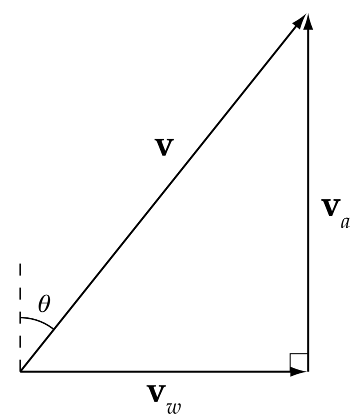
Problem 1 An airplane flies at speed $v_a = 150$ mi/hr relative to the air. A wind is blowing at speed $v_w = 120$ mi/hr toward the east, relative to the ground. (a) If the plane flies due north according to its onboard compass, find the speed and direction (relative to north) of the plane as measured by observers on the ground. (b) If the plane flies due north according to ground observers, how fast does the plane move relative to the ground? In this case, at what angle to north must the pilot fly, according to the onboard compass?

Solution:

(a) The plane flies at $v_a = 150$ mi/hr with respect to the air, in a northward direction. The air moves at $v_w = 120$ mi/hr eastward with respect to the ground. Hence, the plane's velocity with respect to the ground is the sum of the wind's velocity and the plane's velocity with respect to the wind, $\mathbf{v} = \mathbf{v}_a + \mathbf{v}_w$. By the Pythagorean theorem, therefore, the plane's speed with respect to the ground is

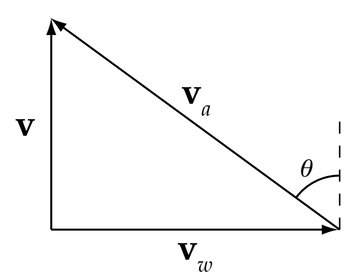


$$v = \sqrt{v_w^2 + v_a^2} = (30 \text{ mi/h})\sqrt{5^2 + 4^2} = (30 \text{ mi/h})\sqrt{41}$$
$$= 192 \text{ mi/h}$$

and it flies at an angle θ east of north, where

$$\theta = \arctan v_a/v_w \approx 38.7^\circ \quad \text{east of north}$$

(b) Now the plane flies due north with respect to the ground, so $\mathbf{v}$ is due north. Most of the plane's velocity with respect to the air is given over to "fighting the eastward drift," but some is left over to move north. Again, $\mathbf{v} = \mathbf{v}_a + \mathbf{v}_w$, but application of the Pythagorean theorem to the triangle at the right gives



$$v = \sqrt{v_a^2 - v_w^2} = 90 \text{ mi/h}$$

In this case the plane flies west of north at an angle

$$\theta = \arctan v_w/v = \arctan 4/3 \approx 53.1^\circ \quad \text{west of north}$$

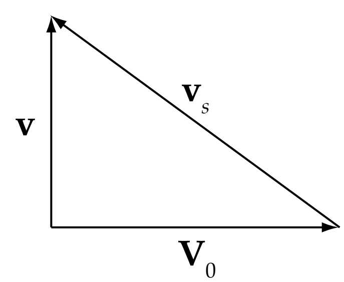
Problem 2 A river of width D flows at uniform speed V_0. Swimmers A and B, each of whom can swim at speed v_s relative to the water, decide to have a race beginning at the same spot on the shore. A swims downstream a distance D relative to the shore, and immediately swims back upstream to the starting point. B swims to a point diametrically opposite to the starting point on the other shore (i.e., "straight across"), and then swims back to the same point. Assume $v_s > V_0$. Find the total time for each swimmer. Who wins the race?

Solution: Consider first swimmer A. When swimming with the current, his speed with respect to the shore is $v_s + V_0$, so the trip "out" takes $\frac{D}{v_s+V_0}$. On the return trip, his speed is the difference, so the round-trip time for A is

$$\Delta t_A = \frac{D}{v_s + V_0} + \frac{D}{v_s - V_0} = D\frac{(v_s - V_0) + (v_s + V_0)}{v_s^2 - V_0^2} = D\frac{2v_s}{v_s^2 - V_0^2}$$

Now consider swimmer B. In order to arrive at the point directly opposite, she must swim somewhat against the current, yielding a velocity $v = \sqrt{v_s^2 - V_0^2}$ across the river. Since her speed is the same going and coming, her round-trip time is

$$\Delta t_B = \frac{2D}{\sqrt{v_s^2 - V_0^2}}$$



To compare the two times, let's form the ratio:

$$\frac{\Delta t_B}{\Delta t_A} = \frac{\frac{2D}{\sqrt{v_s^2 - V_0^2}}}{D\frac{2v_s}{v_s^2 - V_0^2}} = \frac{\sqrt{v_s^2 - V_0^2}}{v_s} = \sqrt{1 - V_0^2/v_s^2} < 1$$

For the given condition that $v_s > V_0$, this ratio is always less than 1. Hence, $\Delta t_B < \Delta t_A$, and the swimmer who goes across the current always takes less time than the swimmer who goes with and then against the current.