your instructor, you may decide it is desirable to take supplementary data for your report. For this purpose, and for consultation with your instructor on the content of your report, a dedicated meeting date is provided in the semester schedule for Tech Report preparation. Check with your laboratory instructor for the due dates of the first draft (for comments) and of the final tech report. Allow sufficient time for several revisions after you receive comments on your draft. Appendix D of this manual contains some discussion of writing style guidelines for your report, which should be prepared in the manner of a more-or-less formal publication. Your instructor can provide you with further guidance and examples of proper technical report style.

Philosophical Musings

Some Notes on Scientific Method

from *Zen and the Art of Motorcycle Maintenance* by Robert M. Pirsig

Actually I've never seen a cycle-maintenance problem complex enough really to require full-scale formal scientific method. Repair problems are not that hard. When I think of formal scientific method, an image sometimes comes to mind of an enormous juggernaut, a huge bulldozer–slow, tedious, lumbering, laborious, but invincible. It takes twice as long, five times as long, maybe a dozen times as long as informal mechanic's techniques, but you know in the end you're going to get it. There's no fault isolation problem in motorcycle maintenance that can stand up to it. When you've hit a really tough one, tried everything, racked your brain and nothing works, and you know that this time Nature has really decided to be difficult, you say, "Okay, Nature, that's the end of the nice guy," and you crank up the formal scientific method.

For this you keep a lab notebook. Everything gets written down formally so that you know at all times where you are, where you've been, where you're going, and where you want to get. In scientific work and electronics technology, this is necessary, because otherwise the problems get so complex you get lost in them and confused and forget what you know and what you don't know and have to give up. In cycle maintenance, things are not that involved; but when confusion starts, it's a good idea to hold it down by making everything formal and exact. Sometimes just the act of writing down the problems straightens out your head as to what they really are.

The logical statements entered into the notebook are broken down into six categories: (1) statement of the problem, (2) hypotheses as to the cause of the problem, (3) experiments designed to test each hypothesis, (4) predicted results of the experiments, (5) observed results of the experiments, and (6) conclusions from the results of the experiments. This is not different from the formal arrangement of many college and high-school lab notebooks, but the purpose here is no longer just busywork. The purpose now is precise guidance of thoughts that will fail if they are not accurate.

The real purpose of scientific method is to make sure Nature hasn't misled you into thinking you know something you don't actually know. There's not a mechanic or scientist or technician alive who hasn't suffered from that one so much that he's not instinctively on guard. That's the main reason why so much scientific and mechanical information sounds so dull and so cautious. If you get careless or go romanticizing scientific information, giving it a flourish here and there, Nature will soon make a complete fool out of you. It does it often enough anyway, even when you don't give it opportunities. One must be extremely careful and rigidly logical when dealing with Nature: one logical slip and an entire scientific edifice comes tumbling down. One false deduction about the machine and you can get hung up indefinitely.

In Part One of formal scientific method, which is the statement of the problem, the main skill is in stating absolutely no more than you are positive you know. It is much better to enter a statement "Solve Problem: Why doesn't cycle work?" which sounds dumb but is correct, than it is to enter a statement "Solve Problem: What is wrong with the electrical system?" when you don't absolutely know the trouble is in the electrical system. What you should state is "Solve Problem: What is wrong with cycle?" and then state as the first entry of Part Two:

"Hypothesis Number One: The trouble is in the electrical system." You think of as many hypotheses as you can; then you design experiments to test them to see which are true and which are false.

This careful approach to the beginning questions keeps you from taking a major wrong turn which might cause you weeks of extra work or can even hang you up completely. Scientific questions often have a surface appearance of dumbness for this reason. They are asked in order to prevent dumb mistakes later on.

Part Three, that part of formal scientific method called experimentation, is sometimes thought of by romantics as all of science itself, because that's the only part with much visual surface. They see lots of test tubes and bizarre equipment and people running around making discoveries. They do not see the experiment as part of a larger intellectual process, and so they often confuse experiments with demonstrations, which look the same. A man conducting a gee-whiz science show with fifty thousand dollars' worth of Frankenstein equipment is not doing anything scientific if he knows beforehand what the results of his efforts are gong to be. A motorcycle mechanic, on the other hand, who honks the horn to see if the battery works is informally conducting a true scientific experiment. He is testing a hypothesis by putting the question to nature. The TV scientist who mutters sadly, "The experiment is a failure; we have failed to achieve what we had hoped for," is suffering mainly from a bad script-writer. An experiment is never a failure solely because it fails to achieve predicted results. An experiment is a failure only when it also fails adequately to test the hypothesis in question, when the data it produces don't prove anything one way or another.

Skill at this point consists of using experiments that test only the hypothesis in question, nothing less, nothing more. If the horn honks and the mechanic concludes that the whole electrical system is working, he is in deep trouble. He has reached an illogical conclusion. The honking horn only tells him that the battery and horn are working. To design an experiment properly, he has to think very rigidly in terms of what directly causes what. This you know from the hierarchy. The horn doesn't make the cycle go. Neither does the battery, except in a very indirect way. The point at which the electrical system directly causes the engine to fire is at the spark plugs; and if you don't test here at the output of the electrical system, you will never really know whether the failure is electrical or not.

To test properly, the mechanic removes the plug and lays it against the engine so that the base around the plug is electrically grounded, kicks the starter lever, and watches the sparkplug gap for a blue spark. If there isn't any, he can conclude one of two things: (a) there is an electrical failure or (b) his experiment is sloppy. If he is experienced, he will try it a few more times, checking connections, trying every way he can think of to get that plug to fire. Then if he can't get it to fire, he finally concludes that a is correct, there's an electrical failure, and the experiment is over. He has proved that his hypothesis is correct.

In the final category, conclusions, skill comes in stating no more than the experiment has proved. It hasn't proved that when he fixes the electrical system, the motorcycle will start. There may be other things wrong. But he does know that the motorcycle isn't going to run until the electrical system is working and he sets up the next formal question: "Solve problem: what is wrong with the electrical system?"

He then sets up hypotheses for these and tests them. By asking the right questions and choosing the right tests and drawing the right conclusions, the mechanic works his way down the echelons of the motorcycle hierarchy until he has found the exact specific cause or causes of the engine failure and then he changes them so that they no longer cause the failure.

Part II

Experiments

Experiment 1

Basic DC Circuits

Abstract

At an MIT commencement a few years ago, graduates were handed a battery, some wire, and a lightbulb, and asked to light the bulb. Their fumblings and confusions were videotaped and presented in the documentary *Minds of Our Own.*[1] By the end of this experiment you should be well-equipped to shine at graduation time, bringing fame and glory to Harvey Mudd College.

1.1 Overview

This experiment is an introduction to simple direct-current (DC) circuits, which consist of wires (good conductors), resistors (lousy conductors), and voltage supplies (batteries and DC power supplies). You can think of the flow of electric current in a wire much like the flow of water in a pipe. Water flow is measured as an amount of water per unit time (liters/second, for example) that passes a particular point in a pipe; an electric current is an amount of electric charge per unit time (coulombs/second) that passes a particular point in a circuit. This combination is given a special name for convenience's sake, the **ampere**: 1 A = 1 C/s. A device that measures electric current is called an **ammeter**.

Water flows through a pipe because of a pressure difference. Electricity flows through a wire because of an **electric potential difference**. Potential differences are measured in volts (V). The work done on a charge Q by the electrostatic field as the charge moves from point A to point B is

$$W = Q(V_A - V_B) = QV_{AB} \tag{1.1}$$

It is easier to pump a lot of water quickly through a short pipe with a large cross section than through a long, skinny pipe. Similarly, it is easier to pump a lot of current through a short fat wire than a long skinny one. The long skinny one has a greater **resistance** to the flow of current. A **resistor** is an electrical component made from a poor conductor (see Appendix B for more details) that functions like a very skinny section of pipe to limit the flow.

1.2 Theory

Aristotle (384 – 322 BCE) held that objects on Earth fall at steady speed, and heavier objects fall faster than light objects. Think about dropping a stone and a piece of paper at the same time from the same height and you may be willing to muster some charity for the seemingly naive Aristotle, who was said to have performed his experiments with small stones in water. Galileo Galilei (1564 – 1642) argued persuasively against this doctrine, holding that insofar as air resistance could be neglected, all objects fall the same way: at a steadily increasing rate. That is, they accelerate. You know all about that.

Electrons in a wire behave a whole lot more like Aristotelian pebbles than Galilean cannonballs. In response to an applied push (the electric field associated with the applied voltage), the free electrons in the metal drift with an average speed that is proportional to the strength of the push. In consequence, the electric current that flows in the conductor is proportional to the applied voltage; double the voltage and you double the amount of current. (If the current gets too large, this law breaks down. The resistor may, too!)

The **resistance** is given by dividing the voltage by the current,

$$R = \frac{V}{I} \tag{1.2}$$

Resistances are measured in **ohms**: 1 Ω = 1 V/A. The resistance of a conductor depends on the material of the conductor, as well as the conductor's size and shape. Georg Simon Ohm (1789 – 1854) described the proportionality implied by Eq. (1.2) in 1827, and it is known as **Ohm's law** (usually written in the form $V = IR$).

Why don't the electrons accelerate under the influence of an applied force? Because they are continually colliding with atoms fixed in the metal of the wire. The collisions transfer momentum and energy from the electrons to the wire, preventing the electron drift velocity from increasing, and heating the atoms of the

[1] In fairness, MIT exacted a certain revenge on the Harvard folks responsible for this little experiment the following year by asking Harvard graduates to explain why the Earth has seasons. You can appreciate their "insights" in *Private Universe*.

wire. This effect is called **Joule heating**. The rate at which electrical energy is converted to heat (dissipated) is

$$P = IV = I^2 R = \frac{V^2}{R} \tag{1.3}$$

where in the final two forms we have used Eq. (1.2) to eliminate either V or I. If I is in amperes, V in volts, and R in ohms, then the power P is in watts (W).

Two conservation laws govern the flow of charges in conducting circuits and are known as **Kirchhoff's laws**:

(a) Because electric charge is conserved, the algebraic sum of currents from any point in a circuit is zero. That is, the sum of the currents arriving at a point in the circuit is equal to the sum of the currents leaving the same point.

(b) Because the electrostatic force between any pair of charges is conservative, we can define a unique value of electrostatic potential to each point in a circuit (up to an overall constant). Therefore, the sum of voltage changes across all elements forming a closed loop in a circuit is zero.

These laws are illustrated in Fig. 1.1.

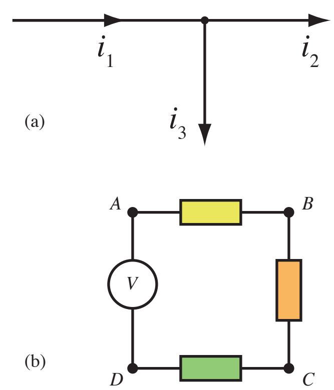


Figure 1.1 (a) The sum of currents entering the junction is equal to the sum of currents leaving it: $i_1 = i_2 + i_3$. (b) The sum of voltage drops around a closed circuit vanishes: $V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$.

1.3 Precautions

Warning: Improperly connecting a circuit can fry equipment, so please read the following guidelines carefully.

Measuring current

To measure the current flowing through a component, insert an ammeter *in series* so that all the current that flows out of the wire *must* flow through the ammeter (see Fig. 1.2). By design, an ammeter has very low resistance to avoid changing the current it measures. If the ammeter is connected in parallel, so current can choose whether to pass through the ammeter or the resistor you are investigating, **you will destroy the ammeter!**

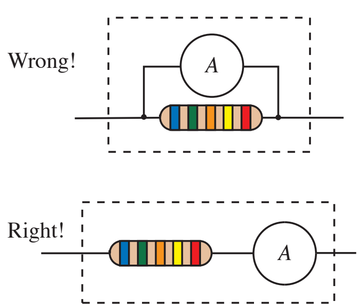


Figure 1.2 To measure current passing through a circuit element you must insert the ammeter *in series* with the element, as shown in the lower figure.

Meters have a maximum current or voltage they can handle. Carefully respect this limit or you will damage the equipment.

Measuring voltage

To use a voltmeter to measure the voltage drop across a component, wire the voltmeter *in parallel* with the component, as shown in Fig. 1.3. This is *opposite* to the case of the ammeter. A voltmeter ideally has a very high resistance, to avoid changing the voltage drop it is measuring. If you hook it up in series with the component you are investigating, you probably won't damage the meter, but you will disrupt the flow of current through your circuit and get nonsensical readings.

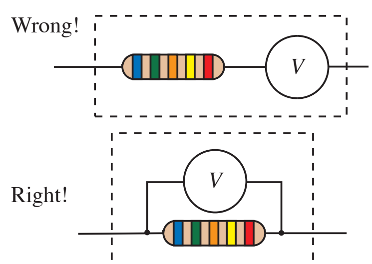


Figure 1.3 To measure the voltage drop across a circuit element, you must wire the voltmeter *in parallel with* the element, as shown in the lower figure.

Resistors have a maximum allowable power dissipation. You can use Eq. (1.3) to compute the power that will be produced as heat for a given current I.

1.4 Procedure

This part of the experiment is intended for students with little or no experience with DC circuits. While these exercises are elementary, they should familiarize you with the basic equipment and procedure to be used in later experiments. It is recommended that you team up with a partner of comparable experience and that you go through the experiment at a pace appropriate to your background. Consult your instructor if you have questions or if you need to verify that a circuit is properly wired.

You must document your work carefully in your lab notebook. Be sure to reproduce schematics of the circuits you construct and describe your observations and measurements in sufficient detail that they could be reproduced by someone else at a later time. Neatness and organization are worth cultivating!

When making measurements of current and voltage, the accuracy of the meters will usually not be a limitation. Digital multimeters can have accuracies better than 1 part in 1000. All measurements will be uncertain in at least the last digit displayed, but you should consult the back of the meter for more detailed error estimates. Write down in your lab notebook not only the value of the estimates but how you have determined them.

When making current measurements in the experiments below, use the 20-mA scale on the multimeters. If the 2-mA scale is used, you will get an inaccurate reading due to a phenomenon called **loading**.

1.4.1 Resistors in Series

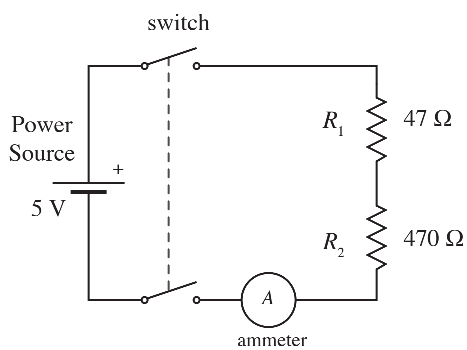


Figure 1.4 A series circuit

Set up the circuit shown in Fig. 1.4. Ask your instructor how to set the meters to read voltage and current if you have any questions. Before powering any circuit, it is important to verify that no components will be damaged when the circuit is completed. For this experiment, 1 V should be safe, since the current in the circuit will not violate the power rating of the resistors.

1. With the switch on, measure the voltage at the switch outputs and the voltages across R_1 and R_2. Test Kirchhoff's voltage law.
2. Measure the current through the circuit. Is it the same in all segments?
3. Calculate the product of current and voltage for each of the resistors to determine the power being dissipated.
4. Does R_1 or R_2 dominate in determining the current in the circuit? Think of replacing both resistors with either R_1 or R_2 — which more closely approximates the circuit you have already built? Test your prediction by rebuilding the circuit.
5. Calculate the values for R_1 and R_2 from your current and voltage measurements (neglect error measurements). Compare these results with the given value of those resistors. Writing $R_{\mathrm{tot}} = R_1 + R_2$, do your current measurements agree with the result that $I = V_{\mathrm{tot}}/R_{\mathrm{tot}}$?

1.4.2 Resistors in Parallel

Now arrange R_s, R_1, and R_2 in a parallel configuration as shown in Fig. 1.5.

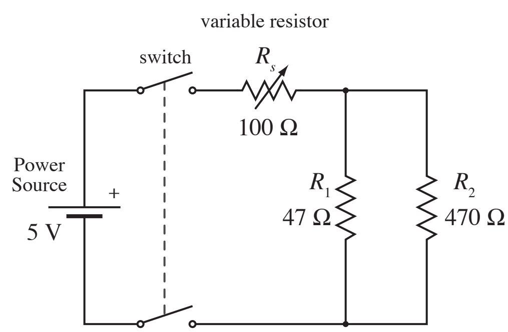


Figure 1.5 A parallel circuit

1. Measure the voltages across R_1 and R_2. Are the results equal within uncertainty?
2. Measure the currents through all the resistors, and compare your measurements to the predictions of Kirchhoff's current law.

3. Writing your result as $I_s = I_1 + I_2$ or

$$\frac{V}{R_{||}} = \frac{V}{R_1} + \frac{V}{R_2}$$

we see that

$$\boxed{\frac{1}{R_{||}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Are your resistance values consistent with this relation?

4. Does R_1 or R_2 dominate in determining the current through R_s? Think of replacing both of these resistors with either R_1 or R_2. Which of the new circuits would more closely approximate the one you have already built? Test your prediction by rebuilding your circuit. Compare your results to what you found for the series circuit.

1.4.3 Circuit Loading

If time permits, measure the resistance of a nominal 4.7-MΩ resistor in two ways. (The color code for a 4.7-MΩ resistor is yellow–violet–green; see Appendix B.) First, using clip leads on banana jacks, connect the resistor directly to a multimeter set to measure resistance. Record the value and use information on the back of the meter to estimate the uncertainty in the value.

Then wire up the circuit illustrated in Fig. 1.6 and measure the voltage across the resistor, and the current in the circuit. From these values, compute a value for the resistance and compare to the value you obtained with the more straightforward method.

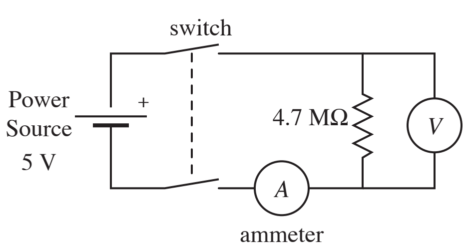


Figure 1.6 Measuring the voltage across a resistor to determine its resistance.

1.4.4 Charge and Discharge of a Capacitor in an RC Circuit (Optional)

If time allows, connect the circuit illustrated in Fig. 1.7. The voltage source ε is a function generator outputing a square wave. The variable resistance and capacitance are provided by decade boxes. Set the frequency of the square wave generator slightly higher than 100 Hz, and adjust the value of the resistance and capacitance to produce a rounded square wave.

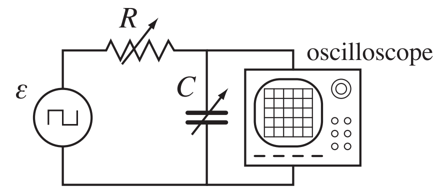


Figure 1.7 A basic RC circuit.

A capacitor is a pair of conducting plates separated by a small gap. Current may flow onto one plate, where it accumulates and establishes an electric field in the region between the plates, whose effect is to oppose the further flow of current onto the plate. The greater the capacitance C, the more charge must flow onto the plate to change the capacitor's voltage.

If a steady voltage is applied to the circuit, current will flow through the resistor onto the top capacitor plate until the voltage drop across the capacitor just matches the applied voltage. The current decays exponentially to zero, and the voltage across the capacitor approaches the steady state value exponentially with a characteristic time τ.

Play around with the values of R and C to determine how each affects the rise and fall time of the voltage across the capacitor. Do the dependencies make sense? Can you determine a quantitative relationship between the values of R and C, and the exponential "decay" time τ? Hint: the voltage across a capacitor satisfies

$$V_C = \frac{Q}{C} \tag{1.4}$$

whereas the voltage across the resistor satisfies

$$V_R = IR = R\frac{dQ}{dt} \tag{1.5}$$

Experiment 2

Magnetic Fields and an Absolute Determination of Current

Abstract

Electric currents produce magnetic fields, which in turn exert torques on permanent magnets. By using a simple geometry to permit the field to be readily calculated from the current, and by carefully balancing the torques from the current and from the magnet, it is possible to measure the current (an *electrical* quantity) by making measurements solely of the *mechanical* quantities of mass, length, and time. Your mission: to measure a current *as accurately as possible* using a ruler, some compasses, a bar magnet, a stopwatch, and a scale.

2.1 Overview

1. Current in a coil produces a magnetic field $\mathbf{B}_{\text{coil}}$ along the axis at the coil's center. The field is proportional to the current i flowing in the coil and given by Eq. (2.2) below. Measuring $\mathbf{B}_{\text{coil}}$ will allow you to measure i.

2. To measure $\mathbf{B}_{\text{coil}}$, you will produce an equal and opposite magnetic field with a bar magnet, as determined using a magnetic compass needle. To quantify the field of the bar magnet, and therefore $\mathbf{B}_{\text{coil}}$, you will measure the strength of the bar magnet's dipole moment M. The magnetic field of the bar magnet is proportional to M and falls off with distance according to Eq. (2.3) below.

   The calibration requires two measurements. First, you will use a compass needle to compare the strength of the bar magnet's field to that of the Earth. Since the compass needle aligns with the horizontal component of the total magnetic field at its center, by orienting the bar magnet's field perpendicular to the Earth's, you can deduce their relative strength from the angle of rotation from local magnetic north. Measurements at several magnet positions yield a value for M/B_h, where B_h is the local value of the horizontal component of Earth's magnetic field.

3. Second, you will suspend the bar magnet in a sling and make it oscillate about the Earth's field. The rate of oscillation is proportional to both M and B_h, allowing you to deduce their product, MB_h. Since you already know M/B_h, you may calculate both the magnet strength, M, and the local value of Earth's magnetic field, B_h.

4. Using the compass needle to make $B_{\text{bar}}(x)$ equal to $B_{\text{coil}}(i)$ for different currents i, you can deduce the current i from the magnet position x.

As the above list makes plain, this experiment involves many individual measurements which are combined to determine the current flowing in the coil. They may be done in any order. Each measurement contributes to the uncertainty of that determination; *your challenge is to make the combined uncertainty as small as possible.*

2.2 Background

The great mathematician, astronomer, and physicist Johann Carl Friedrich Gauss first realized in 1833 the possibility of determining electromagnetic quantities, such as electric current, by measuring only mechanical quantities, such as mass, length, and time. To honor this work a common set of electromagnetic units are called **gaussian units**, and the gaussian unit for magnetic field strength is called the **gauss**.[1] This experiment is a variant of Gauss's original design.

Later work on electric currents focussed on resistance standards as the practical means of quantifying current, and the common unit of current, the **ampere**, was defined to have a convenient magnitude. The Italian physicist J. Giorgi showed in 1901 that it was possible to combine the mechanical units of the metric system (meters, kilograms, and seconds) with an electrical unit (ohms or amperes) to produce a "rational" system of units. In 1954 the CGPM[2] officially added the funda-

[1] Gaussian units use the gram, centimeter, and second as base units for mechanical quantities, although Gauss himself used the millimeter. See `http://www.bipm.org/en/si/history-si/`

[2] General Conference on Weights and Measures, *Conférence générale des poids et mesures*, in French.

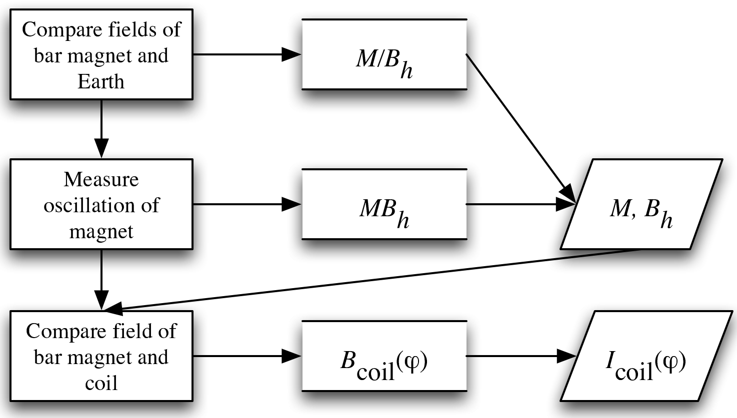


Figure 2.1: Logical flow of the experiment.

mental unit of the ampere to the *Système international* (SI), using the definition

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to* 2×10^{-7} N/m *of length.*

The reconciliation of mechanical and electrical units was accomplished through the definition of the constant μ_0, the **magnetic permeability of free space**; it is defined to have the exact value

$$\mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{kg\,m\,A^{-2}\,s^{-2}}$$

and is used in expressions such as

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i d\mathbf{l} \times \hat{\mathbf{r}}}{r^2} \tag{2.1}$$

which relate currents and geometric quantities to magnetic fields. Notice that the odd factor of 10^{-7} in the definition of μ_0 is chosen to give the ampere a "reasonable" size.

Recent work on circuits that permit one to transfer one electron at a time across a conducting bridge may lead in the future to a more refined definition of the SI unit of charge, the coulomb, in terms of the electron charge. In the meantime, the coulomb is defined as $1\ \mathrm{C} = 1\ \mathrm{A} \times 1\ \mathrm{s}$, and the accepted number of electrons in a coulomb is $6.241\,506 \times 10^{18}$ (1996 value).

Background information on the theory of magnetic dipoles in external fields can be found in HRK Chapter 32, sections 5 and 6. Review on the theory of magnetic fields produced by current loops will be found in HRK Chapter 33, sections 1 and 2.

2.3 Theory

The magnetic field produced at the center of a circular coil of N turns, radius R, and current i comes directly from applying the Biot-Savart law, Eq. (2.1), and results in

$$B_{\text{coil}} = \frac{\mu_0 N i}{2R} \tag{2.2}$$

The magnetic field from a bar magnet on its symmetry axis is also fairly simple. A magnetized object like a bar magnet is well approximated as a magnetic **dipole** at distances large compared with the dimensions of the dipole, where the magnetic field drops off in magnitude proportional to $1/r^3$. However, the behavior is a bit more complicated at distances comparable to the size of the dipole, as is the case in this experiment. An adequate description is to think of the magnet as consisting of two point-like poles of equal and opposite strength, separated by a distance $2b$. [Note: Such magnetic "monopoles" do not actually exist, as far as anybody knows. They are however, a convenient fiction to use here for the purpose of describing the magnet's close-in field.] You can readily show that the resulting approximate magnetic field at a distance r from the center of the magnet is

$$B_{\text{bar}} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - b^2)^2} \tag{2.3}$$

In the above expression, $M = 2bp$ is the **magnetic dipole moment**, and p is the monopole strength of each pole. In SI units, dipole moment has the units of A m^2. The geometry is illustrated in Fig. 2.2.

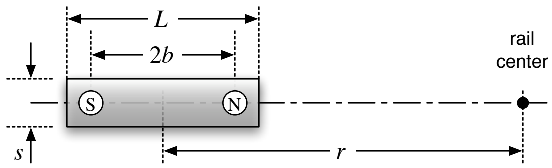


Figure 2.2 Bar magnet dimensions and locations.

When the magnetometer rail is oriented as shown in Fig. 2.3, perpendicular to the Earth's horizontal magnetic field B_h, the compass magnet points in the direction of the net horizontal vector sum of the Earth's field and that of the bar magnet. Accordingly, the angle of deflection ϕ is given by

$$\tan\phi = B_{\text{bar}}/B_h \qquad (2.4)$$

Combining equations (2.3) and (2.4), we obtain an expression for the *ratio* M/B_h :

$$\frac{M}{B_h} = \frac{2\pi}{\mu_0} r^3 \left(1 - \frac{b^2}{r^2}\right)^2 \tan\phi \qquad (2.5)$$

By measuring the angle ϕ for a variety of distances r (on both sides) we obtain a best estimate of the ratio M/B_h. If we knew the value of B_h, we would be almost finished, since the dipole moment and therefore the magnetic field B_{bar} would be known. However, the Earth's magnetic field is highly spatially variable, especially in a building environment filled with ferromagnetic materials, so we only know its value roughly in advance. Measuring b and the oscillation frequency of the bar magnet in Earth's field gets us around this problem.

When the bar magnet is immersed in the Earth's magnetic field, it experiences a **torque**, given by $\boldsymbol{\tau} = \mathbf{M} \times \mathbf{B}_h$. Note that the magnitude of the torque is proportional to the *product* MB_h. Once we know both the product and the ratio of M and B_h, we know their separate values, since $M = \sqrt{(M/B_h)(MB_h)}$.

If suspended by a thread of negligible torsion, the magnet will oscillate back and forth under the restoring torque of the magnetic field. The magnet's equation of motion is given by

$$I\frac{d^2\theta}{dt^2} = -MB_h \sin\theta \qquad (2.6)$$

where I is the rotational moment of inertia of the bar magnet about its center. This is the same equation that governs a pendulum. For oscillations of small amplitude, the oscillation frequency f is related to $M\,B_h$ by the expression

$$MB_h = 4\pi^2 f^2 I \qquad (2.7)$$

Once M and B_h have been determined, the final result of the experiment can be obtained. Knowing M, we can use Eq. (2.3) to calculate B_{bar} at any distance r. For a variety of current values i (both positive and negative), we have measured the distance r at which the compass deflection is zero, at which point $|\mathbf{B}_{\text{bar}}| = |\mathbf{B}_{\text{coil}}|$. Then from Eq. (2.2) we obtain i, a measurement of the current which we have obtained without reference to external electrical standards.

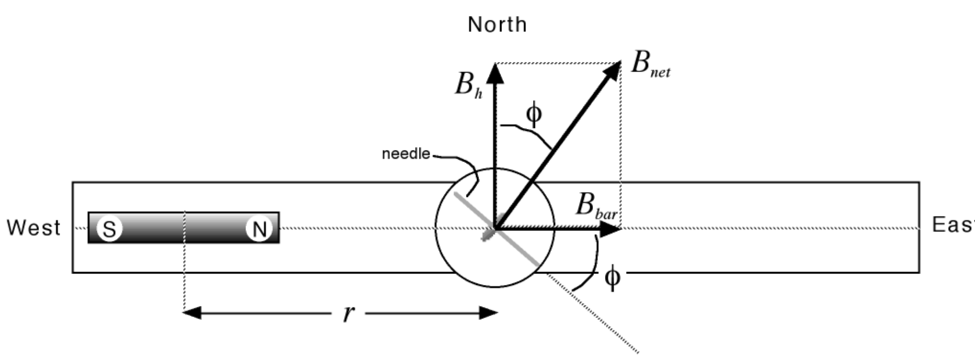


Figure 2.3: Top view of the magnetometer rail assembly.

2.4 Experimental Equipment and Procedures

2.4.1 Preliminary setup

Figure 2.4 shows another layout view of the magnetometer rail. As seen in the photo of Fig. 2.5, the scale on the side of the rail and the scribe mark on the side of the plastic magnet holder allow the distance r between the magnet and rail centers to be measured. Throughout the experiment, all superfluous magnets and magnetic materials should be removed from the vicinity of the apparatus. Carefully orient the rail per-

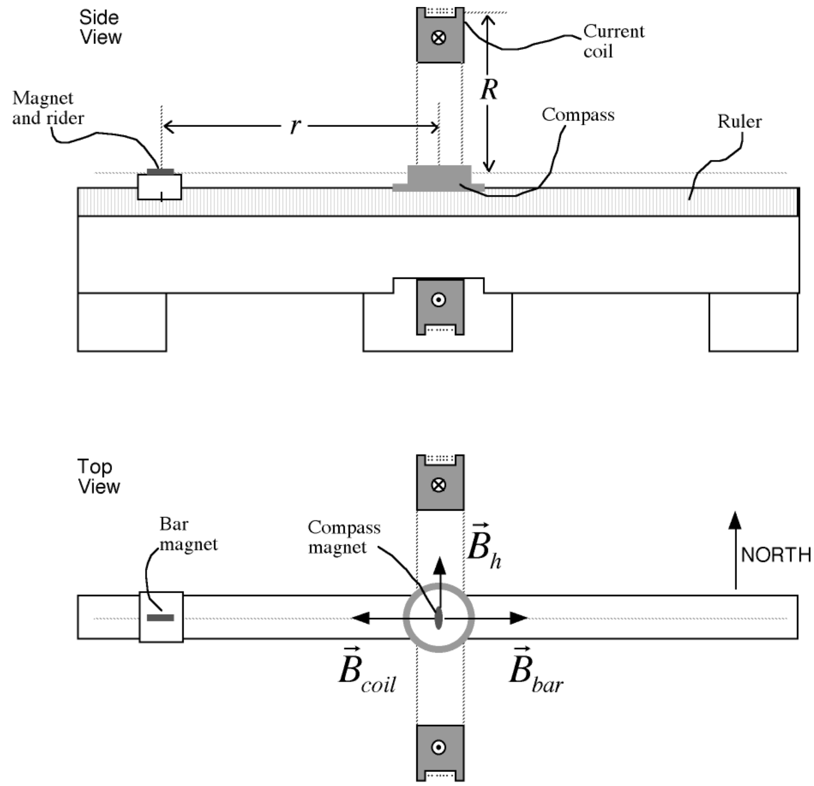


Figure 2.4: Schematic views of magnetometer rail assembly and current coils. Compass orientation shown is for $\mathbf{B}_{\text{coil}}$ exactly cancelling $\mathbf{B}_{\text{bar}}$.

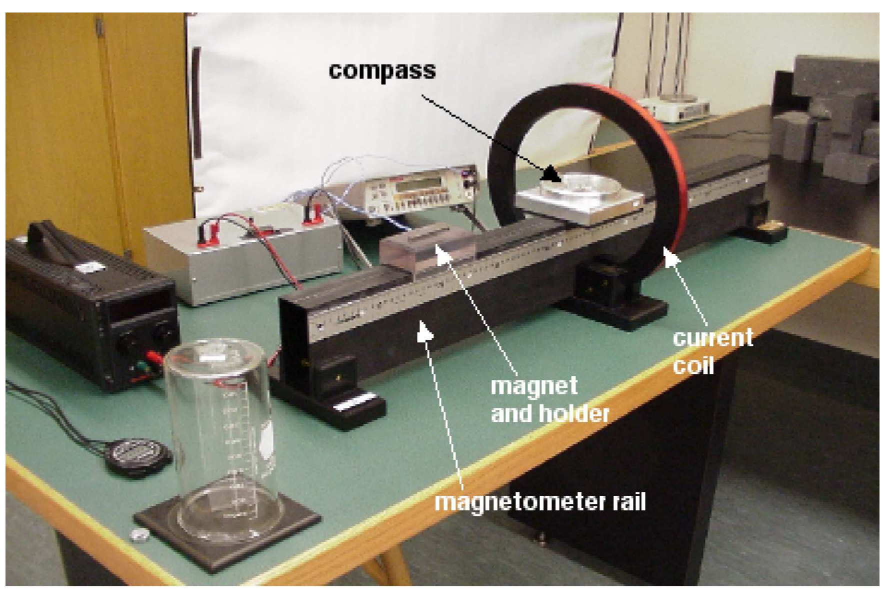


Figure 2.5: Photograph of the magnetometer apparatus.

pendicular to the ambient magnetic field, so that the indicator needle reads zero degrees and is parallel to the rail axis. The position of the rail on the laboratory bench should be marked with tape so that it can be returned to the same position when it has to be moved later in the experiment.

Before taking data, you should as always familiarize yourself with the equipment and take some important preliminary measurements. After selecting a bar magnet to use, mark its poles so that you always use it in the same orientation throughout the experiment. (*You should also identify the magnet you are using if you need to use the same one the following week. As a courtesy to the other students, please try not to drop or damage the magnets in any way that would require them to retake their data from the previous week!*)

To determine the true magnetic center of your magnet, compare the distances r measured on the right and left sides of the compass where the compass deflection is 45°. If these distances differ slightly (as they probably will), the magnet's effective center is offset from its geometric center. You should carefully record this offset distance and use it during the rest of the experiment to correct all your measured values of r.

2.4.2 The Current coil

In this part of the experiment, current from a variable source is connected to the circular coil. If you have not already done so, you should determine the number N and radius R of the coil's turns, for use in Eq. (2.2).

Note: Large and potentially hazardous electric currents are employed in this part of the experiment. Reread Appendix A on this topic!

The circuit used to supply the current to the magnet coils is shown in Fig. 2.6. The reversing switch allows you to change the direction of current, so that you can perform trials with the magnetic field pointing in both directions. **To protect this switch from arcing damage, please reduce the current to zero before reversing direction.** The load resistor is located inside the switch box, and its purpose is to limit the current flowing in the circuit. Make sure that the meter (labeled "A" in the diagram) is set to ammeter mode. In order to minimize the effects of stray magnetic fields, the switch box, the multimeter and power supply should be kept several feet from the compass, and twisted pair wire leads are used to connect the coil to the supply and switch box.

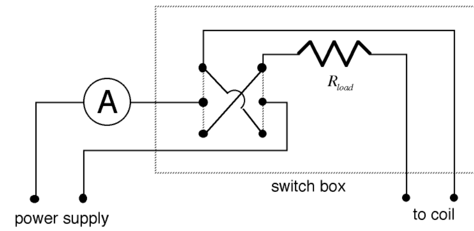


Figure 2.6 Circuit diagram of coil current supply with reversing switch

You will adjust the current using the knobs on the power supply, and measure the "accepted" value of the current with the multimeter. At the conclusion of the experiment, these current values will be compared with your computed current results, in order to validate them and quantify any errors present.

In taking measurements, explore several current settings, both positive and negative, in the range of 0.4 to 0.7 A, as read by the multimeter.

> **Warning: Do not exceed a current of 0.7 A.**

The more measurements of this kind that are made, the better you'll be able to characterize any systematic errors or asymmetries in the measurements. After selecting a current reading, the magnet is placed so that its field exactly cancels that of the coil. Note that for each current setting, there are *two* magnet positions (on the right and left) that can be measured.

2.4.3 Compass deflection measurements

When the magnet is placed on the rail, the compass deflects to a new equilibrium position and the angle ϕ is measured from it. Although in theory a single measurement of r and ϕ would suffice to determine M/B_h from Eq. (2.5), in practice a larger number of measurements is needed to determine the ratio with good precision. Measurements should be taken at a generous number of locations on both sides of the center, at distances that probe both the near-dipole field and the far field $1/r^3$ regions. In this way, it is possible to capture and account for any non-idealities in the bar magnet (more on this subject below).

> **Warning: When handling the bar magnet in this and subsequent parts of the experiment, take care not to subject it to mechanical shocks or magnetic influences that could change its magnetization.**

Note that in Eq. (2.5), r is assumed to be a positive number. However, it will be convenient for us to define r as negative on one side of the rail in order to separate our north pole and south pole measurements from one another; we just need to remember to take the absolute value of r when using Eq. (2.5). In moving the magnet from the positive to negative side, take care to retain a consistent orientation of the magnet, with the north pole facing center on one side and the south pole facing center on the other side. Also, in selecting positions to take measurements, bear in mind that the least relative error in the quantity $\tan\phi$ is obtained for angles near 45°. So while it is desirable to obtain measurements over a wide range of angles, those obtained near 45° will contribute with the most weight to determining M/B_h.

2.4.4 The Bar magnet and torsional oscillations

As shown in Fig. 2.7, in the experiment's third part the magnet is suspended from a slender thread beneath an inverted glass beaker. In the presence of the external field $\mathbf{B}_h$, the magnet oscillates about its equilibrium position. The measurement of the frequency of oscillation is accomplished with a stopwatch, and can be performed with quite high precision by timing the duration of several consecutive periods.

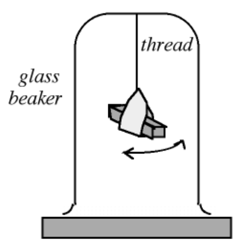


Figure 2.7 Torsional magnet suspension.

In order for the measurement to be meaningful, it must be carried out in the *same location* that the M/B_h measurement was made. In order to do this, the coil and compass are temporarily removed from the rail assembly, allowing the beaker and magnet to be placed on the rail in their place. (*Note: The aluminum mounting base for the compass can be replaced with a non-metallic one, which eliminates the eddy currents that tend to damp out the magnet oscillations rather rapidly.*)

It is also necessary to make accurate physical measurements of the bar magnet itself. The moment of inertia of the uniform bar of length L and square cross section s^2 is,

$$I = \frac{m(L^2 + s^2)}{12} \tag{2.8}$$

The mass m is measured with a lab balance and the dimensions are measured with calipers (along with accompanying uncertainties).

Consideration of equations (2.3) and (2.5) also shows that it is necessary to determine the distance $2b$ between the effective poles of the magnet. There are different possible approaches to getting this quantity. A simple preliminary approach is to estimate the positions of the poles visually using small compasses, and measure the distance $2b$ between them with a ruler. This approach will give a fairly good "ballpark" value for initial estimates, but often systematically differs by several millimeters from the results of mathematically finding the best fit to Eq. (2.5). Measure and record this *provisional* value for b by this method, but keep in mind that a better value will be obtained in later analysis by determining b to best fit the data with Eq. (2.9) below.

2.5 Experimental Analysis

The principal goal of this experiment is to calculate the current i in the coil as accurately as possible, and to compare these calculations to the "accepted" value read from the multimeter.

Perhaps the most significant challenge here is in distilling the best value of M from all of the rail measurements. You should first carry out a quick "ballpark" calculation to verify that your results make sense, before getting too deeply into the detailed analysis. It is at this stage that it is usually easiest to catch a simple mistake, such as incorrect unit conversion or a missing factor of 2, etc. *You should set up an organized spreadsheet to help with the analysis.*

Pick a single representative measurement of r vs. ϕ near $\sim$45° and using Eq. (2.5), get a provisional value of M/B_h. Then using Eq. (2.7), get a provisional value for $M\,B_h$; you can defer any error propagation until later, since your purpose here is to perform a quick check. If everything is correct up to this point, the dipole moment M should be on the order of $0.5 - 1.0$ A m^2 and the horizontal magnetic field B_h should be roughly 2×10^{-5} T. If your results are not in this range, go back and find the mistake before proceeding further. Finally, set equations (2.2) and (2.3) equal to one another using your provisional values for M and b to obtain the current i for one of your trials in the first part of the experiment. If all is well, it should agree with the multimeter reading within about 10% or so. Next we'll discuss how the final analysis will hopefully refine this result.

The simplest approach would be to simply obtain separate estimates of M/B_h from Eq. (2.5) for every measurement, and then take their average. However, this approach has some disadvantages. Some of the important sources of error, such as errors in the pole spacing b for example, tend to *systematically* push the M/B_h estimate off in one direction, rather than randomly. As a result, averaging over a lot of measurements yields a result that retains a bias from the "true" value, that cannot be averaged out.

A more sophisticated and potentially better approach is to *fit* the expression of Eq. (2.5) to the unknown parameters

M/B_h and b, given the experimental measurements of ϕ vs. r. Solving the expression for ϕ, we want to fit the experimental data to the function,

$$\phi = \arctan\left\{\frac{\mu_0}{2\pi}\frac{(M/B_h)}{|r|^3\left(1-\frac{b^2}{r^2}\right)^2}\right\} \tag{2.9}$$

Using a plotting/fitting computer utility such as Origin or Kaleidagraph and proper error weighting, you can solve for the unknowns M/B_h and b, as well as their uncertainties. Fig. 2.8 shows schematically how we might expect this fit to look when plotted.

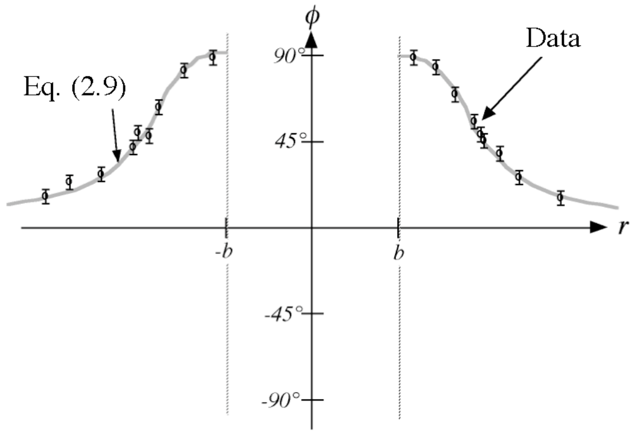


Figure 2.8: ϕ vs. r data fitted to Eq. (2.9).

2.5.1 Fitting

The use of a plotting program to perform such a parameter fit is a great labor-saving convenience, and pretty easy. However, it pays to be familiar with the software and how it works before you need to use it. Here are some hints and reminders for using **Kaleidagraph** and **Origin** to analyze this experiment:

Kaleidagraph

When defining your fitting function, Kaleidagraph requires the independent variable (i.e., r) to be named `m0`. It requires the parameters (i.e b and M/B_h) to be named `m1` and `m2` (and so on). So with the program set to compute in degrees (rather than radians), the definition of your fitting function should look something like this:

```
invtan(m2/(5e6*abs(m0^3)*(1-(m1/m0)^2)^2));
m1=0.03;
m2=32000
```

Note that it is necessary to provide the initial guess values following the function definition, or the program won't be able to get a solution. Also remember that you *must* specify a **weighted** fit, or else the χ^2 goodness-of-fit figure you obtain will be meaningless. In Kaleidagraph, you need to divide this number by the number of degrees of freedom (# of data pts. − # of parameters) in order to get χ^2 per degree of freedom.

Origin

When defining your fitting function, Origin allows you to name the parameters whatever you like. For simplicity here, we will use the names **b** and **M** and call the independent variable **R**. Origin works only in radians, so we must convert into degrees. The fitting function definition for Origin would look like:

```
(180/pi)*atan(M/(5e6*abs(R^3)*
  (1-((b^2)/(R^2)))^2))
```

Origin also requires you to provide starting guess values for the fitting parameters, but these are specified in the 'Start Fitting Session' dialog, rather than in the function definition. In Origin, the reported χ^2 value is already normalized to χ^2 per degree of freedom.

Igor

Like Origin, Igor allows you to name the fit parameters anything you like. For simplicity here, we will use the names **b** and **M** and call the independent variable **r**. Igor works only in radians, so we must convert into degrees. The fitting function definition for Igor would look like:

```
f(r) = (180/pi)*atan(M/(5e6*abs(r^3)*
  (1-((b^2)/(r^2)))^2))
```

Igor also requires you to provide starting guess values for the fitting parameters, but these are specified in the 'Coefficients' pane of the fitting dialog. Igor reports the value of χ^2, so if you would like the reduced χ^2 value, divide by the number of degrees of freedom.

All

In the case of both Kaleidagraph and Origin, the fitting programs do your error analysis for you automatically. The fitted values of the parameters are reported along with their propagated uncertainties. However, before you use these values uncritically, remember that these uncertainties are only valid to the extent that the residuals of the fit are randomly distributed *and* χ^2 per degree of freedom for the fit is approximately unity. See `http://www.physics.hmc.edu/analysis/fitting.php` for more information.

Experiment 3

RLC Resonance

Abstract

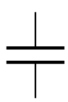
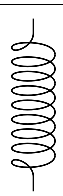
Current in a series circuit comprising a resisitor, capacitor, and inductor tends to oscillate at a frequency that depends on the inductance L of the inductor and the capacitance C of the capacitor, although resistance damps out the oscillations. You will study how the oscillating current generated by an applied AC voltage depends on frequency, observing the maximum response (**resonance**) at the natural frequency set by L and C.

3.1 Overview

When you push a child on a swing, the amplitude of the child's motion builds up swing by swing: with each push you pump energy into the child-swing system. Unless you are warped, you tend to push the child at the **natural frequency** of the system, which is the frequency at which it oscillates when you stop pushing and the child swings by herself.

In this experiment, you get to be warped and push at all sorts of frequency, from the pathetically low to the ridiculously high. Between these extremes lies the natural frequency, the one the system likes the best, as judged by the amplitude of the motion it experiences. In the case of the swing, it's the child that moves. In an RLC circuit, it's the electric current.

To help you gain an intuition for the three circuit elements involved, here's a brief introduction:

Element	Equation	Properties
 R	$V_R = IR$	A **resistor** is a piece of poorly conducting wire, a pipe through which electric current flows. The magnitude of the current depends on the voltage across the resistor: double the voltage and you double the electric current. The mechanical analog of a resistor is a viscous liquid, like honey, through which you are trying to push an object. Resistors turn good, clean electrical energy into heat, a process that is called **Joule heating**.
 C	$V_C = \dfrac{Q}{C}$	A **capacitor** is a pair of conducting plates facing one another across a gap. Electric charges flowing onto one plate produce an electric field in the gap between the plates, causing the voltage of one plate to differ from that of the other. The mechanical analog of a capacitor is a spring: as you pile up charge, it creates an electric force that makes it harder to pile up more charge. As charge builds up, capacitors store energy in the electric field between their plates.
 L	$V_L = L\dfrac{dI}{dt}$	An **inductor** is a coil of wire. As current flows in the coil, it generates a magnetic field inside the coil, which opposes a further change in the current. The mechanical analog of inductance is mass: the greater L the more the inductor opposes a change in the current that flows through it. Inductors store energy in the magnetic field generated by the current flowing through them.

1. Before lab — Read this chapter of the lab manual. The theoretical introduction is significant, so devote adequate time to understanding it. It will make your experience in lab much easier! If needed, you may read HRK Chapter 37 for more detail.

2. Choose a specific experiment option in consultation with your instructor.

3. You must analyze the results and present a rigorous discussion of your findings. Often, the experience gained (and mistakes made) during the first week's meeting will allow you to be that much

more efficient and effective in completing the experiment the following week.

3.2 Theory

Resonance is a ubiquitous and very important phenomenon in physics, which is not restricted to electronic circuits. "Pumping" a swing and tuning a radio are both examples you are familiar with (although you may not have realized you were "resonating" at the time!). The key feature of all resonance effects is that the size of the effect (for example, the height the swing goes or the volume of the the sound) depends on the frequency of the excitation. Several Nobel Prizes in physics have been awarded over the years for studies of resonance — for example, nuclear magnetic resonance (the basis of magnetic resonance imaging, or MRI, an important medical tool of recent vintage). In this experiment you will study the resonance phenomenon in a simple AC (alternating current) electrical circuit.

The electrical circuit you will study is composed of a resistor, a capacitor and an inductor. The resonance effect in this experiment stems from the fact that the voltage drop across the individual circuit elements depends on the frequency of the sinusoidal voltage applied to the circuit. In some cases, the voltage drop can be of larger magnitude than the applied voltage (just as the swing can go much farther than you push it, given the right frequency). To understand this effect, we begin with the basics of elements in an AC circuit, and how their frequency-dependent behavior is mathematically described.

Briefly, the three topics we should understand for the purposes of this experiment are:

- AC voltages represented as complex numbers (what are amplitude and phase?)
- Impedance of discrete circuit elements (relationship between voltage and current)
- Analysing a circuit with more than one element (how to apply the Kirchhoff laws)

3.2.1 AC voltages as complex numbers

A steady AC voltage is represented as a sinusoidal function of time, with a fixed amplitude, frequency and phase shift. We call the angular frequency ω (ordinary frequency $\nu = 1/T = \omega/2\pi$); amplitude we call and the phase . Therefore we write the time-dependent voltage as

$$V = V_0 \cos(\omega t + \phi) = V_0 \mathrm{Re}\left[e^{i(\omega t+\phi)}\right] \tag{3.1}$$

In the latter half of the above equation, we make use of Euler's identity to express the sinusoid in terms of complex exponentials. In this notation the imaginary unit is i and the real part of the expression gives us the cosine part.[1] At first this seems an unfriendly complication, but upon reflection we see that the exponential expressions are much easier to manipulate algebraically than sines and cosines. In practice, we typically leave the taking of the real part as tacitly understood, and just leave the expression in its complex form, that is, $V_0 e^{i(\omega t+\phi)}$.

Another simplification is realized if we assume that the only time dependence in the system is in the sinusoidal voltage at the frequency ω (that is, the system is in steady state). Then without any loss of generality we can simply factor out the time dependence in the expressions, and focus our attention on the amplitude and phase of the voltage. Schematically we have replaced real voltages with a complex number as in,

$$V = V_0 e^{i\phi} \tag{3.2}$$

We can perform all needed calculations using this complex representation. When working with a quantity like Eq. (3.1) that has explicit time dependence, the observable instantaneous voltage is always obtained by taking the real part. When we leave out the time dependence as in Eq. (3.2), the absolute value of the complex number tells us its amplitude, while the argument or phase tells how much the sinusoid lags or leads in its cycle relative to the cosine function.[2]

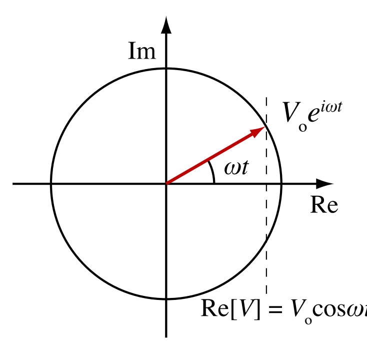


Figure 3.1 Phasor representation of a sinusoidally varying voltage at time t. The actual measured voltage is the projection of the phasor on the real axis.

[1] Note that engjneers sometjmes use $j = \sqrt{-1}$ as the jmagjnary unjt, presumably so jt wjll not be confused wjth electrjcal current.

[2] Recall that if a complex number $z = x + iy$ is not already expressed in polar form, its absolute value and argument are given by $A = |z| = \sqrt{x^2 + y^2}$ and $\phi = \arg(z) = \arctan(y/x)$. So $x + iy = Ae^{i\phi}$ in this example. If the phase is defined as positive for a *lag*, as is the case with our LabView program !!!, then $x + iy = Ae^{-i\phi}$ and $\phi = -\arctan(y/x)$.

This algebraic representation of the voltage has a geometrical analog called a phasor diagram. A phasor is a vector in the complex plane whose magnitude is the amplitude of the voltage and whose direction gives the phase.[3] For example, we show the phasor diagram for an AC voltage in Fig. 3.1. In the illustrated case, the angle the phasor makes with the real axis is just ωt; if the waveform were delayed in phase by an amount ϕ (as it is in Eq. (3.1)), then this angle would be $\omega t - \phi$.

The locus of points traced out by the phasor as a function of time is a circle as shown in Figure 2. The true voltage oscillates between $\pm V_0$.

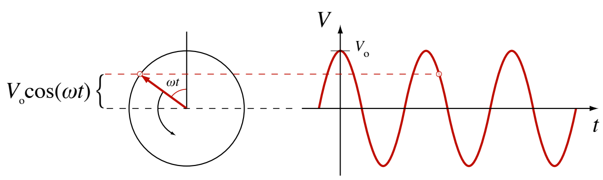


Figure 3.2 Time dependence of relationship between phasor representation and real observed sinusoidal voltage.

Now consider two AC voltages, both at the same frequency, as shown in Fig. 3.3. The voltages have different phases. In Fig. 3.3 (a), the phase difference is shown as the angular separation of the phasor vectors. In Fig. 3.3 (b) this phase difference is appreciable in the different instantaneous magnitudes of the two voltages, even though they have the same amplitude.

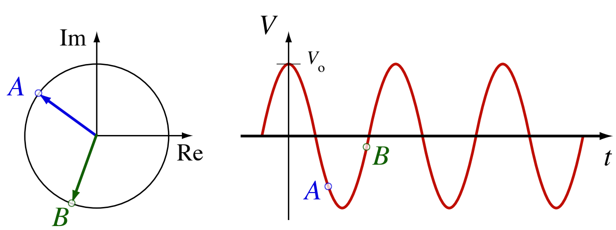


Figure 3.3 Relationship between phasors of different phase.

3.2.2 Complex impedance

Impedance is just a slight generalization of the concept of resistance, with which you are already familiar in the context of Ohm's law. Just as in the case of the resistance of a resistor, impedance is the ratio of voltage to current for a given element, be it a resistor, capacitor or inductor. How this differs from resistance is that unlike the case of a pure resistor, the ratio of voltage to current may depend on frequency, and there may be a phase difference (lag or lead) between the sinusoidal voltage and current. For this reason, the impedance of a circuit element is in general a complex number (to quantify the phase lag) and depends on frequency (rather than being a fixed constant). In general we define impedance Z by the relation, $V = IZ$, where all the quantities may be complex. If for example V and I have different phases, then

$$V = V_0 e^{i\phi_1} \quad \text{and} \quad I = I_0 e^{i\phi_2}$$
$$\text{implies that} \quad Z = \frac{V_0}{I_0} e^{i(\phi_1 - \phi_2)} \tag{3.3}$$

Impedance of a resistor

This is the simple and familiar case of Ohm's law. As we discussed above, for a pure resistor there is no phase lag or frequency dependence, so the impedance reduces to the simple (real) result,

$$V_R = IR \quad \text{implies} \quad \boxed{Z_R = R} \tag{3.4}$$

Thus the impedance for a resistor is just same as its DC resistance.

Impedance of an inductor

The relationship between the voltage and current for an inductor is

$$V_L = L\frac{dI}{dt} \tag{3.5}$$

Applying the complex representation of the AC voltage and current, this becomes,

$$V_L e^{i\omega t} = i\omega L I e^{i\omega t} \quad \text{or} \quad V_L = i\omega L I \tag{3.6}$$

Thus the impedance for an inductor is

$$\boxed{Z_L = i\omega L} \tag{3.7}$$

If the current is assumed real across an inductor (cosine-like) then the corresponding voltage is purely imaginary (sine-like), or different in phase by 90°. Also, we see that for low frequencies, an inductor presents little obstruction to the flow of current, but at large ω, it takes a large voltage to cause current to flow; hence the inductor will then act somewhat like a large resistor, neglecting phase effects.

Impedance of a capacitor

The relationship between voltage and current for a capacitor is

$$V_C = \frac{Q}{C} = \frac{\int I\, dt}{C} \tag{3.8}$$

Again, going to complex representation we find,

$$V_C e^{i\omega t} = \frac{I e^{i\omega t}}{i\omega C} \quad \text{or} \quad V_C = \frac{I}{i\omega C}$$

[3] The "arrows" Feynman uses in *QED* to discuss photons are phasors that represent the complex probability amplitude.

Thus, the impedance for the capacitor is

$$\boxed{Z_C = \frac{1}{i\omega C} = -\frac{i}{\omega C}} \tag{3.9}$$

So here we see that the voltage and current of the capacitor also differ in phase by 90°, but in the sense opposite to that of the inductor. The capacitor is also opposite to the inductor in that it presents the greatest obstacle to current flow at low frequencies, and little obstruction as ω grows large. Much insight into asymptotic behavior can be achieved by considering these components to be large or small resistors in the appropriate frequency limit.

These are the basic concepts to understand; the detailed use of these impedances to derive the behavior of an RLC circuit is found in the Appendix of this experiment.

3.3 Experimental Procedure

3.3.1 Equipment and Software

Having covered the theory behind this experiment, we now turn our attention to the equipment you will use to carry it out. Although in principle you could measure all the needed voltage amplitudes and phases manually with an oscilloscope, this would be very tedious and limit the amount of data you could take in a reasonable time. Instead, you have an Igor-based computer data acquisition system which greatly simplifies the procedure.

The computer acquisition system has four analog signal input channels; one for the driving voltage and the remaining three for the voltages across the three circuit components (R, L, and C). The application interface provides a virtual oscilloscope window, in which you can view up to all four waveforms simultaneously in real time. In addition to displaying the waveforms, the program displays the frequency of the applied sinusoidal voltage signal, as well as the amplitude and phase of the voltages across the resistor, inductor, and capacitor (measured with respect to the driving voltage).

The application also produces plots of the amplitude/phase results, and exports the data for use in other graphing and analysis programs. Within the Igor application, you can view or print graphs of amplitude and phase vs. frequency on either linear or logarithmic axes. The application can superimpose theoretical curves for the RLC circuit [as described in equations (3.15), (3.17), and (3.19]. There is also an option to plot data and theory in phasor form on the complex plane. More detailed step-by-step instructions on the use of the program are provided at the end of this section.

In addition to the computer equipment, you are provided with a variety of components and hook-up wires with which to construct your circuit and connect it to the computer. Figure 3.4 shows a schematic diagram of the arrangement of components for the standard RLC circuit measurement. Attention should be paid to observing the correct polarity in the connection of the computer input channels; if reversed the resulting voltage will be negative and therefore 180° out of phase with theory. Also note that you are provided with four digital multimeters which when hooked up in parallel with each channel provide valuable additional diagnostic information.

Choosing an Experiment

This experiment provides you with the choice of several options to pursue. Depending on your particular interests and previous electronics experience, you may find one of the options more attractive than the others. The following description details the procedure for the standard experiment option, followed by descriptions of other possible alternative experiments:

(a) Standard option: series RLC circuit

Construct a series RLC circuit like the one shown in Fig. 3.4. The usual nominal values to use for the capacitance and inductance are $C = 5\ \mu$F and $L = 500$ mH. With these values fixed, you will repeat the measurements twice for two different values of the resistance, $R = 50\ \Omega$ and $500\ \Omega$. Note that you can readily use one of your digital multimeters to directly measure the actual values of the resistances R and R_L. It is a good idea to do this before assembling the series circuit. However, the actual values of L and C may differ from their nominal manufacturer's values, so one of your objectives in this experiment will be to extract from your results refined estimates of these component values. **Pay attention to the polarity of the banana-to-BNC connectors: the outside conductor on the BNC cable goes to the marked side of the banana jacks.**

Using the LabView application, collect amplitude and phase data for each of the components. Vary the

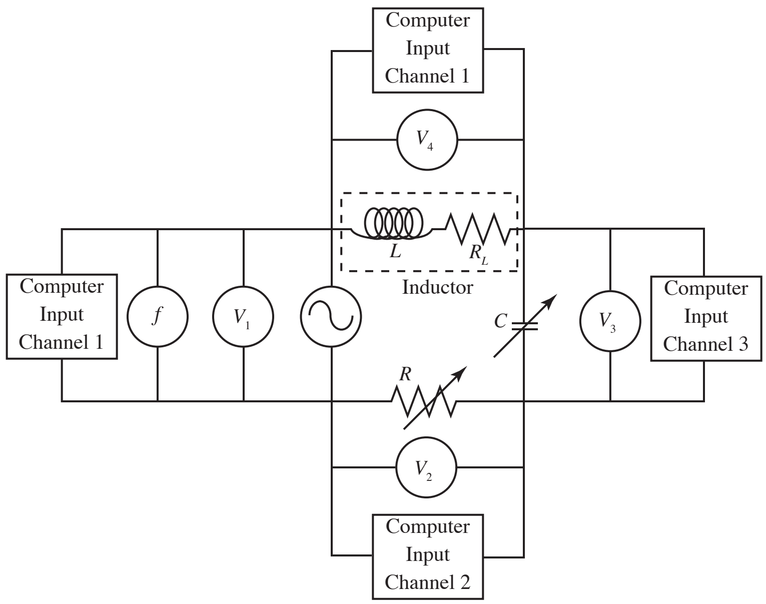


Figure 3.4: An RLC circuit driven by an oscillating voltage. The voltmeters shown as circles are the digital multimeters. The boxes showing channels are the connections to the analog-to-digital converter in the computer allowing the computer to function as an oscilloscope. The (unavoidable) internal resistance of the inductor is explicitly shown.

input frequency from the function generator over the range of roughly 20 to 400 Hz. During or after collecting data points, you can display and print graphs of your measurements on either a linear or logarithmic frequency scale. When deciding at what frequencies to take data, keep in mind that a spacing that looks even on a linear scale will not look evenly spaced on a logarithmic one. Plan your data taking to capture the important features of the functions without taking an excessive number of data points (20 to 30 usually suffices.)

Unless you have made a mistake somewhere, your amplitude and phase results should appear quite close to the theoretical curves using the nominal values of R, L, C, and R_L. Compare and contrast the results you get for the two different choices of the damping resistor, R. In particular, try to estimate the sharpness of the resonance peak with the "**quality factor**" defined as $Q = \omega_0/\Delta\omega$, where $\omega_0 = 1/\sqrt{LC}$ is the resonant (peak) frequency and $\Delta\omega$ is the peak's full width at half-maximum. It is possible to show that the quality factor of the voltage resonance should be related to total resistance, L, and C through

$$Q = \frac{\omega_0 L}{\sqrt{3}(R + R_L)} \tag{3.10}$$

Check the consistency of your experimentally determined values of Q and ω_0. The culmination of your analysis will be in determining the best values of L and C consistent with your data. Although you could try a series of trial-and-error guesses for the values in question, this approach can be a tedious and unwieldy.

Adjust this to allow fitting within Igor The preferred approach is to use "inverse model" or fitting techniques. To do this, use the Igor application to save your data to a text file for import into Kaleidagraph or Origin. You can then use the plotting and fitting functions of those programs to find the best fit to equations (3.15), (3.17), and (3.19) for the 50-Ω data. A word about uncertainties: there's no obvious value of uncertainty produced by the LabView RLC application for the output voltages or phases, however in order to get meaningful uncertainties from either fitting or forward modelling, you have to have some uncertainties to start from. This is an example of when to use your scientific intuition. Start off with a "reasonable" estimate, say for example about 0.2% of full scale. Then if the resulting reduced χ^2 value for your fit differs a lot from unity, you can adjust your first estimate up or down as needed until you have (in your judgment) realistic uncertainties. **something about residuals**

(b) Parallel RLC analysis option

This option may appeal to students who enjoy the mathematical theory side of things. Because it involves some mathematical derivations, the experimental data-taking is reduced in this option. Construct a circuit as in the standard option, but instead of a series circuit, connect two of the three elements in parallel with one another and the third element in series with that pair. Which of the three elements, R, L, or C, to make the series element is up to you; making the resistor the series element $[R(L||C)]$ is a choice that leads to a particularly interesting resonance behavior. The theory of the parallel-series circuit is not derived in this lab manual, although the method to do so is similar. The challenge of this option is to derive the appropriate equations describing your chosen circuit, and then compare them with experimental data you take for a single choice of R, L, and C. To keep the amount of work to a reasonable level, only derive and measure the voltage amplitude and phase for the single series element. (When taking data with the LabView set-up, the two idle data channels can be turned off and disregarded.)

Obviously, the LabView application's built-in theory applies only to the standard circuit and is useless for this option. You can check with your instructor if you think you need some assistance in getting the equation derivations right. The expressions are similar, but a little hairier than equations (17), (19) and (21). Keep in mind that because of the way the arctangent function is defined, you may have to add or subtract π radians to or from your phase predictions to avoid unphysical discontinuities, but this is usually obvious on a plot. The same approaches described in the standard option are used to find the optimal values and uncertainties for L and C that best explain your data.

(c) Unknown "black box" determination option

For those who find the standard experiment option a little too cut-and-dried, this option provides a little "mystery." Your instructor has a number of sealed boxes that contain a resistor, inductor and capacitor connected in series inside. In addition to the two end terminals (to which you connect the driving voltage), there is a pair of test terminals that gives you access to the voltage across one of the unknown components (you're not told which one of the three). In addition to measuring the DC resistance across any pair of the four terminals, you gain information about this system by driving it with voltages of various frequencies, and using the computer system to measure and record the amplitudes and phases across the test terminals. Do not drive the system with voltages greater than 0.1 V or the inductors will not act as linear devices.

Using the methods and theory described above, you should be able to deduce the values of L, C, R and inside the box, and which of the three components the test terminals are connected to. When using the computer to assist you in fits of your data, it is best to fix the value of R since you have measured it directly and the fit is not tightly constrained otherwise. You may have time to analyze more than one unknown box.

(d) Design your own experiment option

Using your own ideas and/or ideas from the above three options, you may want to devise your own custom experiment. It should use the basic methods and apparatus of the experiment as described above, but may emphasize some different aspect or circuit design that you find particularly interesting. Consult with your instructor for his or her approval of your experimental plan.

Appendix — Analyzing a circuit with more than one element

The payoff for our development of the ideas of complex impedance is realized now in the ease with which we can analyze circuits consisting of combinations of these three elements. Series and parallel impedances add in a manner exactly analogous to DC resistances. As a result, analyzing a circuit composed of inductors, capacitors and resistors is no harder conceptually than dealing with the same circuit composed of all resistors.

As an illustration, we'll look at the series RLC circuit and compare it with a series circuit of three resistors. Figure 5. (a) An RLC circuit shown driven by an oscillating voltage. (b) An analogous DC circuit composed of three resistors.

The analogous resistor circuit is quite easy to analyze. The current through each resistor must be the same, and follows from the series resistance of the three,

$$I = \frac{V_0}{R_1 + R_2 + R_3} \tag{3.11}$$

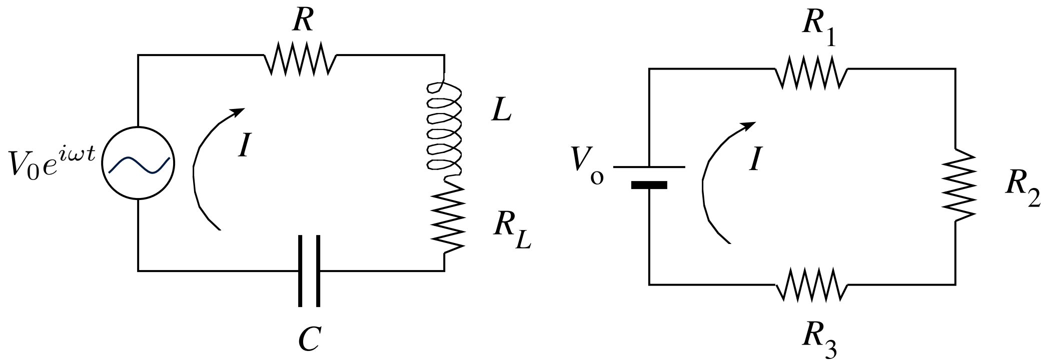
Since the three resistors form a simple **voltage divider**, we can also easily write the voltage drop across each one,

$$\begin{aligned} V_1 &= V_0 \frac{R_1}{R_1 + R_2 + R_3} \\ V_2 &= V_0 \frac{R_2}{R_1 + R_2 + R_3} \\ V_3 &= V_0 \frac{R_3}{R_1 + R_2 + R_3} \end{aligned}$$

Now we apply the same reasoning to the series RLC circuit. In this case the current through each element of the circuit is given by,

$$I = \frac{V_0}{Z_R + Z_L + Z_C} \tag{3.12}$$

Now in this case, the impedances are not all real, so we expect that the resulting current I will be some amount out of phase with the driving voltage. The voltages across each of the elements are now simply given



(a) AC circuit (b) Resistor circuit

Figure 3.5: (a) An RLC circuit shown driven by an oscillating voltage. (b) An analogous DC circuit composed of three resistors.

by,

$$V_R = V_0 \frac{Z_R}{Z_R + Z_L + Z_C}$$
$$V_L = V_0 \frac{Z_L}{Z_R + Z_L + Z_C}$$
$$V_C = V_0 \frac{Z_C}{Z_R + Z_L + Z_C}$$

We must take note of the fact that by virtue of their construction as coils of wire, inductors generally have a non-negligible resistance, in addition to their designed inductive impedance. As a result, we have to replace the expression in equation (7) with one that takes the finite resistance of the inductor into account:

$$Z_L = i\omega L + R_L \tag{3.13}$$

In this expression, R_L is the resistance in ohms of the inductor that would be measured at DC ($\omega = 0$). Armed with these expressions, we can now predict all of the voltages and phases across every element of our circuit. For the **resistor**, we obtain,

$$V_R = V_0 \frac{R}{R + i\omega L + R_L + \frac{-i}{\omega C}} \tag{3.14}$$

which yields the amplitude and phase (lag),

$$|V_R| = V_0 \frac{R}{\sqrt{(R+R_L)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$
$$\phi_R = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R + R_L}\right) \tag{3.15}$$

For the **inductor**, we obtain,

$$V_L = V_0 \frac{i\omega L + R_L}{R + i\omega L + R_L + \frac{-i}{\omega C}} \tag{3.16}$$

which yields the amplitude and phase,

$$|V_L| = V_0 \left[\frac{R_L^2 + (\omega L)^2}{(R+R_L)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\right]^{1/2}$$
$$\phi_L = -\arctan\left(\frac{\omega L}{R_L}\right) + \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R + R_L}\right) \tag{3.17}$$

And finally, for the **capacitor**, we obtain,

$$V_C = V_0 \frac{-i/\omega C}{R + i\omega L + R_L + \frac{-i}{\omega C}} \tag{3.18}$$

which yields the amplitude and phase,

$$|V_C| = V_0 \frac{1/\omega C}{\sqrt{(R+R_L)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$
$$\phi_C = \frac{\pi}{2} + \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R + R_L}\right) \tag{3.19}$$

These derived expressions for the amplitude and the phase lag depend on frequency, and contain the mathematical essence of the resonance phenomena you will be observing. Although it will be up to you in the experiment and analysis to explore these functions, we present in Fig. 3.6 some typical plots to show you what you might expect.

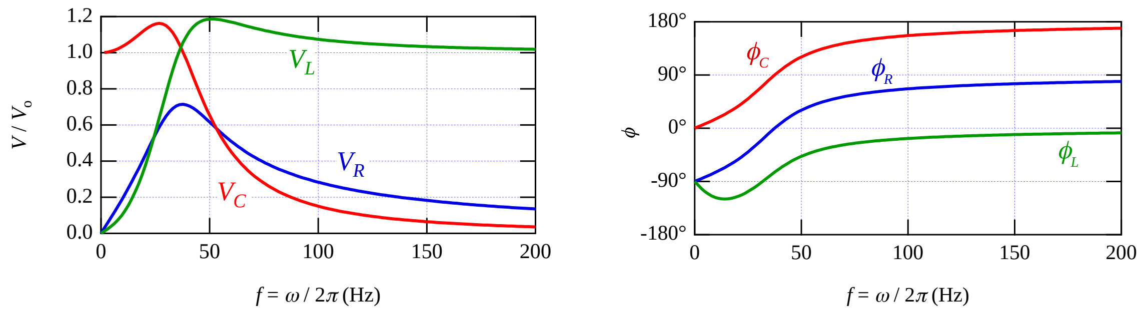


Figure 3.6: Example RLC voltage amplitudes as a function of frequency. For the illustrated case, $R = 100\ \Omega$, $L = 0.6$ H, $C = 30\ \mu$F, and $R_L = 40\ \Omega$.

In pondering these equations and their graphs, you should ask yourself the following questions:

- What is significant about the system's response at the **resonant frequency**, $\omega_0 = \sqrt{1/LC}$?
- Why do V_L and V_C approach the limits of zero and V_0 at opposite extremes?
- What influences the width of the peak in V_R, and the "overshoot" in V_L and V_C?
- Why do the phases always differ from each other by 90°, *except* for ϕ_L as $\omega \rightarrow 0$?
- How is this system analogous to resonance of a mechanical spring-mass-damper system?

Experiment 4

Geometrical Optics

Abstract

Light bends on passing from one medium to another. By analyzing this refraction you will measure the factor by which light is slowed on entering acrylic or water. Curved surfaces cause the magnitude of the bending to depend on surface position, producing lenses which can be used to form images. You will study image formation using both converging and diverging lens systems.

4.1 Overview

This experiment consists of two independent investigations, both involving the refraction of light. On the first day, you will study the relationship between the angle at which light arrives at an interface between two dissimilar media and the angle at which it leaves the interface on the opposite side. This relationship is known as **Snel's law**.[1] Light bends at the interface because its speed of propagation is different in the two media. The ratio of the speed of light in the vacuum to its speed in a medium is called the **index of refraction** n of the medium. For isotropic media, this single number is all that is required to characterize the bending, although the index of refraction typically depends on the frequency or wavelength of light.

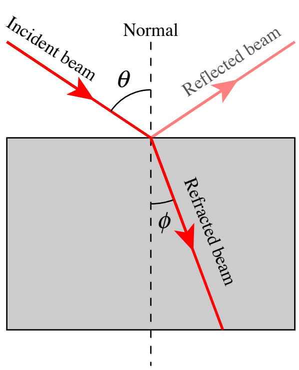


Figure 4.1 Refraction at the interface between two transparent media.

Snel's law may be derived from Fermat's principle, which holds that the time taken by a light ray to pass between two points should be an extremum. As discussed in Feynman's *QED*, the "arrows" (probability amplitudes) corresponding to paths near the extremal path predicted by Fermat's principle all point in the same direction (add in phase), and so reinforce one another. Arrows from other paths tend to point equally in all directions, thus canceling each other out. This behavior leads to the simple law of refraction. The index of refraction of each medium generally depends on the wavelength λ of the light, although this dependence may be weak. You may compare the refraction of different materials or the refraction by the same material at different wavelengths.

The second investigation focuses on lenses, both converging and diverging. Converging lenses can form **real images** of **real objects**; that is, the image of a tangible object can be viewed by placing a paper or screen "downstream" of the lens. Converging lenses are thicker in the middle than at the edges, which causes light rays to bend towards the axis of cylindrical symmetry (to converge). **Diverging lenses** are thinner in the center and cause light to bend away from the axis of symmetry. If you look at something through a diverging lens, it appears smaller. A diverging lens produces a **virtual image** of a real object, which means that the object is located on the "upstream" side of the lens. Therefore, you cannot view the image on a screen, since it would block the light before it gets to the lens! However, using a combination of converging and diverging lenses, you *can* produce a real image from the diverging lens that you can view on a screen. You will use various lenses both alone and in combination to study image formation and to measure the focal length of the lenses.

[1]Before you reach for your red pen, this is how Willebrord Snel spelled his name. See C. F. Bohren, *Clouds in a Glass of Beer: Simple Experiments in Atmospheric Physics* (Dover, New York, 1987/2001) 100. See also `http://www.snellius.tudelft.nl/snelvanroyen.html`.

4.2 Background and Theory — Refraction

The angles of incidence θ and refraction ϕ are illustrated in Fig. 4.1, and are related to each other by Snel's law

$$n_1 \sin\theta = n_2 \sin\phi \tag{4.1}$$

where n_1 and n_2 are the **indices of refraction** of the two media. By definition, the index of refraction is the ratio of the speed of light in the vacuum to the speed of light in the medium. The refractive index of air may be taken equal to one to the accuracy of this experiment.

The law of refraction has an interesting history. The ancient Greeks knew a version of this law that worked only near normal incidence. Credit for the discovery of Eq. (4.1) is customarily given to the Dutch mathematician and scientist, Willebrord Snel van Royen (1580–1626), who was the son of a University of Leiden mathematics professor. He followed in his father's footsteps at the University of Leiden, focusing his research on mathematics, including trigonometry; cartography; and optics. He discovered the law of refraction in 1621,[2] but did not publish his work. The first published account was given by the French philosopher, mathematician, and scientist René Descartes (1596–1650) in *La Dioptrique* (1637). Descartes did not acknowledge Snel's work, although many believe that he had seen it. Notwithstanding this bit of awkwardness, the law of refraction is known as Descartes' law in French-speaking countries.

4.3 Experimental Procedures — Refraction

In this part of the experiment you are provided with a helium-neon laser of wavelength 632.8 nm, attached to a circular fixture, the center of which is occupied by a semicircular piece of transparent optical material. The optical sample can be rotated so that the entering laser beam is presented at any desired angle of incidence to the flat face of the sample. A measuring tape has been affixed to the circular outside screen to permit you to measure the position of the reflected and refracted laser spots.

Before beginning to take data, verify the alignment of the beam through the center point of the sample

[2] M. Born and E. Wolf, *Principles of Optics*, 7th edition (Cambridge, 1999) xxvi.

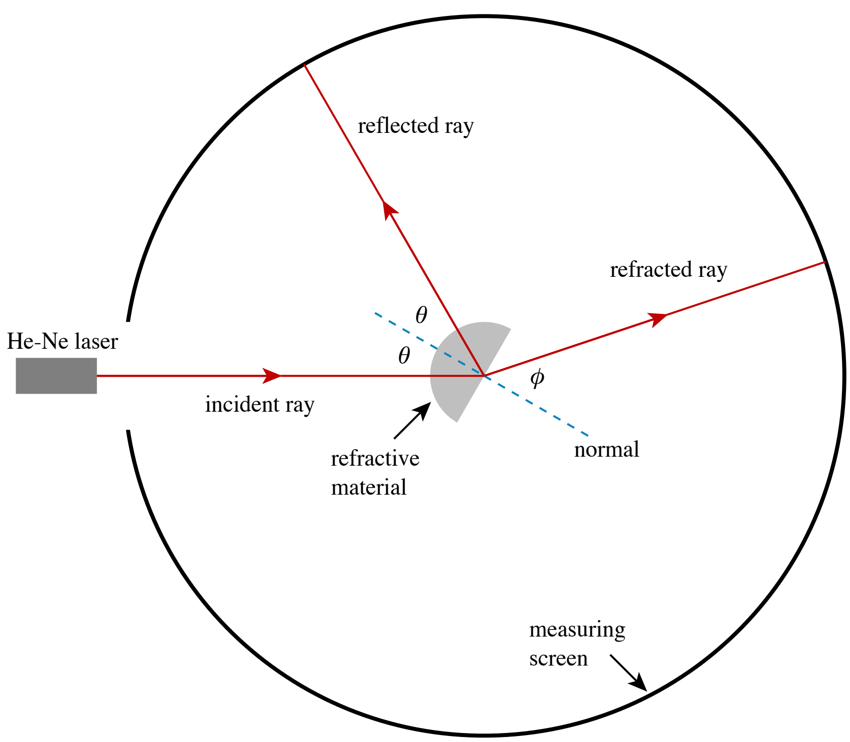


Figure 4.2: Index of refraction experimental setup.

Refl (mm)	Unc (mm)	Refr (mm)	Unc (mm)		θ	$\delta\theta$	ϕ	$\delta\phi$		n	δn
2738.5	1	987	2		0.071	0.003	0.048	0.002		1.463	0.060
2450	1	936	2		0.320	0.003	0.209	0.002		1.513	0.013
2503.5	1	945	2		0.274	0.003	0.179	0.002		1.520	0.016

Figure 4.3: A portion of a spreadsheet to record raw angular position data and to compute angles of reflection and refraction.

piece, and re-check this centering periodically throughout the session. Think carefully about how you will convert distance measurements around the circumference of the measuring screen into the angles of reflection and refraction. As suggested by Fig. 4.2, knowing the angular position of the zero point and the reflected and refracted rays allows you to determine the angles ϕ and θ. Prepare a spreadsheet to hold the raw position data, as well as to compute the angles and their uncertainties. An example is shown in Fig. 4.3.

One convenient way of checking the alignment of the incident beam is looking for the faint reflection of the beam from the curved first surface of the sample; if the beam is entering the sample normal to its surface, this reflection will be directed straight back toward the source.

Warning: Do not look directly into the laser beam.

Before starting the experiment, describe the beam you see and give approximate dimensions. Use these observations to estimate the uncertainty in your angular measurements.

4.3.1 Measurements

Using the optical acrylic (looks like glass) sample, measure the angles θ and ϕ at several positions. Make enough well-spaced measurements that you will have a good basis for performing a fit to determine the index of refraction, n, of optical acrylic.

Once you have completed these measurements for the acrylic sample, select one of the following options:

1. **Index dependence on material**: Replace the sample with the liquid sample container filled with distilled water, or another liquid provided by the instructor. This will allow you to contrast the refractive indices of different materials.

2. **Index dependence on wavelength**: Replace the 633-nm (red) He-Ne laser with a 532-nm (green) diode laser. Repeat the measurements on the acrylic sample. This will allow you to observe the phenomenon of dispersion, which is the subtle dependence of the refractive index on wavelength. Note: this is a small effect and requires very careful measurements!

Many everyday applications of optics make use of these two important phenomena. Considering the other parts of the experiment you have to complete in lab, you may find it an effective use of your time in lab to record the beam positions on the paper strip attached to the screen, but leave the detailed analysis until later. Discuss issues of time management with your lab partner and instructor. Also, note that a small lateral offset of the incident beam from being perfectly centered on the sample will result in a systematic (one-sided) error in the resultant index of refraction. If you are alert, you can readily detect this effect and figure out how to account for it during your later analysis.

4.3.2 Analysis

There are two principal ways to analyze your measurements. Pick one or the other.

1. Prepare a plot of $\sin\phi$ vs. $\sin\theta$. According to Eq. (4.1), you should obtain a straight line with slope equal to the index of refraction of the semicircular prism (either acrylic or water). Note that if you choose this option, you will need to compute $\delta(\sin\phi)$ from $\delta\phi$ via

$$\delta(\sin\phi) \approx \left|\frac{d\sin\phi}{d\phi}\right|\delta\phi = |\cos\phi|\delta\phi \qquad (4.2)$$

   This expression is valid only when $\delta\phi$ is measured in radians.

2. Plot ϕ vs. θ and fit directly to Snel's law, Eq. (4.1). Use your uncertainties $\delta\theta$ (and $\delta\phi$) on the plot and take advantage of a nonlinear fitting program. In this case, the program takes care of transforming the uncertainties for you.

In both cases, note that fitting routines only account for the errors in the *dependent* variable (plotted on the y axis). If one variable has notably larger uncertainties than the other, put that one on the y axis and invert the fitting function, if necessary. For the curious, see `http://www.physics.hmc.edu/analysis/fitting.html#ErrorsOnBoth` for a method of accounting for *both* x and y uncertainties.

4.4 Background and Theory — Lenses

Both the ancient Greeks and Romans knew that a curved glass surface could focus light. Aristophanes' play *The Clouds* (424 BCE) mentions a "burning glass," which was used to focus the sun's rays to produce fire. The behavior of such a convex lens is illustrated in Fig. 4.4. The Arabian mathematician Alhazen (965–1038) described how the lens in the human eye forms images on the retina, and the use of lenses in spectacles dates from the high Middle Ages in Italy.

The general theory of lenses is quite complicated; lens design is both art and science.[3] However, in many circumstances a simple approximate theory suffices. We assume that the rays of light make only small angles θ with the **optic axis**, which is the symmetry axis of the lens(es), so that we may neglect the difference between $\sin\theta$ and θ. This is called the **paraxial approximation**. We also assume that we may neglect the separation of the two surfaces of the lens (the thin-lens condition). Under these assumptions, the distance from the center of the lens to the object (o) and the distance from the center of the lens to the image (i) are related by the **thin lens equation**,

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \qquad (4.3)$$

where f is the **focal length** of the lens. These distances are illustrated for a converging lens in Fig. 4.5.

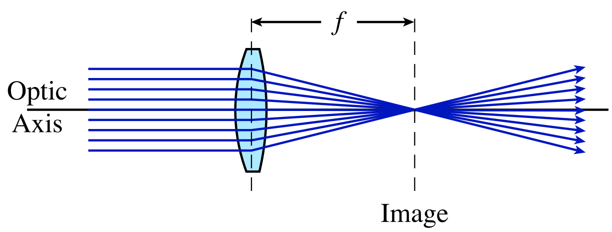


Figure 4.4 A parallel beam of light converges to a point at a distance f from a convex (converging) lens.

Ray diagrams such as Fig. 4.5 and Fig. 4.8 are extremely helpful in understanding the behavior of lenses, and they are easy to construct. Here are the rules:

(a) A ray that passes through the center of the lens is undeviated.

(b) A ray that propagates parallel to the optic axis (the symmetry axis of the lens) emerges from the lens heading for the image focal point, a distance f downstream of the lens.

(c) A ray that passes through the object focal point a distance f before the lens emerges from the lens parallel to the optic axis.

The location of the image can be described in two ways. First, if a screen is placed at the location of the image, a focused picture of the object appears. Second, if you were to look at the system of the object and lens by putting your eye at the location marked with a star at the right of Fig. 4.5, you would see light rays appearing to originate at the location of the image. That is, the object would "appear" to be at the location of the image. As shown in Fig. 4.5, it would be inverted and it may be either magnified or reduced in size. You can readily show from the first principal ray that the magnification is

$$\gamma = \frac{i}{o} \qquad (4.4)$$

4.4.1 Diverging lens

Diverging lenses are thinner in the center and thicker at the edges; they divert incoming parallel rays away from the optic axis, as illustrated in Fig. 4.6. On the downstream side of the lens the rays appear to diverge from a point a distance $|f|$ behind the lens, which is the location of the **virtual image**. It is virtual, not *real*, since you can't image it by placing a paper or screen there, the way you can with a real image produced by a converging lens. Because the focus happens on the upstream side of diverging lenses, their focal lengths are negative.

The same set of three principal rays may be drawn for a diverging lens to locate the virtual image of a real object, as shown in Fig. 4.8. The trick is to remember that the focal points have exchanged places. Therefore, a ray that arrives at the lens parallel to the optic axis emerges from the lens as though it had come from the image focal point, which is $|f|$ *in front of* the lens.

[3] Please note that the singular of *lenses* is *lens*, not *lense*.

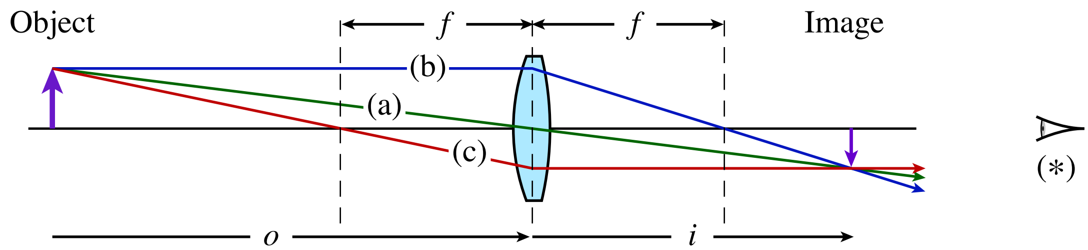


Figure 4.5: Formation of a real image by a converging lens. Light diverges from the object at the left, which is a distance o from the lens. It passes through the lens, and converges to form an inverted image a distance i to the right of the lens. The three principal rays are shown: (a) passes through the center of the lens undeviated; (b) arrives at the lens parallel to the optic axis and leaves passing through the image focal point, a distance f from the lens; (c) passes through the object focal point on the way to the lens, and emerges from the lens parallel to the optic axis.

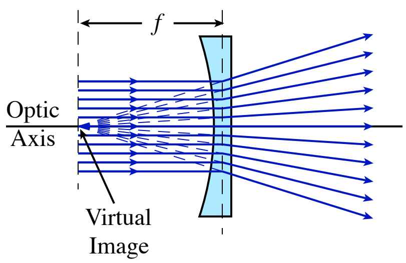


Figure 4.6 Formation of a virtual image from a collimated beam by a diverging lens. The image is virtual because it is on the "wrong" side of the lens; you can't put a screen there and see it.

Similarly, a ray that is heading towards the object focal point a distance $|f|$ *beyond* the lens emerges from the lens parallel to the optic axis.

Just as diverging lenses have negative focal lengths, virtual images have negative image distances i. In the case illustrated in Fig. 4.8, the image distance i is equal to $-\frac{2}{3}o$.

Since there are virtual images, could there be virtual objects? What would they look like? Well, light diverges from a real object, as shown at the left of Fig. 4.5. Light also diverges from the real image formed by the converging lens in that figure. What if you were to stick another lens downstream of that first image? The rays would diverge from the first image and propagate to the second lens; the image of the first lens serves as the object of the second lens. On the other hand, if you were to slide that second lens upstream of the real image, as illustrated in Fig. 4.7, the rays would encounter the second lens *before* they had finished the job of converging to produce the first lens's image (which is the second lens's object). Thus, they constitute a **virtual object** for the second lens. Not surprisingly, virtual objects have a negative object distance o.

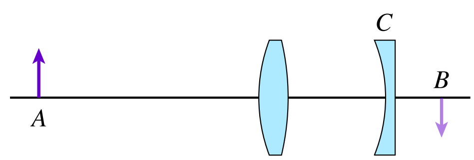


Figure 4.7 A converging lens forms an image of an object at A at position B, which is downstream of lens C. The image at B is a virtual object for lens C.

Amazingly, the thin lens equation, Eq. (4.3), handles all cases for both kinds of lenses, provided you respect the sign conventions outlined above. For easy reference, here's a table summarizing them:

Qty	Positive	Negative
f	converging lens	diverging lens
o	upstream of lens	downstream of lens
i	downstream of lens	upstream of lens

4.5 Experimental Procedure — Image Formation

4.5.1 Preliminaries

Your lab station should have at least two converging lenses and one diverging lens mounted on posts for easy alignment. Select the two converging lenses and make a quick estimate of their focal length by projecting an image of an overhead fluorescent light on the floor. You'll probably need to put the lens closer to the floor than about half a meter to see an image of the glowing tube. In this case, $i \ll o$, and if we neglect $1/o$ compared to $1/i$ in Eq. (4.3), then the distance from the lens to the floor is approximately equal to the focal length.

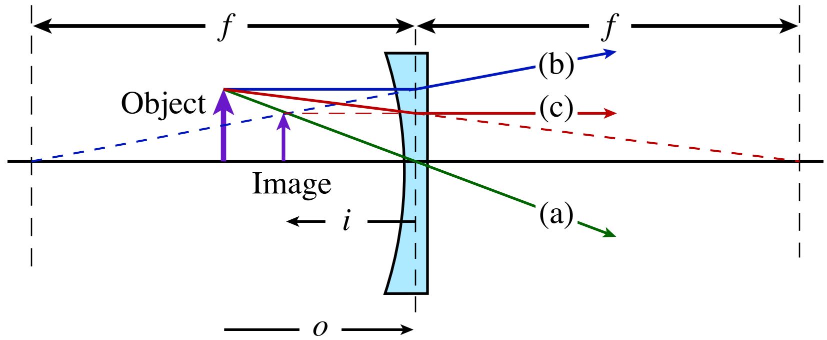


Figure 4.8: Formation of a virtual image by a diverging lens. The image is not inverted, is closer to the lens than the object, and is reduced in size. As in Fig. 4.5, the principal rays are shown: (a) passes through the center of the lens undeviated; (b) arrives at the lens parallel to the optic axis and leaves as though it came from the image focal point, which is $|f|$ from the lens on the upstream side; (c) is heading towards the object focal point, which is a distance $|f|$ downstream of the lens, and emerges from the lens parallel to the optic axis.

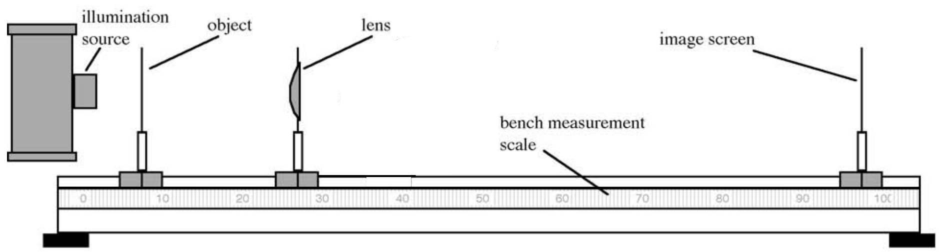


Figure 4.9: Layout of the optical bench and components.

Take a look through each lens to get a feel for what they do to light coming from the opposite side, then select the converging lens with the shorter focal length. You will work with it first. Throughout your work with the optical bench it is critical for you to record carefully exactly what you are doing. You will need to make many sketches, explain tersely how you made your measurements, and record uncertainties.[4] Since you will be forming images on screens and measuring their positions, it will require some judgment on your part to determine where the position of best focus is. Typically the uncertainty you assign to your measurements will be influenced by both your ability to read the bench scale, and by the uncertainty inherent in estimating the position of best focus. Use your scientific judgment here.

Sometimes your ability to find the position of best focus can be improved by adjusting the brightness of the object. For example, if the images formed exhibit flaring that makes it hard to judge sharpest focus, a piece of diffusing tissue paper between the illuminator and the object can dim the images enough to improve their visibility. You may also find it helpful and/or interesting to insert a colored glass filter to reduce chromatic aberration.

Record raw values first, then compute derived quantities. That is, if you are really interested in o, but you measure the position $x_{\rm obj}$ of the object and $x_{\rm lens}$ of the lens so you can compute $o = x_{\rm lens} - x_{\rm obj}$, record both $x_{\rm obj}$ and $x_{\rm lens}$ in your data table, as well as their difference.

Set up the object and image screens on the optical bench such that an image in sharp focus is formed. Place the object screen at the end of the optical bench next to the illuminator and adjust it for some convenient height. The illuminator should be carefully adjusted to obtain bright images:

1. Adjust the height of the illuminator to the same height as the object and position it at the end of the bench.

2. Remove both the white screen and the object screen from the bench, and focus the filament of the lamp on the far wall (which is effectively at in-

[4]Beware the NFL absurdity: the referee carefully peels off gargantuan bodies as they wrestle for the ball, lovingly extracting the pigskin and setting it gently upon the field somewhere in the vicinity of where it might have been at the end of forward progress (but before the post-progress tug-of-war has moved the ball around). The chain gang then trots vigorously out, having carefully marked its location with respect to a yard line, and stretches the chain to see whether the nose of the ball has exceeded the requisite 10 yards. This final measurement is made with great precision as television cameras zoom in for exacting scrutiny. The moral: if you can't tell better than a foot where the lens should go, don't bother measuring its position to the nearest thousandth of an inch!

finity) by sliding the part that contains the socket in and out.

3. Mount the white screen at the opposite end of the bench and adjust the tilt of the lamp so the beam is centered on the screen, at the same height as the illuminator.

4.5.2 Converging lens

Now replace the object screen and place the image screen at the far end of the optical bench. Place the converging lens with the shortest focal length between the object and screen, and align the system. Adjust the position of the lens to produce as clear an image as possible on the screen. How reproducible is the position of the lens? That is, if you slide the lens back and then reposition it to produce the clearest image, how much variation in the optimal position of the lens do you get from iteration to iteration? Do you and your partner agree? Does the sensitivity depend on where the screen is located?

Pick a particularly good data point and compute the focal length of the lens, as well as its uncertainty. You may find the following formulas helpful in making uncertainty calculations:

$$\delta\left(\frac{1}{x}\right) = \left|\frac{\partial(1/x)}{\partial x}\right|\delta x = \frac{\delta x}{x^2} \tag{4.5}$$

$$\delta\left(\frac{1}{f}\right) = \sqrt{\left[\delta\left(\frac{1}{o}\right)\right]^2 + \left[\delta\left(\frac{1}{i}\right)\right]^2}$$

$$\delta f = f^2\sqrt{\left(\frac{\delta i}{i^2}\right)^2 + \left(\frac{\delta o}{o^2}\right)^2} \tag{4.6}$$

Note that Eq. (4.6) assumes that the error in o is uncorrelated with the error in i, which may or may not be the case, depending on how you measure.

Repeat this procedure with the longer focal length lens.

Even better measurements

Suppose that we plot a series of image and object distance measurements in the form of a graph with $1/o$ on the x-axis and $1/i$ on the y-axis. Rewriting the lens equation,

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} \tag{4.7}$$

we see that if the thin lens formula is obeyed, our graphed data should describe a straight line with a slope of -1 and a y-intercept of $1/f$, as shown in Fig. 4.10. Notice, as well, that your data points up to this point all reside in the first quadrant of this plot. Your challenge, now, is to add *at least one point* in quadrants II and IV. After you have added these, you will be able to make a very fine fit for the focal length f of the lens. Note that you will need to use the other converging lens somehow to obtain points in quadrants II and IV. Be sure to document your work very carefully. If you are stuck, you might skip to the following section on the diverging lens, and then come back.

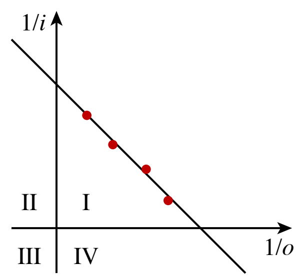


Figure 4.10 A plot of $1/i$ vs. $1/o$ should yield a straight line with slope -1 whose x and y intercepts are $1/f$.

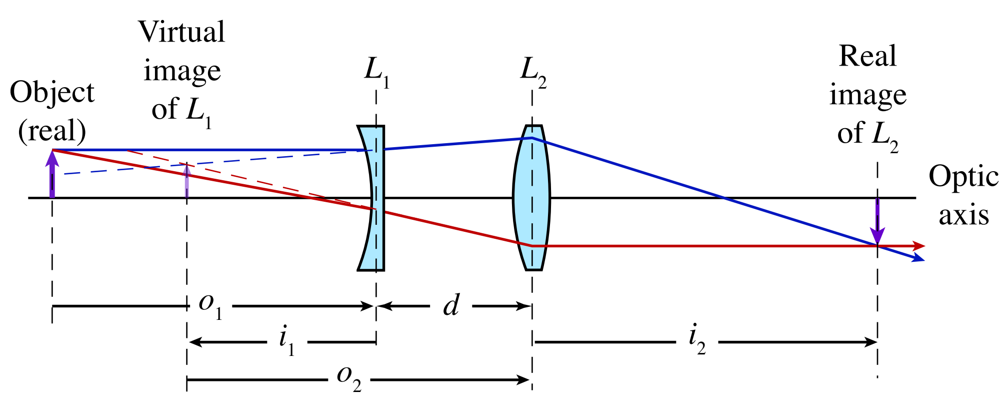


Figure 4.11: A method for forming a real image from the virtual image of a diverging lens.

4.5.3 Diverging lens

In this part of the experiment, you will investigate the properties of a diverging lens, again by forming images on a screen, this time using the diverging lens in combination with one or more converging lenses. A possible geometry is shown in Fig. 4.11. The virtual image of the diverging lens L_1 is located a distance $|i_1|$ upstream of the lens, and a distance $o_2 = |i_1| + d$ upstream of lens L_2.

How could you determine the position of the image of L_1? With both lenses and the screen in place to produce a sharp image of L_2, remove L_1. The image on the screen will disappear (or grow diffuse). Now slide forward the object until the image on the screen is once again sharp. At this point, the object's position is the same as the virtual image of L_1 before the lens was removed.

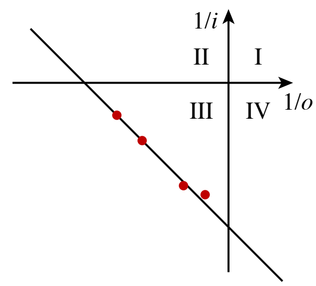


Figure 4.12 A plot of $1/i$ vs. $1/o$ should yield a straight line with slope -1 whose x and y intercepts are $1/f$ (which is negative).

Once you have obtained a rough estimate of f for the diverging lens, you can explore quadrants II, III, and IV, as illustrated in Fig. 4.12, using one or both of the converging lenses to produce virtual objects or images, as needed.

Experiment 5

Fraunhofer Diffraction

Abstract

Diffraction refers to the fanning out of a wave upon encountering a constriction or obstacle. You will study Fraunhofer diffraction from single and multiple slits illuminated with a laser beam. Each point in the opening serves as a source of secondary waves, whose interference produces the diffraction pattern. Comparing the observed patterns to theoretical curves permits you to make very accurate measurements of the slit widths and slit separations.

5.1 Overview

As discussed by Feynman in *QED*, the probability for a photon to pass from the source to the detector may be calculated by adding up the "arrows" (probability amplitudes) for each path from the source to the detector, and then taking the square of the resulting arrow (amplitude). For a laser beam normally incident on one or more slits, the arrows within the slits all point in the same direction (the amplitudes are in phase with one another), and we may consider this the starting point for the interference effects. (The alignment of the arrows within the slit is part of the simplification that constitutes *Fraunhofer* diffraction.) For a finite slit width, the distance from a point within the slit to the detector depends upon position within the slit. Therefore, arrows arriving at the detector from different positions within the slit point in different directions and interfere with one another.

You will investigate how the width of the slit influences the pattern seen far from the slit, as well as the influence of multiple slits on that pattern. You should observe the patterns both qualitatively on a piece of paper and quantitatively using a photodetector, which is mounted behind a narrow window to yield a well-defined diffraction angle. By translating the window/detector with a micrometer, and by knowing the distance between the slit and the window, you can record the diffracted intensity as a function of diffraction angle. A computer data logging program will assist you in this effort, but you will have to supply the distances it needs to convert a voltage proportional to micrometer rotation into a diffraction angle. From this calibration, you will then be able to make a very precise determination of the slit width (and separation, in the case of multiple slits).

5.2 Background and Theory

The first mention of diffraction phenomena appears in the work of Leonardo da Vinci (1452–1519). Over a century later, a professor of mathematics and physics from Bologna, Francesco Maria Grimaldi (1613–1663), made more careful observations of the phenomenon by illuminating a thin rod by a pencil of sunlight admitted into a darkened room. He was surprised to see a shadow behind the rod that exceeded in width the geometric shadow, and that it was fringed with colored bands. Grimaldi coined the term *diffraction*, which means "breaking up," to describe the phenomenon and he developed a theory that light was a fluid that exhibited wave-like motion. His observations were published soon after his death, and this work stimulated Isaac Newton (1642–1727) to study optics.

Newton took up Grimaldi's observations in the third book of his *Opticks* (1704). However, Newton's insistence on a corpuscular theory of light prevented him from developing a satisfactory theory of diffraction. A century later Thomas Young (1773–1829) reported a simpler and more convincing demonstration of "double-slit" diffraction at the 24 November 1803 meeting of the Royal Society of London, in which he split a narrow "pencil" of light into two parts using a card held

[1]See Walter Scheider, *The Physics Teacher* **24** (1986) 217. Thomas Young's interests were many and varied; the fact that his contemporaries weren't particularly interested in his diffraction experiments was perhaps not terribly troubling to him. He took up other pursuits. A child prodigy, he was reading fluently at age two and by age 16 was proficient in Greek, Latin, and eight other languages. He put to good use this facility with languages by helping decipher the Rosetta stone, which had been discovered in 1799 by French troops preparing the foundation for a fort in the Nile Delta. Young provided the key breakthrough for deciphering ancient Egyptian hieroglyphs: cartouches on the stone contained a phonetic representation of the royal name Ptolemy. Jean-François Champollion took up where Young left off, applying his linguistic skills and knowledge of Coptic to decipher the rest of the stone. Besides working as a

edge-on into the beam.[1] However, British science was unprepared to admit that the lion, Newton, could have erred, and further development of the wave theory of light came on the Continent with the work of François Jean Arago (1786–1853) and especially Augustin Fresnel (1788–1827). Fresnel produced a mathematical theory of diffraction, for which he was awarded the *Grand Prix* of the French Academy of Sciences in 1819.[2]

5.2.1 Single slit

To compute the diffraction pattern for a single slit, we must add the amplitudes arising from each point in the slit. Let x be the position in the slit, which has width a, as indicated in Fig. 5.1. If we center our coordinates on the slit, $-\frac{a}{2} \le x \le \frac{a}{2}$, then the extra distance traveled by a wave originating at point x in the slit, compared to the wave originating at $x = 0$, is $-x \sin\theta$. Its amplitude is therefore proportional to $e^{-ikx\sin\theta}$, where $k = 2\pi/\lambda$. We can thus add amplitudes (arrows) across the slit with the integral

$$\begin{aligned}\psi(\theta) &\propto \int_{-a/2}^{a/2} e^{-ixk\sin\theta}\, dx \\ &= \frac{e^{-ika/2\sin\theta} - e^{ika/2\sin\theta}}{-ik\sin\theta} \\ &= a\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda} \end{aligned} \tag{5.1}$$

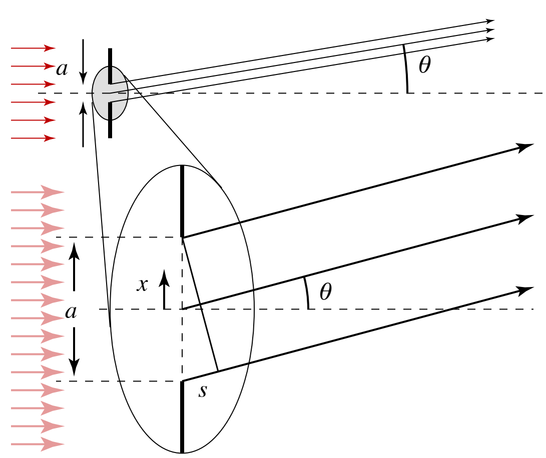


Figure 5.1 Geometry of Fraunhofer diffraction. A uniform wave illuminates a slit of width a at normal incidence. The diffracted light is observed on a screen far from the slit, so that we may approximate as parallel the rays coming from each point in the slit.

Squaring and normalizing, we get the intensity

$$\boxed{I(\theta) = I_0 \,\mathrm{sinc}^2\left(\frac{\pi a\sin\theta}{\lambda}\right)} \tag{5.2}$$

where

$$\mathrm{sinc}\,\phi \equiv \frac{\sin\phi}{\phi} \tag{5.3}$$

The single-slit intensity distribution of Eq. (5.2) is shown in Fig. 5.2. Of course, when the angle θ is small, it is possible to approximate $\sin\theta \approx \theta$, which simplifies this expression somewhat. Is the small-angle approximation appropriate in this experiment?

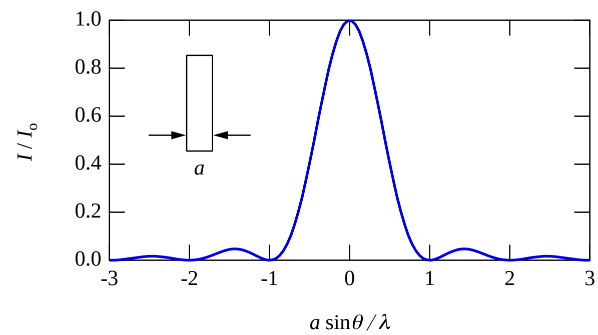


Figure 5.2 Diffracted intensity $I(\theta)/I_0$ far from a single slit, as given by Eq. (5.2).

5.2.2 Double slit

It is actually slightly easier to compute the diffraction pattern from a double slit, at least crudely. If we can neglect the width of the slits, then there are only two "arrows" to sum, giving

$$\begin{aligned}\psi(\theta) &\propto e^{-i\frac{1}{2}kd\sin\theta} + e^{i\frac{1}{2}kd\sin\theta} \\ &= 2\cos\left(\frac{\pi d\sin\theta}{\lambda}\right)\end{aligned} \tag{5.4}$$

where d is the distance between the slits, and we have used $e^{i\phi} + e^{-i\phi} = 2\cos\phi$.

However, a more realistic calculation takes into account the finite width a of each slit. We have already computed the sum of all the arrows from one slit, in Eq. (5.1), assuming the slit is centered on the origin. All we need to do is slide one slit up $d/2$ (which multiplies the integral by $e^{-i\frac{1}{2}kd\sin\theta}$) and slide the other

medical doctor, Young also investigated the elastic properties of materials—Young's modulus relates the stretching of a solid (the strain) to the applied force per unit area (the stress).

[2] There's a great story about this prize. The prize committee included such notables as Arago, Siméon Poisson, Jean-Baptiste Biot, and Pierre-Simon Laplace. Poisson, in particular, was sure that Fresnel's theory was hogwash and set about a calculation of the shadow behind an opaque disk to see if he could identify a fatal flaw. Indeed, Poisson deduced that along the axis of the disk, where the shadow "should" be darkest, Fresnel's theory predicted a bright spot, which was obviously absurd. Arago requested that Poisson's prediction be tested, and to the general astonishment of the jury, the spot duly appeared! Fresnel won the prize, but credit for the spot is conventionally awarded to the skeptical Poisson.

down $d/2$ (which multiplies the integral by $e^{i\frac{1}{2}kd\sin\theta}$), to get

$$\psi(\theta) \propto e^{-i\frac{1}{2}kd\sin\theta}\,\mathrm{sinc}\,\Phi + e^{i\frac{1}{2}kd\sin\theta}\,\mathrm{sinc}\,\Phi = 2\cos\left(\frac{\pi d\sin\theta}{\lambda}\right)\mathrm{sinc}\,\Phi \qquad (5.5)$$

where

$$\Phi = \frac{\pi a\sin\theta}{\lambda} \qquad (5.6)$$

Squaring and normalizing gives the observed intensity pattern,

$$\boxed{I(\theta) = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)\mathrm{sinc}^2\left(\frac{\pi a\sin\theta}{\lambda}\right)} \qquad (5.7)$$

which is plotted in Fig. 5.3.

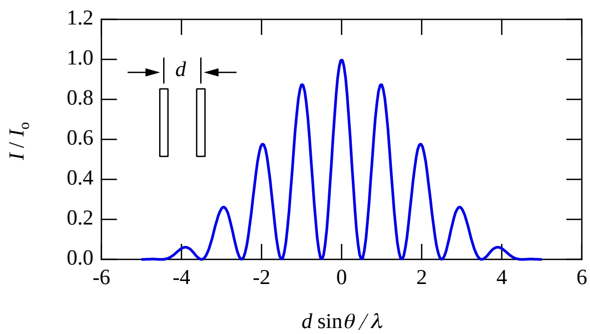


Figure 5.3 Diffracted intensity from two identical slits whose centers are separated by distance d.

5.2.3 Multiple slits

You can work out the theory for N slits by generalizing the double-slit case. There is one more mathematical trick that helps tidy up the answer. With N equally spaced slits the phase difference between adjacent slits is $r = e^{-ikd}$ and we must sum

$$S_N = 1 + r + r^2 + \cdots + r^{N-1} \qquad (5.8)$$

which is a **geometric series**. Multiply Eq. (5.8) by r and subtract from Eq. (5.8) to get

$$\begin{aligned} S_N &= 1 + r + r^2 + \cdots + r^{N-1} \\ rS_N &= \phantom{1 +{}} r + r^2 + \cdots + r^{N-1} + r^N \\ (1-r)S_N &= 1 - r^N \end{aligned} \qquad (5.9)$$

Substituting the definition of r and symmetrizing gives the result

$$S_N = \frac{e^{i\frac{1}{2}Nkd\sin\theta} - e^{-i\frac{1}{2}Nkd\sin\theta}}{e^{i\frac{1}{2}kd\sin\theta} - e^{-i\frac{1}{2}kd\sin\theta}}$$

$$S_N = \frac{\sin\left(\dfrac{N\pi d\sin\theta}{\lambda}\right)}{\sin\left(\dfrac{\pi d\sin\theta}{\lambda}\right)} \qquad (5.10)$$

An example of the application of this expression to calculate a 5-slit diffraction pattern is shown in Fig. 5.4.

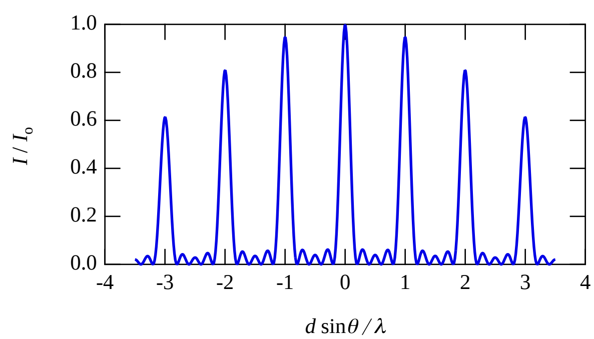


Figure 5.4 Diffracted intensity from five identical slits separated by d.

5.3 Experimental Procedures

Make sure the laser is turned on, as it needs at least 30 minutes before it is stable enough to permit reasonable intensity measurements. Also check that the 5-volt power supply is on. *Do not turn on the photocell power supply yet.*

5.3.1 Alignment procedure and preliminary observations

Warning: The laser mount does not handle vibrations and bumping very gracefully. Take care not to bump it and to keep vibrations of the photometer to a minimum.

It is likely that the system is still reasonably well aligned from the previous lab meeting and will need only minor adjustments. To align the system, first use the alignment target. Put the target close to the laser and translate the laser perpendicular to the optic axis until the beam falls on the target spot. Do this by adjusting the control screws on the laser mount. To align the laser beam along the optic axis of the apparatus, place the target just in front of the photocell box and rotate the laser about vertical and horizontal axes to

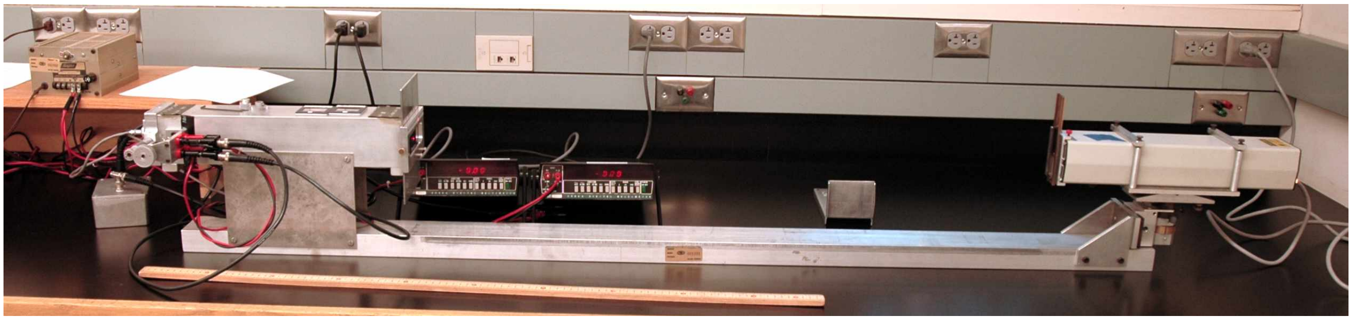


Figure 5.5: Fraunhofer photometer.

center the laser spot again. Repeat these two steps until no further adjustments are necessary to center the laser on the target spot at both locations.

Before continuing with the final alignment, you should observe a few examples of the single- and double-slit diffraction/interference patterns you will be measuring, and determine the effects of varying the slit width and separation. To do this, remove the photocell mount at the rear of the photometer box and project the pattern on a piece of paper after mounting a slide of single slits in the holder on the front of the box.

5.3.2 Removing the photocell mount

The photocell mount is secured to the photometer by means of a set screw. The photocell mount is located at the rear of the light-tight box and the removal of the photocell mount is a simple matter of loosening a set screw and then sliding the photocell mount from the backside of the light-tight box. (See Fig. 5.6.)

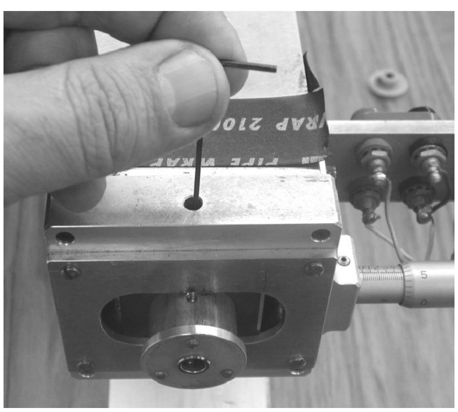


Figure 5.6 Removing the detector.

The set screw securing the photocell mount to the light-tight box is accessed via a hole drilled through the top of the aluminum plate that is attached to the rear of the light-tight box. Move the photocell mount to the midpoint of its travel. Look into the hole in the above-mentioned aluminum plate. You should be able to see the set screw. Use the hex wrench provided to loosen the screw.

Warning: Do not remove the set screw. Simply back it out a single turn.

If you're unable to see the set screw when the photocell mount position is set to the midpoint of its travel, try moving the photocell mount back and forth a short distance while looking into the access hole. See your instructor for help.

Center the broadest slit in the laser beam. (Note: The clamping screw holding the slide in place should be tightened only very lightly by hand. Over tightening can damage the slide.) Sketch the diffraction pattern seen when projected onto a piece of paper.

Warning: Do not look directly into the laser beam!

Try some of the other single slits; then change to the double slits. From your sketches write down how varying the distances a and d affect the patterns. Next remove the slide with the slits from the front of the photocell box. You should now determine carefully for later use the distance from the slits on the slide to the photocell window when it is remounted in its box.

Here are some things to worry about while the photocell is still out:

- How wide is the window (the opening slit mounted in front of the photodiode)?
- How should the window be oriented?
- What is the distance between the target slits and the window in front of the detector?

When you have sufficiently pondered these questions, replace the photocell in the mounting bracket on the back of the photometer box and tighten the set screw that holds the photocell in place.

The photocell is positioned by turning a micrometer drive attached to the back of the photometer box. The micrometer drive is connected by a gear arrangement to a 10-turn potentiometer. With 5 V applied across the entire potentiometer, the relative position of the photocell is given by the voltage (0 – 5 V) between the center tap and one end of the potentiometer, as read by a voltmeter.

5.3.3 Potentiometer

A potentiometer (as we are using the term) is nothing more than a resistor that allows one to make connections across the ends of the resistor and between one end of the resistor and the movable center tap (Fig. 5.7). If 5 volts is set up across the ends of the resistor, then the voltage between the center tap and one end of the resistor will vary from 0 to 5 volts linearly with the position of the tap.

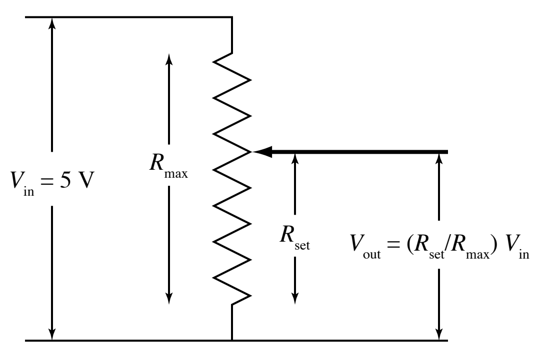


Figure 5.7 Potentiometer position circuit.

If you haven't already done so, now would be a good time to sketch the experimental apparatus and explain how it works; *i.e.*, how you can use it to measure a diffraction pattern.

For the final alignment, you will use two voltmeters to monitor the photocell position and the photo-voltage. To avoid saturating the photocell keep $V <$ 5 V; you need to reduce the laser beam intensity by adjusting the partially silvered mirror ("beam attenuator") mounted on the rail near the laser **(Photo 5-3)**. Position the mirror so that the laser beam passes through it about one-fourth of the way from the top.

Finishing the alignment

Turn on the photocell power. Using the same hex wrench that you used to remove the photocell mount, remove the gear from the end of the potentiometer. Rotate the micrometer handle while watching the reading on the voltmeter connected to the photocell output. Adjust the photocell position so that the voltage obtained is a maximum. Then adjust the beam attenuator so that the photocell has an output just less than 5 V so it is not saturated by the laser beam. Next, adjust the potentiometer until the voltmeter connected across the center tap of the potentiometer reads 2.5 volts. Without rotating either shaft, reinstall the gear onto the end of the potentiometer so that the two gears line up and are flush with each other. This should center the maximum intensity peak.

At this point you should make fine adjustments of the vertical position of the laser beam spot by adjusting the tipping screw on the laser's seesaw bracket. Adjust for maximum photo-voltage. Now, again adjust the beam attenuator to give a non-saturating peak photo-voltage close to, but not exceeding, 5.0 V.

Change the micrometer position and note the intensity profile of the laser beam. If you did everything correctly, you should see a steep but smoothly varying profile with a peak photo-voltage of a bit less than 5.0 V at a micrometer position voltage near 2.5 V.

5.3.4 Beam profile

You will be using a computer program to aid you in taking data and in comparing your results with the theoretical expressions of section 5.2. The voltages from the potentiometer and photodetector are converted to digital values by an analog-to-digital converter (ADC), which is connected to a USB port on the computer. During an acquisition, these voltages are sampled several times per second and plotted in real time as photo-voltage vs. position voltage, allowing you to monitor the data as you slowly rotate the micrometer or potentiometer gear to translate the detector window.

To get a feel for how the acquisition program works, you should first take a scan of the laser beam itself, with no target slit mounted at the front of the photometer box. If you haven't already done so, launch Igor by clicking on the Igor icon in the Dock at the right of the screen. Then select the **Fraunhofer Diffraction** program.

Rotate the micrometer to one edge of its range, note the position of the micrometer (see Fig. 5.8), then click the **Take Data** button. Smoothly turn the micrometer shaft or gear to advance the detector. Watch the computer screen as you go to make sure that the data are smooth and quiet. *Do not reverse the direction of the photocell motion while collecting data.* If you do, you will see the effects of "screw backlash" in your data. Scan through the complete 5-V range of the potentiometer, then click the **Done** button to finish the acquisition.

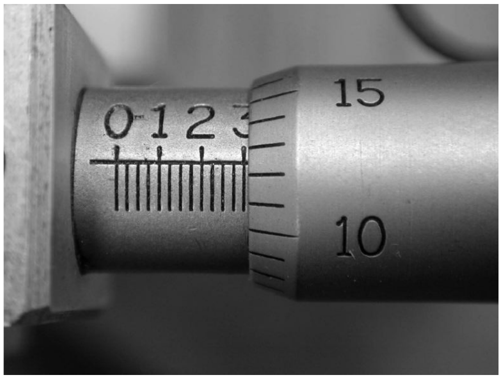


Figure 5.8 Reading the micrometer: it takes four revolutions to advance by one "major" unit. What do you suppose those units are? How could you check?

You may save and print these data, if you wish, but you should at least note the width of the beam, which you can readily convert from volts to distance. Determine the distance the micrometer has advanced and use this information to calculate the width of the laser beam. Is your value reasonable?

5.3.5 Taking data

At this point you are ready to take diffraction data. Select the slide of single slits and note the labeling of the slide and the slits (it will be handy later to have identified each slit before installing the slide). Place the slide in the holder at the front of the photocell box and center the slit you have selected under the laser spot. Advance the micrometer until the potentiometer voltage is about 2.5 V, and scan a short ways in either direction to look for the maximum intensity. At the maximum, adjust the attenuator slide to produce a peak intensity slightly lower than 5 V. Then set the micrometer at the edge of its travel.

Start the data acquisition and carefully scan the micrometer across the full range of its motion, watching the computer screen to monitor the data acquisition as you go. Click the **Done** button when you have finished.

When you are satisfied with your data, click the **Normalize** tab. To convert your position data (in volts) to angles, you must enter the complete *angular* range of motion of the detector window (corresponding to a 5-V change in the potentiometer voltage). Make a sketch of the relevant geometry in your notebook, and use the distance from the target slit to the detector window to compute the *full* angular range, in radians. Enter this value in the box, then click the **Normalize** button. The Normalize routine both scales and shifts the data to produce a maximum peak height of 1 centered at $\theta = 0$. For best results you should make sure to go smoothly and rather slowly over the central maximum so the program can make an accurate determination of the peak height and position.

Comparing to theory

To compare your data to Eq. (5.2), click the **Slit Dimensions** tab and enter the slit width. Best estimates of the various slit widths and separations are provided on a sheet near the photometer set-up. Then select **1 slit** from the popup menu. You should see the computed curve superimposed on your data, as well as a panel of residuals at the top of the graph. The residual of a data point is its vertical distance from the computed curve. When a curve accurately describes experimental data, the residuals should be randomly distributed with respect to the zero line. When there are significant disagreements between the curve and the data, the residuals will exhibit a pattern.

If the calculated curve is either fatter or skinnier than the central peak in your data, click on the **Hypothetical Dimensions** tab and enter a different value for the slit width, adjusting that value until you get the best agreement. Note that when a hypothetical curve is plotted, the residuals panel is adjusted too show residuals for the hypothetical curve. To narrow the peak do you have to increase or decrease the value of the slit width? Why?

When you have found the best value for the hypothetical slit width, save and print your data. Click the **Save & Print** tab and click the **Save** button. You will be asked for each partner's name and your section number so your data can be stored in your own local directory (you will be able to copy the files to your own computer or your Charlie account later). A dialog will tell you the name of the data file, which you should record in your lab notebook.

Then click **Print**. Two copies of the graph will be printed, one for each partner. Cut the page in half, tape the graph into your lab notebook, and comment briefly on the extent of agreement between your data and the calculated curve(s) on the same notebook page. To test repeatability, you should realign the apparatus and record a second run.

If all has gone well in the first week, you should be well into computer data taking by your second lab period. Repeat the experiment using another single slit, two different double slits and the three-, four-, and five-slit configurations. Readjust the laser optics to eliminate any saturation of the photocell response at intensity maxima and for a maximum as large as possible but not to exceed 5 V. Carefully re-center each set of slits in the laser spot. There is a second set of gears you can use to change the range of the photocell positions. These gears should be used for some of the slits to record a useful portion of the pattern.

In analyzing your results, there are a number of opportunities for quantitative comparison with theory. Your experimental curves should have the correct general qualitative form, but they will unavoidably differ from the ideal theory in some particulars. The positions (and uncertainties!) of the experimentally observed maxima and minima should be used together with different trials for the hypothetical parameters to derive refined estimates of the slit dimensions and their uncertainties for one particular single slit and one particular pair of double slits. (These can be compared with direct microscopic measurements.)

Discrepancies in the heights, depths and symmetries of the peaks should give you food for thought and further analysis. The theory section makes certain simplifying assumptions about the illumination and geometry that may not describe your experiment accurately. Many of these "non-idealities" can be profitably simulated and studied at length in the context of a laboratory report. Finally, you should use the trends of your plots for the 3, 4, and 5 slit patterns to describe the effect of increasing the number of slits, and explain the cause of this effect.

5.3.6 Saved data

The program saves your data in two different places. First, it is copied into a data folder within the Igor experiment itself. Second, it is exported to a tab-delimited text file in a subfolder of the **Data** folder on the Desktop of the computer. Open this folder, then open your section's folder, and then open the folder named for you and your partner. Each file is named **Fraun_xxxx**, where **xxxx** represents the time at which the data were saved.

The file format is tab-delimited text, which can be opened by any data analysis package. The first row holds column labels, and subsequent rows hold the data. There is a potential gotcha with text files: different operating systems use different characters to indicate the end of a line. UNIX computers use the linefeed character (ASCII 10), Macintosh computers use the carriage return character (ASCII 13), and DOS/Windows computers use both! If your data analysis program has trouble reading the file, you can use Excel to open it first and handle the conversion of line terminators.

To copy the data to your Charlie account, make sure you are in the Finder (check the first text item in the menubar at the top of the screen; if it isn't **Finder**, then click the first icon in the Dock) and press Command-K or select **Connect to Server...** from the **Go** menu. Enter **charlie.ac.hmc.edu** and you will be presented a login dialog.

Experiment 6

The Grating Spectrometer and the Balmer Series

Abstract

The colors emitted by many sources of light provide a spectral fingerprint that uniquely identifies them. You will study the discrete line spectrum of two elements having a single electron in their outermost shell: sodium and hydrogen. Using literature values for the wavelengths of the yellow sodium doublet, you will calibrate the period of a diffraction grating. You will then measure the wavelengths of four visible hydrogen lines and compare them to the Balmer equation, deducing a value for the Rydberg constant.

6.1 Overview

A **diffraction grating** is a transparent or opaque substrate on which a series of fine parallel, equally spaced grooves are patterned. When a light wave illuminates a grating, each groove scatters light in all directions. However, the scattering will be appreciable only in directions for which the light from adjacent grooves arrives in phase. The behavior of diffraction gratings can thus be readily explained using the wave theory of light.

Diffraction gratings may be used to disperse the colors of a polychromatic source and to identify its component wavelengths. To make a quantitative identification of the wavelengths, you will need to calibrate the grating, which means to determine the distance d between rulings. The grating you will use has a *nominal* period of 600 lines/mm, but you will determine d much more accurately than the 1 significant figure implied by this value by observing the yellow light of a sodium lamp. You have probably seen the bright yellow light of a sodium lamp in parking lots, where they are used because of their energy efficiency and power. By taking as given the literature values for the wavelengths of the two adjacent sodium lines,[1] and by measuring precisely the angles at which the yellow light is diffracted by the grating, you can determine d.

Having calibrated the grating, you will then study the spectrum of a Balmer hydrogen discharge tube, which allows you to investigate the energy levels of a hydrogen atom. When an atom in an excited electronic state relaxes to a lower-energy state, it emits the energy difference between the states in the form of a photon of light. In the case of hydrogen, most of these transitions produce photons either in the ultraviolet or the infrared. For four transitions ending in the $n = 2$ level, the emitted photons are visible and you can study them by eye using the grating you have calibrated with the sodium lines.

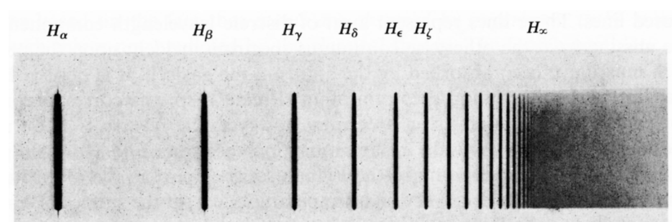


Figure 6.1 The line spectrum of hydrogen.

6.2 Theory

In 1666 while Isaac Newton (1642–1727) was home in Lincolnshire because the plague had closed Cambridge University down, he discovered that whereas a single prism of glass would cause sunlight to fan out into a rainbow of colors, a subsequent prism caused no further color changes. Newton reasoned that sunlight was a mixture of all colors, and that a prism caused the various colors to bend (or refract) differently, thereby producing a smooth, continuous band of gradually varying appearance.

[1] The discrete wavelengths at which a particular atom or molecule emits light are called "lines" because this is how they appeared on photographic plates in the early days of spectroscopy (see Fig. 6.1). A diffraction grating fans the light out in one direction, but doesn't affect the extent of the beam in the perpendicular direction. Hence, the "signal" on the photographic plate looks like a line segment.

A century and a half later in 1814, an orphaned glassmaker, Joseph von Fraunhofer (1787–1826), discovered that the smooth solar spectrum was actually interrupted by a series of dark lines.[2] To study these dark lines quantitatively, Fraunhofer invented the diffraction grating.[3] By mid-century, scientists had determined that the dark (absorption) lines and the identically located bright (emission) lines serve as spectral fingerprints, uniquely identifying the atomic species involved. Comparing the yellow light of sodium to a prominent dark line in the solar spectrum allowed Gustav Kirchhoff (1824–1887) and Robert Bunsen (1811-1899) to deduce the presence of sodium in the solar atmosphere.

The positions of the spectral lines of different substances made a challenging puzzle to nineteenth century scientists—and to modern quantum theory, for all but the simplest atoms! In 1885, J. J. Balmer (1825–1898) showed that the wavelengths of the visible spectral lines of hydrogen were given by the expression

$$\frac{1}{\lambda} = R\left(\frac{1}{N^2} - \frac{1}{n^2}\right) \tag{6.1}$$

where n is an integer and $N = 2$ for the visible lines of the Balmer series. The hunt was on to find a theory that could explain Balmer's relationship.

At the end of the century, Max Planck (1858–1947) found he could explain the smooth (dominant) part of the solar spectrum with the help of the relation

$$\Delta E = h\nu = \frac{hc}{\lambda} \tag{6.2}$$

between the frequency and the quantum of energy emitted or absorbed by an atom. Niels Bohr (1885–1962) combined the Planck relation with his own idea of angular momentum quantization to produce a simple theory for the hydrogen atom that reproduced Balmer's relation, Eq. (6.1). Bohr's model described unphysical microscopic solar systems, but the principal idea of angular momentum quantization survived the birth of modern quantum mechanics in 1925–26 and the hydrogen atom calculations of Heisenberg (1901–1976) and Schrödinger (1887–1961).

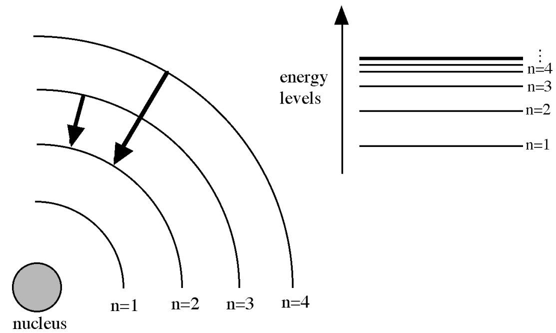


Figure 6.2 A schematic representation of quantized atomic energy levels.

Quantum mechanics also tells us that electrons in orbit around a nucleus cannot have just any energy (like planets orbiting the Sun), but are restricted to certain quantized energy levels (Fig. 6.2). This prediction of quantum mechanics explained the previously mysterious fact that elements emitted light only at discrete wavelengths. Quantum mechanics was able to explain and quantitatively predict the wavelengths of these spectral lines for hydrogen, the simplest element. This confirmed the relation that had been empirically discovered by Balmer for the wavelength of the visible hydrogen lines,

In Eq. (6.1), N and n are integers corresponding to the lower and upper energy levels, respectively. For the visible Balmer series, the lower level always corresponds to $N = 2$. However, n is different for each spectral line in the series, each corresponding to a different upper energy level. The constant R is known as the Rydberg constant, with an accepted value (1998) of $R = 1.097\,373\,157 \times 10^7\ \text{m}^{-1}$. In this experiment you will first observe and carefully measure the wavelengths of these spectral lines, and then determine best values for n and R.

The technique used in this experiment to measure the wavelengths makes use of diffraction gratings. When *parallel* light is incident upon an array of many equally spaced rulings on a transparent substrate, light emerges on the other side at certain specific angles where constructive interference occurs among the simultaneously illuminated rulings. These intensity maxima occur at angles θ for which

$$d \sin\theta = m\lambda \qquad m = 0, \pm1, \pm2, \ldots \tag{6.3}$$

[2]Fraunhofer's childhood augured anything but a scientific career. Orphaned at age 11, he was not strong enough to become a wood turner and was apprenticed to a glassmaker, Philipp Anton Weichselberger. Weichselberger did his level best to frustrate the intellectual curiosity of the boy for the next two years, but Fraunhofer's life took a sudden turn when Weichselberger's house/workshop collapsed in 1801. After being buried in the rubble for several hours, Fraunhofer was rescued and came to the attention of Prince Elector Max IV Joseph, who directed the rescue operation. The prince took an interest in Fraunhofer and made sure that he had books and time to study while he learned the craft of lens grinding.

At age 22 he was appointed head of the glass factory in Benediktbeuern, where he worked to develop new types of glass and on methods to avoid streaking and inhomogeneities. He expanded the product line to include telescopes, binoculars, microscopes, magnifying glasses, and the best and largest telescopes at the time. He invented the spectroscope in 1814 and the diffraction grating in 1821.

[3]Evidently, the American D. Rittenhouse preceded Fraunhofer by 28 years, but his work was hardly noticed. See M. Born and E. Wolf, *Principles of Optics*, 7th Edition, Cambridge, 1999, p. 453.

Here, d is the spacing between rulings on the grating, and the integer m is referred to as the order of the observed interference maximum. The theory of diffraction gratings is a straightforward extension of the Fraunhofer diffraction theory presented in Sec. 5.2. In particular, the behavior of the grating is roughly as indicated in Fig. 5.4, which shows the diffracted intensity from 5 identical slits, except that the transmission peaks for the grating are much narrower and the small wiggles at the bottom are much more rapid and imperceptibly tiny.

As indicated in Fig. 6.3, the purpose of the grating spectrometer is to measure the angles θ at which lines are observed, allowing their wavelengths to be determined. The light from some source enters the instrument at a slit opening on the collimator tube. We need the light striking the grating to be parallel rather than diverging, so the lens in the collimator tube serves this purpose.

After striking the grating and emerging at some angle θ, the light is collected for viewing by the telescope assembly. The lens on this assembly focuses the parallel light into an image which is viewed by the observer through the eyepiece. The principal observational challenge in conducting this experiment is learning to use the instrument to measure θ most effectively with a minimum of systematic error and uncertainty.

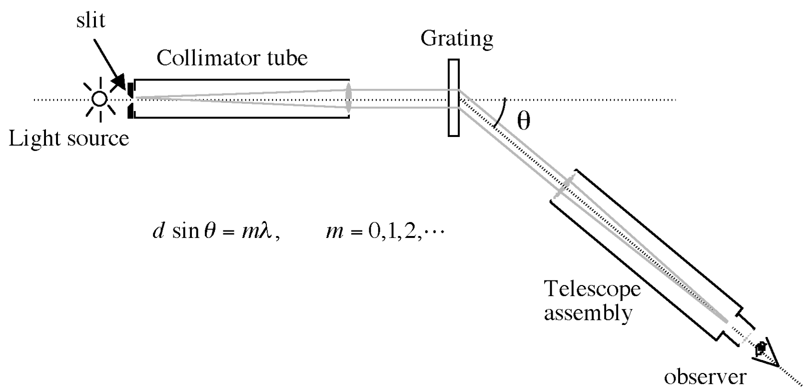


Figure 6.3: A schematic view of the operation of a grating spectrometer.

6.3 Using the Spectrometer

The grating spectrometer is shown in Fig. 6.4, with its important components and adjustments labeled. You should begin by familiarizing yourself with location and function of all the parts, so that you can use them comfortably while working in the dark. The important moving parts are the telescope assembly, the rotating grating table, and the circular measurement scale. Each of these can be moved by hand, and locked in place by tightening a lock screw. Avoid damaging the instrument by forcing the components to move when they are locked. Only moderate "finger tight" pressure should be required to hold adjustments in place, once they are made.

In addition to the rotational adjustments, you will adjust the focus of the telescope assembly, the collimator tube, and the width of the slit opening. Take time now to find the following adjustment locations and see how they work:

- Telescope arm lock screw — *under the circular scale on the right side of the center post*
- Telescope arm fine adjustment — *under the rim of the scale on the front of the telescope*
- Circular scale lock screw — *under the circular scale on the left side of the center post*
- Grating table lock screw — *above the circular scale on the center post*
- Reticle (cross hairs) focus adjustment — *outer black ring of telescope eyepiece*
- Telescope image focus lock — *knob on top of telescope near eyepiece*
- Collimator focus lock — *knob on top of collimator near slit*
- Slit width adjustment — *black knob on side of slit assembly*

6.3.1 Focusing procedure

For the spectrometer to work properly, the light that diverges from the input slit must be collimated by the collimator tube, the grating normal must be aligned with the collimated input light, the telescope assembly

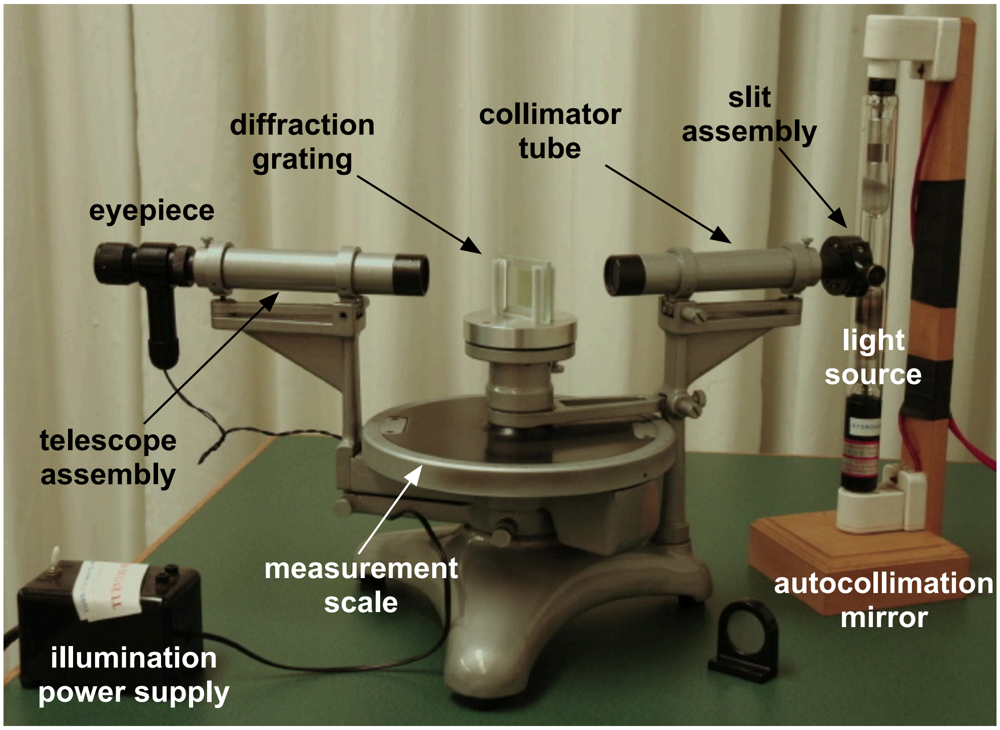


Figure 6.4: Photo of grating spectrometer and supporting equipment.

must be focused to image collimated light correctly, and it must respect the grating axis. The following procedure will allow you to achieve these conditions.

> **Warning: Never touch a grating! Gratings cannot be cleaned, and are significantly degraded by fingerprints and dirt.**

1. First you must adjust the telescope assembly to image collimated (parallel) light properly. Look through the eyepiece and rotate the ocular until the black crosshairs are clear and sharp.

2. Turn on the illumination power supply and place the autocollimation mirror on the grating table, aimed to reflect light back into the telescope. Rotate the mirror slowly while looking through the ocular until you see a green cross. Light from the green cross reflects from a mirror inside the eyepiece assembly, is collimated by the telescope lens, bounces off the autocollimation mirror, and then back down the telescope to be viewed through the eyepiece.

   When the green light is collimated at the mirror, the green cross will appear sharp and clear. By sliding the eyepiece in its sleeve, adjust the focus of the green cross until it is sharpest. Lock the eyepiece in place, keeping the reticle line parallel to the slit image. You should now see *both* the black crosshairs and the green cross sharply focused.

3. At this point it would be helpful to make sure that both partners can see both the green cross and the crosshairs simultaneously in focus. Either partner is allowed to adjust the ocular *only*. This accommodates for differences between eyeballs; no other adjustment should be necessary. If it is, go back to the previous step.

4. You must now make sure that the normal to the grating is aligned with the axis of the telescope (which is fixed). Remove the autocollimation mirror and rotate the grating table so that the grating can reflect (like a mirror) the green cross light back down the telescope. Locate the brightest image of the green cross, rotate the grating to center the image in the eyepiece, and adjust the tilt of the grating table to put the strongest green cross at the horizontal crosshair in the top half of the field of view. [Since the source of the green cross is at the bottom of the field of view, its image should appear symmetrically in the top part.]

5. Looking through the telescope eyepiece, line up the telescope and collimator tube so that the slit image is centered, vertical, and focused. You may slide the slit assembly in or out of the tube slightly to achieve best focus. Then lock it in place.

6. With the slit image carefully centered in the reticle adjust the circular scale until it is zeroed, and lock it in place.

7. Rotate the grating table so that the ruled face of the grating faces the telescope and make sure that the grating is centered in the holder. Lock the grating table to prevent further rotation, and turn off the green illumination source.

6.4 Experimental Measurements

6.4.1 Grating calibration

From Eq. (6.3), it is evident that the observed position θ of a spectral line allows its wavelength λ to be determined provided that the grating spacing d is known. The grating in this experiment is known to have a nominal spacing of about 600 lines/mm, but a more precise determination of d is required. For this purpose, we will take the wavelengths of the yellow sodium doublet (588.995 nm and 589.592 nm) as accepted standards.

Turn on the sodium vapor lamp and place it at the slit opening of the spectrometer. It will take several minutes for the light to change from a pinkish mixture of contaminant wavelengths to the pure yellow color characteristic of Na. While it is warming up, rotate the telescope arm of the spectrometer and look for a series of red lines, as well as the yellow lines of the sodium doublet. [Until you have calibrated the grating, you are not in a position to measure the wavelength of some of the red lines; however, an investigation of these lines to determine the contamination gas could be an interesting topic for a technical report.] Using the nominal value for d, estimate the angle(s) at which you expect to find the sodium doublet, and use the spectrometer to look for them.

During this step, you'll be learning how to use the spectrometer, which will help you find the more difficult lines of the hydrogen spectrum. One of the adjustments available to you is the slit width. The wider the slit opening, the brighter is the light seen, which may be helpful in trying to view particularly faint lines. (You can also see fainter lines more easily by darkening the room, using a black cloth to shield outside light, and allowing 10 minutes or so for your eyes to become dark adapted.) A wider slit opening however, has the undesirable effect of making the line image wider, and therefore making the uncertainty of its position greater. You'll note that if you make the slit wide enough, the images of the two doublet lines will merge together into one wide line.

Make adjustments until you achieve a good compromise between maximum sharpness and adequate visibility. Now is a good time to make and record estimates of the uncertainties in θ and λ associated with your choice(s) of slit width. Also note that in addition to the uncertainty caused by finite slit width, the spectral lines will have a finite width due to the resolving power of the grating. The smallest resolved wavelength difference $\Delta\lambda$ is

$$\frac{\lambda}{\Delta\lambda} = Nm \tag{6.4}$$

where m is the order and N is the number of grating rulings illuminated. If you use a piece of paper to cover part of the width of the grating, you should see the apparent line width increase accordingly. Using standard error propagation, this effect should be incorporated into your uncertainty budget.

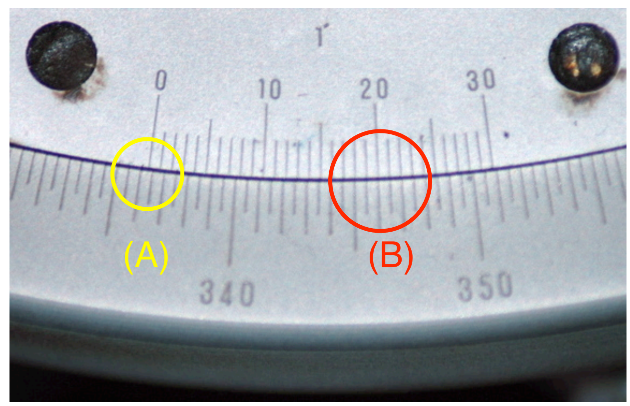


Figure 6.5 The vernier allows you to read angles to the nearest minute of arc. Start at (A) to determine $336° < \theta < 336°30'$. Then look for the lines that match up best at (B) to obtain the final value $336°20'$.

Make careful measurements of θ for both the sodium doublet lines on both sides of zero, for as many orders as you can. Figure 6.5 shows how to read the vernier to about 1 minute of arc. At this point you may wish to thank the ancient Sumerians for the sexagesimal system we use for telling time and measuring angles. If you have been careful in the alignment of the spectrometer and grating, the right and left angles of deflection should agree with one another to better than $10'$; if they do not, now is the time to tweak the grating position to get symmetric readings, in preparation for the hydrogen measurements.

From your Na measurements, determine the best value for d to be used in the study of the hydrogen spectrum in the next part. You will probably note that some orders appear brighter than others, and that the lines may appear brighter or clearer on one side than the other. This is because the grating is blazed or optimized for maximum output in a particular direction. Make note of this information, as some of the fainter hydrogen lines may be hard to observe in any but the brightest and best conditions. Consider carefully the uncertainty of your data in deciding whether to use the first- and/or second-order measurements in obtaining a calibration for d and also in determining the hydrogen wavelengths.

6.4.2 Hydrogen observations

Replace the Na lamp with the Balmer hydrogen discharge tube lamp.

> **Warning: Be sure to turn on the cooling fan and direct it toward the base of the Balmer tube, or overheating may result.**

Observe the hydrogen spectrum. You will be looking for four visible wavelength lines, corresponding to colors of deep violet, violet, blue-green, and red. (The shortest wavelength line can be particularly hard to see, and may require some patience and observation skill to locate and measure.)

Record the color and angular position of all the spectral lines in all of the orders in which they can be adequately observed. Include appropriate uncertainties for each measurement, so that the correct relative weighting can be assigned to observations of differing quality in coming up with a final set of "best results" for the hydrogen wavelengths. Calculate the final values and propagated uncertainties for the wavelengths of the hydrogen Balmer lines.

6.4.3 Forensic investigation

If time permits, return to the sodium lamp, which should now be cool again, and investigate the pink light it gives off before the discharge warms up. At room temperature, the vapor pressure of sodium in the lamp is too small to operate the electric discharge. Therefore, the lamp is filled with buffer gas to help the lamp get going. For the sodium lamp to work efficiently, the sodium vapor must be heated by the discharge to 260°C. Different buffer gases may be used in sodium lamps to help launch the discharge and heat the sodium. You can figure out what is in the lamp if you use the spectral finger print of the buffer gas(es). Record as precisely as you can the positions of several prominant spectral lines, and convert them to wavelengths. You can then look them up at `http://cfa-www.harvard.edu/amdata/ampdata/kurucz23/sekur.html`.

To use the web-based database, you must enter ranges for several parameters. Here's how to find the sodium lines you have already observed:

1. Set the range from 500 nm to 700 nm.
2. Set the oscillator strength range from -2 to 4. Bright lines have large oscillator strength.
3. Set the lower level range from 0 to 10^5 cm^{-1} and the upper level range from 0 to 4×10^5 cm^{-1}.
4. Select all the Na items in the left column, then click **Search**.

You should see a listing of 8 spectral lines, including the two prominent lines of the sodium doublet. Notice that these are the only entries in this wavelength range that have the ground state as the lower energy level. Notice further that these lines are described by Na I, which is traditional spectroscopic notation for the neutral sodium atom. (Na II indicates singly ionized sodium, Na III is doubly ionized, etc.)

Probably should try this and consider guidance at this and other sites: `http://physics.nist.gov/PhysRefData/Handbook/Tables/findinglist.htm`

6.5 Experimental Analysis

One simple way to evaluate your results is to compare them with accepted literature values, shown in the table below. Statistically characterizing any discrepancies as random or systematic, probable or improbable, is important to demonstrating your understanding of the experiment.

Color	Wavelength
red	656.17 nm
blue-green	486.13 nm
violet	434.05 nm
deep violet	410.17 nm

A somewhat more instructive analysis is obtained by comparing your results with the predictions of the Balmer relation of Eq. (6.1). There are a couple of ways to manage this. Pick either one.

- A plot of $1/\lambda$ as a function of $1/n^2$ should theoretically yield a straight line, whose slope and y-intercept are related to the Rydberg constant and the value for the constant N. (Here, the consecutive integers n index the energy levels corresponding to each line; it will be up to you to determine convincingly what they are for the observed lines by comparing the linearity of the fit of the Balmer formula for several plausible choices of the sequence of n values.) You should prepare the plot giving the best straight line fit, along with appropriate error bars and a properly weighted linear fit determining R and N.

- Plot λ vs. n and fit to Eq. (6.1). You don't necessarily know the values of n, although you may assume that you have measured lines corresponding to consecutive integers. Therefore, you may fit to a modified version of Eq. (6.1):

$$\lambda(n) = R^{-1}\left(\frac{1}{4} - \frac{1}{(n+n')^2}\right) \qquad (6.5)$$

  where n' is a fitting parameter that should come out very nearly equal to an integer.

As before, discuss the results and their agreement (or lack thereof) with theoretical and accepted values. (How does your value of the Rydberg constant constrain other fundamental physical constants?) As quantitatively as possible, ascribe any discrepancies to sources of uncertainty in the experimental equipment and procedures. For example, if there is a lingering systematic bias between your wavelength estimates from the right and left sides, you should quantitatively investigate the origin of this error, and attempt to correct for it in coming up with your final results. Although the results of this experiment are actually well known to the physics world, you should approach the analysis task as if the results are original, and strive to be as precise and rigorous as possible in obtaining them.

Part III

Appendices

Appendix A

Electric Shock – it's the Current that Kills

Naively, it would seem that a shock of 10 kV would be more deadly than 100 V. This is not necessarily so! Individuals have been electrocuted by appliances using ordinary house currents at 110 V and by electrical equipment in industry using as little as 42 V direct current. The real measure of a shock's intensity lies in the amount of current forced through the body, not in the voltage. Any electrical device used on a house wiring circuit can, under certain conditions, transmit a fatal current.

From Ohm's law we know

$$I = \frac{V}{R}$$

The resistance of the human body varies so greatly it is impossible to state the one voltage as "dangerous" and another as "safe." The actual resistance of the body varies with the wetness of the skin at the points of contact. Skin resistance may range from 1000 Ω (ohms) for wet skin to over $500,000$ Ω for dry skin. However, if the skin is broken through or burned away, the body presents no more than 500-Ω resistance to the current.

The path through the body has much to do with the shock danger. A current passing from finger to elbow through the arm may produce only a painful shock, but the same current passing from hand to hand or from hand to foot may well be fatal. The practice of using only one hand (keeping one hand in your pocket) while working on high-voltage circuits and of standing or sitting on an insulating material is a good safety habit.

The Physiological Effect of Electric Shock

Electric current damages the body in three ways:

1. it interferes with proper functioning of the nervous system and heart
2. it subjects the body to intense heat
3. it causes the muscles to contract.

Figure A.1 shows the physiological effect of various currents. Note that voltage is not a consideration. Although it takes a voltage to make the current flow, the amount of current will vary, depending on the body resistance between the points of contact.

As shown in the chart, shock is relatively more severe as the current rises. At values as low 20 mA, breathing becomes labored, finally ceasing completely even at values below 75 mA. As the current approaches 100 mA, ventricular fibrillation of the heart occurs — an uncoordinated twitching of the walls of the heart's ventricles. Above 200 mA, the muscular contractions are so severe that the heart is forcibly clamped during the shock. This clamping protects the heart from going into ventricular fibrillation, and the victim's chances for survival are good.

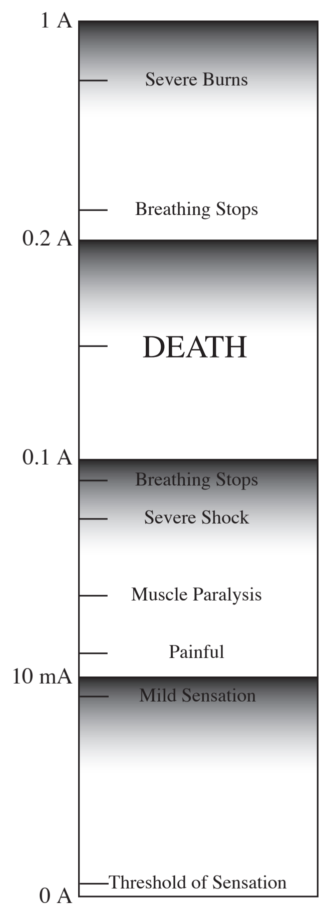


Figure A.1 Common injuries at different currents

AC is four to five times more dangerous than DC, because it stimulates sweating that lowers the skin's resistance. Along that line, it is important to note that the body's resistance goes down rapidly with continued contact. Induced sweating and the burning away of the skin oils and even the skin itself account for this. That's why it's extremely important to free the victim from contact with the current as quickly as possible before the climbing current reaches the fibrillation-inducing level. The frequency of the AC has a lot of influence over the severity of the shock. Unfortunately, 60 Hz (the frequency used in the lab) is in the most harmful range — as little as 25 V can kill. On the other hand, people have withstood 40 kV at a frequency of 1 MHz or so without fatal effects.

A very small current can produce a lethal electric shock. Any current over 10 mA will result in serious shock.

Summary

Voltage is not a reliable indication of danger, because the body's resistance varies so widely it's impossible to predict how much current will be made to flow through the body by a given voltage. The current range of $100 - 200$ mA is particularly dangerous, because it is almost certain to result in lethal ventricular fibrillation.

Victims of high-voltage shock usually respond better to artificial respiration than do victims of low-voltage shock, probably because the higher voltage and current clamps the heart and hence prevents fibrillation. AC is more dangerous than DC, and low frequencies are more dangerous than high frequencies. Skin resistance decreases when the skin is wet or when the skin area in contact with voltage source increases. It also decreases rapidly with continued exposure to electric current.

Prevention is the best medicine for electric shock. That means having a healthy respect for all voltage, always following safety procedures when working on electrical equipment.

In case a person does suffer a severe shock, it is important to free him from the current as quickly as can be done safely and to apply artificial respiration immediately. The difference of a few seconds in starting this may spell life or death to the victim. Keep up the artificial respiration until advised otherwise by a medical authority.

References:

1. Electronics World, p. 50, December 1965
2. Tektronix Service Scope, No. 35, December 1965, The Fatal Current

Appendix B

Resistors and Resistance Measurements

Types of Resistors

Resistance R is defined as the ratio of the potential difference V between the terminals of a conductor to the current I in the conductor, or

$$R \equiv \frac{V}{I}$$

If the resistance of a conductor is independent of the potential difference and the current; that is, if a plot of V vs I is linear, then the conductor obeys Ohm's law.

Almost all conductors offer some resistance to the flow of current and it is often desirable to keep such resistance in a circuit at a minimum. However, special devices called resistors are widely used in electrical circuits solely for their resistive value.

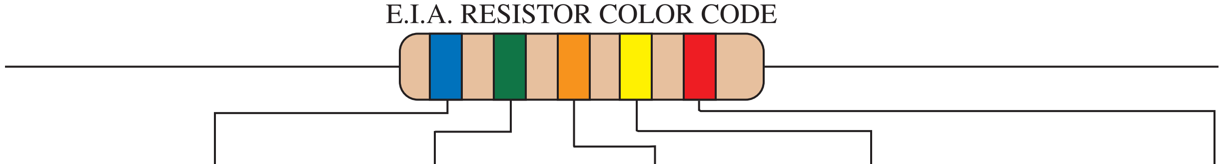
The most common resistor used in electronic circuits is made of carbon molded in the form of a small cylinder with wires attached at each end. The resistance value is generally designated by a set of three colored bands on the resistor. The band closest to the end of the resistor represents the first figure of the resistance; the next band gives the second figure; and the third band gives the number of zeros which must be added to get the total resistance. Figure B.1 below shows the values associated with each color.

For example, a resistor with bands of green-yellow-orange would be 54000 Ω, or 54 kΩ. For resistances ranging 0.1–9.9 Ω, gold is used for the third band. Thus, violet-white-gold is 7.9 W. This labeling system can describe resistances ranging from 0.01 Ω (black-brown-silver) to 99 MΩ (white-white-blue).

The fourth band indicates the tolerance; that is, the accuracy assured by the manufacturer in designating the resistance value. The tolerance is expressed in percent of the nominal resistance value, with silver representing 10 percent and gold 5 percent. The fifth band is a failure rate indicator.

Another commonly used resistor is constructed by depositing a thin metallic film on an insulating form. This type produces little noise in electronic circuits, and obeys Ohm's law over a wide range of currents and temperatures. A third type of resistor is made of a coil of resistance wire wound on an insulated form. These resistors can have very precise resistances, but it must be kept in mind that the inductive effect of the coil will decrease this accuracy when used in a high frequency circuit. Resistive values of these resistors do not follow the colored-band convention described in Fig. B.1, but are instead printed numerically on the resistor's body.

E.I.A. RESISTOR COLOR CODE



1st Significant Figure	
Color	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9

2nd Significant Figure	
Color	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9

Multiplier	
Color	10^n
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Silver	-2
Gold	-1

Tolerance	
Color	Tolerance
Silver	± 10%
Gold	± 5%

Failure Rate Level (%/1000 Hours)	
Color	Level
Brown	M = 1.0%
Red	P = 0.1%
Orange	R = 0.01%
Yellow	S = 0.001%

Figure B.1: Resistor color band values

The accuracy of a resistor may be affected by a number of factors. For resistors of very low values (less than 10 Ω, the resistance of the connection to the resistor can become a significant part of the total resistance. For resistors of very high values (greater than 10 MΩ, leakage of current between the contacts on the surface of the resistor may significantly decrease the resistance, though this effect may be greatly reduced by the use of special insulating materials. The resistance of almost all resistors changes with temperature, and for this reason it is often necessary to hold the temperature of a resistor constant in order to maintain stability in an electronic circuit.

All resistors are limited in the amount of current which they can carry. An excessive current will burn out a resistor, but even a small overload will cause the resistor to become overheated and change its resistance. To determine the maximum current which a resistor can carry, use

$$P = I^2 R$$

The power rating of some resistors is marked on the resistor body; but for the common carbon type, only its physical size indicates the power it can handle. The smallest ones with a diameter of 1/8 inch are rated at 0.5 W, 1/4 inch at 1 W, and 3/8 inch at 2 W.

Resistors designed to operate "hot," such as tube filaments, heating coils, and light bulbs, have a much lower resistance when they are cold than when they are hot. Thus, a heavy surge of current occurs when the circuit is first closed, and at that instant the filament wire is most likely to burn out.

Study the various resistors available to you, determining their type, nominal resistance value, tolerance, and power rating. Some dissected resistors are on display in the laboratory and should be examined for an understanding of their internal construction.

Appendix C

Electrical Techniques and Diagrams

Precautions

Electrical experiments call for special precautions, since applying power to an incorrectly wired circuit can seriously damage or destroy expensive equipment. The key rules are:

1. When in doubt about a circuit, do not apply power. Recheck each branch and consider the function of each element.
2. If still in doubt, ask your instructor or lab assistant to check the circuit.

It is disconcerting to hear a student say, “I don’t know whether this is right or not, but let’s turn it on and see what happens.” Always check the polarity of the circuit before wiring up a DC meter and understand your circuit before “trying” it. Attention should be paid to the power rating of every piece of electrical equipment — make certain never to exceed it. Include an appropriate fuse in your circuit whenever possible.

It will not be necessary for you to have your instructor check every circuit before it is put into operation, especially if it is one with which you are familiar. **However, in the case of a new circuit or one involving expensive equipment, the instructor or lab assistant should be asked to check the circuit before any power connections are made.** Any time that a circuit does not behave as it should, open all switches immediately and correct the problem before applying power again.

Elementary Electrical Techniques

Good results in electrical experiments require that the circuits be correctly and neatly wired and that you understand the reasons for each connection. Translating circuit diagrams to actual circuits quickly and accurately (and vice versa) takes practice. Some students may already be skilled in reading and wiring circuits, while for others it may be a new experience. The following notes are written primarily for beginners.

1. Circuits are usually specified by a schematic diagram using the symbols illustrated in Fig. C.1. Actual laboratory circuits translate these schematic diagrams into wires connected between binding posts or spring clips on the apparatus. Power supplies, batteries, and meters have polarities which must be watched carefully.
2. Every circuit has a main series part in which most of the current flows. Identify this main series circuit and connect it up first, then attach the branches which make up the rest of the circuit.
3. Although in a schematic diagram it is often convenient to indicate a junction point at the middle of a connecting wire, in practice it is necessary to make every junction at a terminal of some apparatus. The results are equivalent if you assume negligible resistance in the connecting wires. Since this assumption is not always admissible, care must be taken not to include a long wire in series with a small resistance. **Never use a meter terminal as a junction point.** Only one wire should be connected to any meter terminal so that it is possible to remove or replace any meter without otherwise disturbing the circuit.
4. A circuit should always be “dead” when it is being made up or when any changes are being made, and the circuit should be checked carefully thereafter before it is powered. A “dead” circuit is one in which at most one terminal of any battery is connected, and no power plugs are inserted into power sockets.

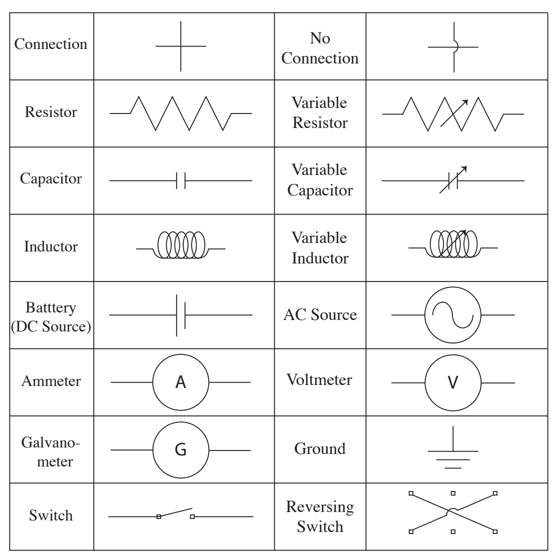


Figure C.1 Common circuit Symbols

Appendix D

Technical Report

One of your requirements for Physics 53 is writing a technical report. You can write the report on any experiment you have completed except for Exp. 1. Pick an experiment which will make an interesting report and one for which your results are reasonable. There is no page limitation for the report, but you should restrict yourself to 1500 – 2500 words. This allows your report to be complete and informative, but also requires concision. Templates in Microsoft Word and LaTeX format are available for download. See the writing section of the physics kiosk at `http://www.physics.hmc.edu/kiosk.php`.

You are strongly urged to submit a complete draft of your paper to your instructor for review. This draft should be a serious attempt at a complete and finished document — the more effort you put into this draft, the more your instructor will be able to help you improve it. Give your instructor at least one week to read the draft (and you will need enough time to rewrite it), so check with your instructor to determine an appropriate schedule for submission.

Your instructor will also announce the due date for the final version of the report, which should be typed and any figures or tables should be neatly drawn. The technical report counts for a significant portion of your course grade (see Introduction), so please take it seriously.

Learning to produce a good technical paper takes time, and for that reason you will be given several opportunities (groan) at HMC to improve your technical writing. We know from alumni surveys that the great majority of Harvey Mudd alumni have found that writing skills were much more important to their careers than they anticipated, and that they wish they had worked harder on writing while in college. Save your reports after they are returned to you to serve as models for future reports.

4.1 A Guide to Technical Report Writing

The writing of technical reports and journal articles is a part of almost all engineering and scientific work. There are no precise rules which an author can apply to the preparation of a scientific paper as rigorously as a cook follows a recipe, but there are some general principles which will help insure a reasonably smooth and understandable presentation of a body of scientific information. Perhaps the best way to pick up those principles is to read reports and articles appearing in various engineering and scientific journals, such as The Physical Review.

One of the most common criticisms of technical reports is that they are not written in a sufficiently brief and concise form. To write succinctly is often difficult. Writing in a rather loose and informal style is much easier, but it simply cannot be tolerated in scientific writing today for some very good reasons. The editor of the Astrophysical Journal wrote the following paragraph:

> The present accelerated growth of this Journal in common with the other scientific journals, makes it imperative that authors (in their own interest) exercise utmost restraint and economy in the writing of their papers and in the selection and presentation of material in the form of tables, line drawings, and halftones. In spite of the obvious need for such restraint, the Editor regrets that authors continue to write in the relaxed style common a century ago; moreover, the temptation to reproduce large masses of IBM printouts and tracings from automatic recording equipment appears too great for most authors to resist. The Astrophysical Journal will enforce stricter standards in the future with respect to these matters.

The present charge for publication in the leading physics journals is over \$70 per page. Thus, it is important that you learn to write your reports in such a way that unnecessary words are eliminated and data is reduced to a minimum. This generally calls for some rewriting.

You should understand that there are also very good reasons for standardizing technical writing. Most readers can afford only the time to scan the articles found in journals or the many technical reports used in industry. In order to facilitate that scanning, a certain degree of standardization is very important. For example, if authors present their data in tables of similar form, it becomes relatively easy for the reader to obtain what he wants from those tables without reading all of the text. Similarly, figures should be in a similar form so that they can be studied without referring to the text. It is for this reason that the title for each figure should be written out in words rather than in symbols and that all figures and tables have a caption which tells the reader what is being presented without his having to read the text. Abstracts are also very important to the reader who is scanning rather than reading in detail. You should look at some of the American Journals of Physics in order to see how abstracts are written, and all of your technical reports should be preceded by a brief abstract. You should note that abstracts are very specific and give quantitative results. A vague abstract which does not tell what you found out as a result of your work is useless to the reader.

4.2 Summary Guide for a Technical Report

4.2.1 Content

Title	
Author	author's name author's institution or company date of submission
Abstract	Summary of the principal facts and conclusions of the paper. Usually less than 100 words.
Introduction	Concise discussion of the subject, scope, and purpose of the experiment. It should be accessible to the educated non-expert.
Theory	Succinct development of the theory related to the experiment.
Experimental methods	Brief description of experimental apparatus and procedures.
Results	Summary of important data and results, using figures (graphs) wherever possible. Include error estimates for all relevant quantities. Note that data are typically presented graphically, not in tabular form, although there are exceptions. If you feel a table is more appropriate, discuss this with your instructor.
Discussion or Conclusions	Further analysis and discussion of results. Specific conclusions. Recommendations if needed. "Graceful termination."
References	(see below)

4.2.2 Mechanics

1. The paper should be divided into clearly labeled sections, for example: ABSTRACT, INTRODUCTION, THEORY, EXPERIMENTAL METHODS, RESULTS, DISCUSSION (or CONCLUSIONS), REFERENCES.

2. Double space *all* copy except the Abstract.

3. Number all pages in sequence, beginning with the title page.

4. Begin the Abstract four or five lines below the Author information.

5. Figures and Tables:
   - Place tables and figures as soon as possible after they are referred to in the text.
   - Give each table and figure a complete title and/or self-descriptive caption.

6. Number all equations. (Place numbers near right-hand margin.) Each equation should be on a separate line. Use complete sentences and fit all equations into the sentences that introduce them.

7. Define all symbols used.

8. See "5. References and Appendices" (below) for methods of citing references in this report.

Comments on "Summary Guide for a Technical Report"

Outline A detailed outline serves as an excellent writing guide. In composing the outline, make as many subdivisions as possible. It is easier to eliminate or combine existing subheadings than to insert new ones. Be sure the outline reflects the emphasis you wish the paper to have. As you write the paper, the outline may be drastically revised, but it is nevertheless a good starting point.

Abstract As stated earlier, the abstract should be a concise and specific summary of the entire paper. It should be as quantitative as possible and include important results and conclusions. Including all this in less than 100 words takes careful thought (and probably considerable rewriting). You should write the abstract after you have written the rest of the paper, even though it appears first.

Introduction Every scientific paper or technical report should contain at least one or two introductory paragraphs. The first paragraph of the Introduction is particularly critical, since it plays a major role in determining the reader's attitude toward the paper as a whole. It is important enough to warrant considerable time and attention. The following steps are suggested as a means of accomplishing a good Introduction.

1. Make the precise subject of the paper clear relatively early in the Introduction. You should assume your reader is someone with essentially your background in physics but with no particular knowledge of the experiment you performed. Thus you should include background material only to the extent necessary for the reader to understand your statement of the subject of the paper and to appreciate the scientific reasons for your doing the experiment to be described.
2. State the purpose of the paper clearly. This statement should orient the reader with respect to the point of view and emphasis of the report and what he should expect to learn from it. A well-done Introduction will also be of great help to you in providing a focus for your writing and in drawing your final conclusions (see below).
3. Indicate the scope of the paper's coverage of the subject. State somewhere in the introductory paragraphs the limits within which you treat the subject. This definition of scope may include such topics as whether the work described was experimental or theoretical (In this particular case, the work is almost certainly experimental), the exact aspects of the general subject covered by the paper, the ranges of parameters explored, etc.

The Main Body This part of the paper generally includes a brief discussion of the theory, a description of the experimental apparatus and procedure, a presentation and analysis of your data and results, and your conclusions. Some suggestions concerning each of these sections follow.

Theory If the major emphasis of your report is the experimental work which you carried out in the laboratory, then the theory may be a reduced version of that given in the laboratory manual. If, on the other hand, the report emphasizes some theoretical aspect of the problem, your section on theory may be an expanded version of the theory given in the laboratory manual. In either case, you should avoid, if possible, simply duplicating the theory given in the laboratory manual. Do not present the details of usual algebraic manipulation, which the reader can easily duplicate. Avoid references to the laboratory manual (find another source!). Note again that the technical report should concentrate on work that you did; a lengthy discussion of the theoretical work of others is not appropriate, in general.

Experimental methods In writing this section you should have in mind a reader who has your background in physics and is familiar with typical laboratory apparatus but has no knowledge of the specific experiment you are describing and probably has no intention of repeating it. Therefore, you should describe your experimental apparatus and the procedures you followed only insofar as it is necessary for him or her to understand how you made your measurements. A careful drawing of the apparatus is often useful here.

In this regard it is important for you to understand the distinction between a technical report and a laboratory manual. The latter is designed primarily to give instructions on how an experiment is to be carried out, while the former describes to the reader what the author has done. Though it is sometimes necessary in a technical report to describe the procedures which were followed in carrying out the experiment, such material should be kept to a minimum, and under no circumstances should it instruct the reader as though he or she were a student in a laboratory.

Data and Results In most cases this part of your report will be very different from your laboratory notebook. The data should be rearranged in the most concise form possible. This generally means a presentation of important data in simple tables which include average values and final results. Avoid presentation of large amounts of raw data; e.g., simply state that the average of 20 measurements was 1.87 seconds with a standard deviation of the mean of 0.08 seconds. You should strive to make the tables self-explanatory, and results should be included in an obvious way so that a minimum of text is necessary. Graphs and figures can take the place of many words. (Think of the figures in Scientific American.)

Some form of error analysis should be included with your results, and the uncertainty of the final result should be stated clearly. The analysis of errors must be thorough and convincing. Your goal should be to convince a skeptical reader that your quanti-tative results are believable. This will often require a more thorough analysis of experimental errors than was done in your lab notebook.

Discussion or Conclusions This section will often contain further analysis and discussion of your results in order to support and clarify your conclusions. Your final conclusions should be quantita-tive, precise, and specific, but they should not be simply a summary of the experi-mental results. Rather, they should be convictions arrived at on the basis of evidence previously presented and analyzed. Also, make sure they fulfill any promise you made to the reader in the Introduction as to what your paper would demonstrate or prove. You may want to include suggestions for further work. The paper should end with what is generally called a "graceful termination."

5. References and Appendices

6. Miscellaneous Comments

Seek a simple, direct style. Avoid long, complicated sentences in the passive voice. Short, active voice sentences are easier to understand and faster to read. You may use the first person singular if you wish, although it is still rare in formal scientific writing. If you do use the first person, be careful that your report does not sound as if you were writing about yourself rather than about your investigation.

Use a logical order of presentation and discussion for clarity; avoid reference to details not presented or explained until later in the report.

The first and last sentences of the paper are often the hardest to write, and they are two of the most important sentences in the paper. Spend some time thinking about them!

Allow yourself enough time to review and revise your paper. It's best if you can wait a few days between revisions so that you can see your paper more clearly. This means you need to plan ahead!

Make sure to add a checklist