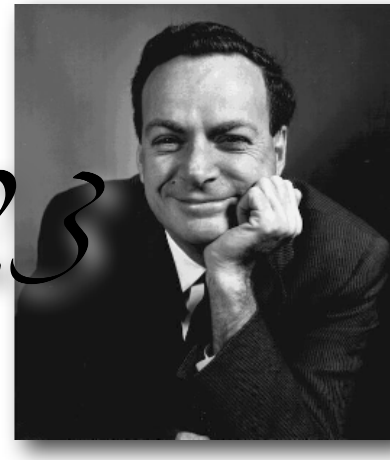
Physics 23



Instructors: J. Eckert, A. Esin, T. Lynn, P. Saeta, J. Townsend

Subject: Just over 100 years ago an obscure Swiss patent officer, third class, revolutionized much of physics. The first half of the course is based on ideas of space and time that Albert Einstein published in the miracle year of 1905. The second half of the course takes off from another of his 1905 papers in which he explained the photoelectric effect by the hypothesis of light particles (photons). We will study the curious behavior of photons and the essence of quantum mechanics.

Objectives: You will learn how a few simple principles, when carefully applied, can explain a rich variety of weird and often counterintuitive phenomena. In particular, you will be able

- to apply basic principles such as the "three rules" of special relativity or the principles of quantum mechanics in a systematic way to new situations;
- to prepare solutions to problems that effectively communicate your work by combining algebra, verbal explanations, and diagrams, where appropriate;
- to explain how the curve of binding energy makes nuclear weapons possible, how the relativistic Doppler effect provides evidence for the expansion of the universe, and how we know that single photons and single particles can exhibit interference.

Texts: *Introduction to Special Relativity* (**Revised** Second Edition, **2007**), *Introduction to Quantum Physics*, available at the physics department office (Keck 1234) for $30. Also *QED* by R. P. Feynman, available at Huntley Bookstore for $10.

Lectures: Thursdays, 12:05 - 1:05 p.m. in Galileo-McAlister. (Get some lunch at 11:30!) Taking notes is highly recommended. Lectures will be in parallel with the text, covering some of the same topics in a different way, and related topics not in the text. *Please ask questions in lecture*. Also, do not hesitate to come to any of our offices (in Keck Laboratories on the middle floor) to ask questions during posted office hours, by appointment, or anytime we are free.

Problem Sections: Tuesdays at 8:00 a.m, 9:00 a.m, or 1: 15 p.m. Discussion of new as well as assigned problems.

Web Site: `http://sakai.claremont.edu/` Handouts, solutions, and other useful information will be available on the course web site on Sakai.

Help: See your section instructor (or any instructor in the course), or one of the tutors. Tutor locations and times will be announced in lecture and will be posted on the course web site.

Problems: You may collaborate on solving problems, but do not merely copy. All problems are due at the beginning of class. Late homework will not be graded. Homework submitted to your section instructor late, but before solutions are posted (for example, at the end of class), will be kept by your instructor, and could make a difference in your final grade. Please read carefully the following page, *On Solving Problems*. Solutions to homework problems will be posted on the course web site.

Exams: There will be a 50-minute midterm, a take-home test problem, a quiz in sections, and a three-hour final exam. With the exception of the take-home test problem, all will be closed books and notes. They will cover material from the lectures, texts, assigned problems, and new problems from section meetings.

Grades: The course grade will be determined from the final exam (35%), a midterm (20%), a take-home test problem (10%), a quantum mechanics quiz (10%), and homework problems and class participation in problem sections (together 25%).

On Solving Problems

A problem solution should be written so that another student in the course, who is unfamiliar with the problem, can readily understand both the problem and its solution.

1. Begin by writing down the problem statement, either verbatim or adequately paraphrased. Then when the homework is graded and returned, you will understand the problem and the solution when you look at it again.

2. Think about the problem before beginning to solve it. Picture the physical situation, think of techniques you might use, and guess how it should come out.

3. Draw as many diagrams of the situation described in the problem statement as are needed to make the problem and your analysis clear.

4. Write in a clear, organized way, using words, mathematics, and figures as appropriate. It is common to use too few words and too few figures. *The solution should be an essay, with mathematics included as part of the English language.*

5. Solve a problem algebraically first; only substitute in numbers at the end. Your approach is more general this way, and you are more likely to catch errors by taking limiting cases or by checking dimensions as you go, for example.

6. Box-in or double-underline the final answer, if it is a reasonably short expression. For numerical answers, be sure to include units (e.g., kg m/s) as appropriate. Don't include more significant figures than is justified by the information given.

7. Check the answer in one or more ways. For example, (a) is the answer reasonable (is it about the size you had expected?) If it is an algebraic expression, is it right in special cases where you already know the answer? (b) Are the dimensions correct?

8. *Ask yourself what you have learned*. Why was this problem assigned? How might it generalize to other situations?

9. It is all right to work by yourself or with other people. If you work with others, be sure everyone is contributing. Do not merely copy someone else's homework, or allow anyone else to copy yours. If some person or book (other than the instructor or textbook) helped you substantially in solving a problem, give credit to the source in a brief note at the beginning or end of the solution.

10. If there are several pages in a homework set, *staple them together*. Put your name, the date, and the course number (and instructor if appropriate) at the top of the first page.

Homework problems are worth 5 points each. A solution that is entirely correct and adheres to the above guidelines will receive 5 points. Solutions with incorrect physics, algebraic mistakes, or stylistic shortcomings will receive less than full credit, down to a minimum of zero points:

A correct and complete solution	5 points
Mistakes in the physics and algebra	1-4 points off
An inadequate statement of the problem	1 point off
Missing diagram(s), where appropriate	1 point off
Failure to work symbolically, where appropriate	1 point off
Failure to use units, where necessary	1 point off
Failure to describe your work using English	1 point off
Obtaining a dimensionally inconsistent result	1 point off

Assignments

Date	Assignment
Tues., Sept. 4	Section 1 — Show up! Go soon to the Physics Office to buy a copy of the text, *Introduction to Special Relativity, Revised Second Edition (2007)*. Note that this edition differs significantly from last year's.
Thurs., Sept. 6	Lecture 1, *Time* — Read Chapters 1–3, *Introduction to Special Relativity, Revised Second Edition* 2007**. Problem due just before the lecture**: [HW01] 2–6 (i.e., Chap. 2, problem 6).
Tues., Sept. 11	Section 2 — Read Chapter 4 and Appendix A. **HW02 due:** (1) Appendix A problem 2 (on p. A-4), (2) 4-4, (3) 4-7, (4) 4-10.
Thurs., Sept. 13	Lecture 2, *The Three Rules* — Read Chapters 5 and 6. **HW03 due**: (1) 5-4, (2) 5-8.
Tues., Sept. 18	Section 3 — **HW04 due:** (1) 5-3, (2) 6-3, (3) 6-7, (4) 6-10.
Thurs., Sept. 20	Lecture 3, *Events and the Lorentz Transformation* — Read Chapters 7 and 8. **HW05 due**: 6-11.
Tues., Sept. 25	Section 4 — **HW06 due**: (1) 6-12, (2) 7-3, (3) 8-2, (4) 8-10.
Thurs., Sept. 27	Lecture 4, *Hubble and the CMB* — **HW07 due**: 6-13. *Take-home test problem handed out.*

Date	Assignment
Tues., Oct. 2	Section 5 — **Take-home test problem due at beginning of section**. **HW08 due**: 7-9.
Thurs., Oct. 4	Lecture 5, *Minkowski Space* — Read Chapters 9 and 10. **HW09 due**: 8-5.
Tues., Oct. 9	Section 6 — **HW10 due**: (1) 9-2, (2) 9-3, (3) 9-10, (4) 10-2, (5) 10-6.
Thurs., Oct. 11	Lecture 6, *Conservation Laws* — Read Chapter 11. **HW11 due**: 10-11 (Hint: the four-vector velocity transforms like a four-vector.)
Tues., Oct. 16	Section 7 — **HW12 due**: (1) 11-2, (2) 11-3, (3) 11-10, (4) 11-13.
Thurs., Oct. 18	Lecture 7, *Fission and the Manhattan Project* — Read Chapter 12. **HW13 due**: (1) 11-17, (2) 12-12.
Tues., Oct. 23	Fall break
Thurs., Oct. 25	**Midterm**
Tues., Oct. 30	Section 8 — Midterm returned; **HW14 due**: The "Realistic" Twin Paradox (problem statement attached at the end of the syllabus).
Thurs., Nov. 1	Lecture 8, *Light is an Electromagnetic Wave* — Read Sections 1.1 and 1.2, *A Modern Introduction to Quantum Physics (MIQP)*.
Tues., Nov. 6	Section 9 — **HW15 due**: *MIQP* (1) 1.1, (2) 1.2, (3) 1.3, (4) 1.7.
Thurs., Nov. 8	Lecture 9, *Light is a Particle* — Read pages 1-15, *QED*; Sections 1.3 and 1.4, *MIQP*. **HW16 due**: *MIQP* 1.4.
Tues., Nov. 13	Section 10 — **HW17 due**: *MIQP* (1) 1.13, (2) 1.14, (3) 1.17, (4) 1.18.
Thurs., Nov. 15	Lecture 10, *Single-Photon Interference* — Read Sections 1.5 and 1.6, *MIQP*, and the rest of Chapter 1, *QED*. **HW18 due**: 1.20.
Tues., Nov. 20	Section 11 — **HW19 due**: (1) 1.21, (2) 1.22, (3) 1.23, (4) 1.25.
Thurs., Nov. 22	**Thanksgiving break**
Tues., Nov. 27	Section 12 — **HW20 due**: (1) 1.26, (2) 1.27, (3) 1.29, (4) 1.30; **In-class quiz.**
Thurs., Nov. 29	Lecture 11, *Interference: Double-Slits and Gratings* — Read Sections 1.6 and 1.7; **HW21 due**: 1.31.
Tues., Dec. 4	Section 13 — Quiz returned. **HW22 due**: (1) 1.32, (2) 1.33, (3) 1.34, (4) 1.37.
Thurs., Dec. 6	Lecture 12, *Sum Over Paths and Single-Atom Interferometery* — Read Sections 1.8, 1.9, and 2.1, *MIQP*, and Chapter 2, *QED*; **HW23 due**: 1.40.
Tues., Dec. 11	Section 14 — **HW24 due**: (1) 1.42, (2) 2.2, (3) 2.10, (4) 2.12.
Thurs., Dec. 13	Lecture 13, *QED* — Read Section 2.3, *MIQP*, and Chapter 3, *QED*. **HW25 due**: 2.9.
TBA	**Final examination**

HW14 — The "Realistic" Twin Paradox

Tuesday, 30 October 2007

We have discussed the twin paradox in which identical twins Al and Bertha are separated soon after birth. Al remains on Earth while Bertha is placed aboard a rocket that blasts away towards a distant solar system. For simplicity we have assumed that somehow Bertha is instantly accelerated to relativistic velocity v for the journey away from Earth, and then she equally suddenly reverses direction when she finds out that nothing interesting is happening on Proxima Centauri. Perhaps you have worried how Bertha could survive such extreme accelerations—I know I have. The aim of this problem is to see how far she can get in a rocket ship that accelerates at a constant rate a, which we may take to be the acceleration due to gravity at Earth's surface, $g = 10\ \mathrm{m/s^2}$, or thereabouts. After all, we know that the human body functions quite well in such an accelerating environment. For the moment we will ignore the problem of devising a ship that can accelerate steadily this way over an extended period of time.

Let's say that Bertha's rocket accelerates at the steady rate a for an interval of time ΔT, then reverses thrust and decelerates at the same rate a for the same time interval ΔT. At the end of this deceleration phase, Bertha and crew are at rest with respect to Al and the Earth and can explore whatever extra-solar objects are in the neighborhood. After some time (say Δt_{exp}) for exploring and refueling, Bertha and company reverse the original trajectory, ending up back on Earth after an absence of $4\Delta T + \Delta t_{\mathrm{exp}}$, *according to Bertha's clocks*. How much time has elapsed on Al's clocks on Earth?

How can we approach this problem which involves the rocket's accelerated reference frame, when we only know how clocks behave in inertial frames? Appendix G discusses some of the problems of handling acceleration, but here we will take a different approach. We will assume that the acceleration a is small enough that the ship's clocks behave sensibly, just as they do on Earth. For $a \approx g$ this seems quite reasonable. Furthermore, provided that the speed of an accelerating object is much less than c in a given reference frame, we may take $\Delta v = a\Delta t$ in that frame.

We will use t_A for the time on Al's clock, and t_B for the time on Bertha's, and assume each is set to zero when Bertha's rocket departs. Bertha's journey separates into four identical segments (why?), plus an interval Δt_{exp} of exploration. Thus, it will suffice to analyze the first interval of length ΔT on the rocket. Knowing how much time has elapsed on Al's clock while Bertha's ticks ΔT we can easily calculate Al's age at the reunion.

We know how to apply special relativity to inertial frames, so we will work in two inertial frames. The first is Al's on Earth. The second is a moving frame that—for a brief instant in time—is the rocket's rest frame. These are illustrated in Fig. 1. Because the rocket accelerates, however, it will begin to develop a nonzero velocity in this second frame. As long as we restrict ourselves to short enough times so that the rocket is never moving very fast in the second frame, we can calculate its motion in the second frame without concern for special relativistic effects. Then we can use the rules of special relativity to transform back to Earth's frame. Let's see how this works.

After a certain elapsed time t_A on Earth, Bertha's ship is moving at speed v_B. According to Al, her clock is running slow. However, we cannot simply write

$$t_A = t_B\sqrt{1 - v_B^2/c^2} \tag{1}$$

because the speed of the ship (with respect to Earth) keeps changing with time. On the other hand, for a tiny interval of time it is fair to say

$$dt_A = dt_B\sqrt{1 - v_B^2/c^2} \tag{2}$$

since for a small enough time interval the ship's speed doesn't change appreciably. If we can find v_B as a function of either t_A or t_B, we could integrate (2) to find the

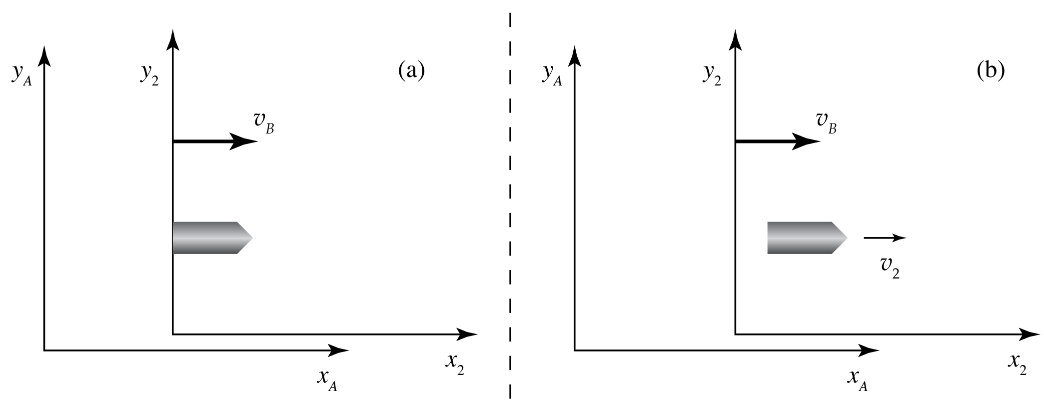


Figure 1: (a) At some point in the first phase of the voyage, the rocket is moving at v_B with respect to the Earth, and is instantaneously at rest in frame 2. (b) Some time later, the rocket has developed velocity v_2 in frame 2 due to its steady acceleration at a.

relationship between times on Earth and times on the rocket ship. In principle, we can manage this integration either analytically—which is challenging but lots of fun if you like calculus—or numerically, which is much easier. In fact, you can set up a solution in a spreadsheet or you can write a short program in Python or some other programming language. Here’s how.

We will break up the trip into a number of equal time steps δt_B onboard the ship. After n time steps, the ship’s clock thus reads $t_B^{(n)} = n\,\delta t_B$. We’ll assume that we know how fast the ship is going (with respect to the Earth) after the nth step, $v_B^{(n)}$, and how much time has elapsed on Earth, $t_A^{(n)}$, too. We now aim to use this information to calculate the ship’s speed and the elapsed time on Earth at the end of the next time step. That is, given the quantities $t_B^{(n)}$, $v_B^{(n)}$, and $t_A^{(n)}$, we would like to calculate $t_B^{(n+1)}$, $v_B^{(n+1)}$, and $t_A^{(n+1)}$, at least approximately.

The first one is easy, since

$$t_B^{(n+1)} = (n+1)\delta t_B \tag{3}$$

That is, we are taking steps of equal length δt_B onboard the ship. To figure out how the ship’s velocity changes, we set up the second inertial frame to be moving at $V = v_B^{(n)}$ with respect to the Earth and use the velocity transformation equation to relate the ship’s velocity in frame 2, v_2, to its velocity in the Earth frame, $v_B^{(n+1)}$. At the beginning of the interval the ship is instantaneously at rest in frame 2. During the time interval δt_2 in frame 2, the ship accelerates at constant rate a. Therefore, the ship’s velocity goes from 0 to $v_2 = a\,\delta t_2$, provided that $v_2 \ll c$ so that we may neglect time dilation effects *in frame 2*. Under this same approximation, $\delta t_B = \delta t_2$, so we will take

$$v_2 = a\,\delta t_B \tag{4}$$

Now you can apply the velocity transformation equation to compute the ship’s speed, $v_B^{(n+1)}$, with respect to the Earth at the end of the time interval of length δt_B onboard the ship.

The final quantity we need is the time elapsed on Al’s clock during the $(n+1)$st step. In principle, this is a straightforward application of (1) in the form

$$\delta t_A \approx \frac{\delta t_B}{\sqrt{1 - v_B^2/c^2}}$$

where the inequality becomes an equality in the limit as $\delta t_B \to 0$. The problem is that the ship’s speed is varying throughout the interval, and so is the time-dilation factor. So, should we use the value of v_B at the beginning of the time step ($v_B^{(n)}$), the value at the end of the time step ($v_B^{(n+1)}$), or some kind of average? I'll leave it up to you to do what you think is most reasonable.

Final hints: while there is nothing wrong with working in SI units, they tend to obscure things. You will simplify the problem tremendously by using years as the time unit and light years as the length unit. Once you work out g in these units, you will have a clearer idea how big you can make δt_B (in years) without violating the assumption that $v_2 \ll c$. Making δt_B too large will undermine the validity of your solution; making it too small will slow down your program and cause it to output too much unnecessary data.

Write a program to solve for t_A as a function of t_B for journeys of various durations and accelerations, and summarize your findings. Assuming that we can devise some means of accelerating a ship this way, how far could an intrepid exploration team get if they wanted to return to Earth before they died? If they were willing to go for a one-way ticket? Your solution should include the well-commented program you write, a discussion of how you handled the problem of the variation of v_B within a time step, perhaps a graph or two, and a discussion of your conclusions.

This assignment is worth 20 points.