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Hairy strings

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Zero modes of the world-sheet spinors of a closed string can source higher order moments of the bulk supergravity fields. In this work, we analyze various configurations of closed strings focusing on the imprints of the quantized spinor vacuum expectation values onto the tails of bulk fields. We identify supersymmetric arrangements for which all multipole charges vanish; while for others, we find that one is left with Neveu-Schwarz–Neveu-Schwarz, and Ramond-Ramond dipole and quadrupole moments. Our analysis is exhaustive with respect to all the bosonic fields of the bulk and to all higher order moments. We comment on the relevance of these results to entropy computations of hairy black holes of a single charge or more, and to open/closed string duality.

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I. INTRODUCTION

Recently, much interest has focused on black holes with hair that arise in more than four spacetime dimensions [1–9]. These new supergravity solutions seem to teach us a great deal about general relativity in higher dimensions, as well as string theory. Hairy black holes may be nonsingular and horizonless; and are often in one to one correspondence with the microscopic states of the corresponding singular black hole of finite horizon area. Mathur has suggested [10,11] that one is to think of the new smooth solutions as replacing the old singular black hole geometries in computing certain physical observables (see for example [12–15]). The traditional singular solution is to perhaps correspond to some yet ill-understood average geometry of the new smooth ones. In this picture, the horizon radius is mapped onto the size of a "fuzz" ball beyond which the hairy solutions become indistinguishable from each other and from the original hole. This exciting proposal has been substantiated with various circumstantial and direct evidence. Furthermore, in a class of such black objects where controlled computations may be performed, dipole charges carried by the hole appear to play an important role in correctly accounting for the proper thermodynamics.

One little explored mechanism of correcting and potentially smoothing out traditional black hole solutions involves giving nonzero expectation value to the zero modes of fermions living on the world volume of branes or strings [16,17]. Such a treatment would be a semiclassical one where the leading effect of the quantization of the spinors is taken into account through their back reaction onto the geometry. The typical length scale for such corrections is set by the world-volume theory, i.e. the brane or string tension. Furthermore, fermion vacuum expectation values (VEVs) would naturally provide hair and multipole moments to the traditional black hole solutions.

In this work, we concentrate on the IIB fundamental string as a source of the bulk supergravity fields. This system in turn may be related by a chain of dualities to various D-brane configurations. Hence, our task is to determine the effect of the spinor VEVs of the IIB Green-Schwarz string onto the IIB supergravity fields. We will approach the problem from a general framework, gradually narrowing onto a class of doubly charged BPS string configurations. Our starting point is the action given in [18], involving the world-sheet couplings to the supergravity fields in the light-cone frame. From these couplings, one is able to fix the multipole charges of the world sheet as measured very far away from the world sheet. We consider in turn: a one dimensional beam of massless closed string states, wrapped strings, and wrapped strings with wave profiles along the world sheet. In each case, we analyze the effect of the spinor VEVs on the asymptotic geometry in preparation to pinning down boundary conditions for new hairy supergravity solutions (for a recent work in this direction, see [19]). This procedure is straightforward, yet leads to certain subtleties; and we find that the final results are indeed very instructive.

In Sec. II, we set up the general formalism and present expressions for the moments in cases where the bulk spacetime is endowed with a set of isometries. In Sec. III, we consider certain world-sheet configurations, and write the multipole moments for such states explicitly. In Sec. IV, we describe the process of giving nonzero expectation values to the spinors and present a concrete example. Section V summarizes and reflects on the results. In the appendixes, we collect the technical details: general conventions in Appendix A, the world-sheet couplings in Appendix B, and the map between world-sheet VEVs and the string ground states in Appendix C.

II. WORLD-SHEET COUPLINGS AND MULTIPOLE MOMENTS

We consider a class of configurations of IIB closed strings arranged such that the spacetime about the strings

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is static, and has a spatial isometry along one of the nine space directions. We work in the light-cone gauge, where the light-cone direction is along the aforementioned isometry of the space—which we label by x^1 in the rest of the paper. And our state is to carry p^+ units of light-cone momentum. We will interchangeably work in scenarios where the light-cone boost direction is a circle of radius R_s, or of infinite extent. We will also refer to the subspace spanned by $x^2, \ldots, x^9$ as *transverse*. Hence, the light cone is defined by $x^\pm \equiv (x^0 \pm x^1)/2$.

Under these circumstances, the couplings of the light-cone Green-Schwarz string to the bulk supergravity fields were worked out in [18] to all orders in the world-sheet spinors—except for couplings to the bulk fermions: the dilatino and the gravitino. For the setup described in the previous paragraph, we may use the results of [18] to compute the multipole moments of the bulk bosonic fields far away from the strings. At first sight, one simply reads off from the world-sheet action the ADM mass in the tail of the metric at infinity, and the Neveu-Schwarz–Neveu-Schwarz, (NSNS) charge of the strings in the tail of the B field. We will however consider going beyond this classical treatment: the world sheet involves fermionic degrees of freedom which, for the configurations in question, will have zero modes. Semiclassically, one sees this effect through nonzero vacuum expectation values (VEV) for fermion bilinears on the world sheet. These VEVs, as we shall see, can source multipole moments for all of the bulk supergravity fields.

The action describing IIB supergravity sourced by a closed fundamental string has the structural form

$$S = \int d^{10}x\mathcal{L} + \int d^8x \int d^2\sigma \delta^8(x - x_0)\mathcal{L}_s[x_0] \to \int d^{10}x\mathcal{L} + \int d^{10}x\mathcal{L}_M, \tag{1}$$

where $\mathcal{L}$ is the supergravity Lagrangian, given by (B3) of Appendix B to linear order in the fields, and $\mathcal{L}_s$ is the world-sheet Lagrangian of the IIB string given by (B8) in its full glorious nonlinear form [18]. We will be interested in configurations of strings confined to the x^1 subspace, and our strings are to be located at some fixed point x_0^a with $a = 2, \ldots, 9$, where the x_0^a's are constants on the world sheet.[1]

The last arrow in (1) involves fixing the light-cone gauge by choosing $x^0(\sigma)$ and $x^1(\sigma)$ using world-sheet reparameterization invariance. This will allow us to write

$$d^2\sigma = Jdx^+dx^- \tag{2}$$

with the Jacobian J treated as a constant on the world sheet for the cases of interest. Hence, this factor is to be included in $\mathcal{L}_M$.

The task is then to linearize $\mathcal{L}_M$ in the supergravity fields in the weak field approximation regime, and write the corresponding equations of motion for the bulk fields. For the metric g_{mn}, one gets

$$(e^a)^n \partial^2 \bar{g}_{nm} = -2\kappa^2 \frac{\delta \mathcal{L}_M}{\delta(e_a)^m}, \tag{3}$$

where $g_{mn} = \bar{g}_{mn} - \eta_{mn}\bar{g}/8$ with $\bar{g} = \bar{g}^m_m$, and $(e_a)^m$ is the vielbein. Note that one needs to carefully vary the action with respect to the vielbein since one has spacetime fermions on the Green-Schwarz world sheet that couple to the spacetime connection. We also have chosen the standard gauge $\partial^m \bar{g}_{mn} = 0$. And ∂^2 involves the eight transverse directions only. The right hand side of (3) is straightforward yet somewhat subtle to compute. Our final results will be collected below. For the dilaton ϕ and axion χ, the procedure is more straightforward; one gets

$$g_s^2 \partial^2 \chi = 2\kappa^2 \frac{\delta \mathcal{L}_M}{\delta \chi}; \tag{4}$$

$$\partial^2 \phi = 2\kappa^2 \frac{\delta \mathcal{L}_M}{\delta \phi}. \tag{5}$$

While for the 2-form gauge field A_{mn}, one gets

$$\partial^2 A^{mn} = 6\kappa^2 \frac{\delta \mathcal{L}_M}{\delta \bar{A}^{mn}} \tag{6}$$

with the gauge condition $\partial^m A_{mn} = 0$. A_{mn} is complex and involves both NSNS and Ramond-Ramond (RR) pieces $A^{(1)}_{mn}$ and $A^{(2)}_{mn}$ respectively

$$A_{mn} = \frac{1}{2\sqrt{g_s}} A^{(1)}_{mn} + i\frac{\sqrt{g_s}}{2} A^{(2)}_{mn}, \tag{7}$$

as described in Appendix B.[2] While the real 4-form gauge field obeys

$$\partial^2 A^{mnpq} = 96\kappa^2 \frac{\delta \mathcal{L}_M}{\delta \bar{A}^{mnpq}} \tag{8}$$

with $\partial^m A_{mnpq} = 0$.

Since we are working at the linearized level in the supergravity fields, at this stage all derivatives may be treated as arising from a flat connection; and there is no need to distinguish between tangent space indices $a, b, c, \ldots$ and spacetime indices $m, n, p, \ldots$. We will hence revert to tangent space indices for the rest of the paper. Furthermore, $a, b, c, \ldots$ will always label the eight transverse directions.

Using Eqs. (B8)–(B27), one finds:

[1] In certain setups that we will be considering, the structural form of the action given in (1) will be reached after a certain averaging prescription. There will be more on this later.

[2] Note that these simplified relations are valid in the linearized approximation scheme that we need.

(i) A dipole moment of the metric of the form

$$\begin{aligned} g^{ab} = &-Ji\pi^2\alpha'^3\sqrt{g_s}V_i^+[-8\sqrt{-h}h^{ij}\bar{\theta}\sigma^{-c(a}\theta V_j^{b)} \\ &+ 8\epsilon^{ij}\theta\sigma^{-c(a}\theta V_j^{b)} \\ &+ \eta^{ab}V_{jd}(\sqrt{-h}h^{ij}\bar{\theta}\sigma^{-cd}\theta - \epsilon^{ij}\theta\sigma^{-cd}\theta)] \\ &\times \partial_c\left(\frac{1}{r^6}\right) + \text{c.c.}; \end{aligned} \tag{9}$$

where

$$V_i^a = \partial_i x^a, \qquad V_i^+ = \partial_i x^+, \tag{10}$$

with $i, j, \ldots$ labeling the world-sheet directions σ^0 and σ^1. The $\sigma^{-ab\cdots}$s are gamma matrices and the θ's are the Green-Schwarz spinors, both defined in Appendix A. In this and similar subsequent expressions, in regard to the spinors θ, one is to take the expectation value of (9), $\langle g^{ab}\rangle$, in the state space of the spinor zero modes. In Sec. IV, we will look at explicit realizations of such VEVs. Note that J is the Jacobian defined in (2). One also finds a quadrupole moment given by

$$\begin{aligned} g^{ab} = &J\frac{\pi^2}{24}\alpha'^3\sqrt{g_s}\sqrt{-h}h^{ij}V_i^+V_j^+ \\ &\times[4\bar{\theta}\sigma^{-c(a}\theta\bar{\theta}\sigma^{b)-d}\theta + 4\eta^{d(b}\bar{\theta}\sigma^{a)-e}\theta\bar{\theta}\sigma_e^{-c}\theta \\ &+ \eta^{ab}\bar{\theta}\sigma^{-ec}\theta\bar{\theta}\sigma_e^{-d}\theta]\partial_c\partial_d\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{11}$$

No higher multipole fields arise in the light-cone gauge from the spinor VEVs. Note that the latin indices label only the eight transverse directions.[3]

(ii) A dipole moment encoded in the dilaton field

$$\phi = 4Ji\pi^2\alpha'^3\sqrt{g_s}V_i^+\epsilon^{ij}V_{ja}\bar{\theta}\sigma^{-ab}\bar{\theta}\partial_b\left(\frac{1}{r^6}\right) + \text{c.c.}; \tag{12}$$

and in the axion field

$$\begin{aligned} \chi = &-4J\pi^2\frac{\alpha'^3}{\sqrt{g_s}}V_i^+[\sqrt{-h}h^{ij}V_j^b\bar{\theta}\sigma^-\theta \\ &- \epsilon^{ij}V_{ja}\bar{\theta}\sigma^{-ab}\bar{\theta}]\partial_b\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{13}$$

Again, no higher moments arise from the spinor VEVs.

(iii) Magnetic and electric dipole moments of the 2-form gauge field

$$\begin{aligned} A^{ab} = &-12Ji\pi^2\alpha'^3\sqrt{g_s}V_i^+[\sqrt{-h}h^{ij}(-2\theta\sigma^{-c[a}\theta V_j^{b]} \\ &- \theta\sigma^{-ab}\theta V_j^c) + \epsilon^{ij}(2\bar{\theta}\sigma^{-c[a}\theta V_j^{b]} \\ &+ \bar{\theta}\sigma^{-ab}\theta V_j^c)]\partial_c\left(a\frac{1}{r^6}\right) + \text{c.c.}; \end{aligned} \tag{14}$$

$$\begin{aligned} A^{+-} = &-48Ji\pi^2\alpha'^3\sqrt{g_s}V_i^+[-\sqrt{-h}h^{ij}V_j^a\theta\sigma^{-ab}\theta \\ &- \epsilon^{ij}V_j^a\bar{\theta}\sigma^{-ab}\theta]\partial_b\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{15}$$

This time, we see explicitly components in the light-cone directions. And one also finds some electric quadrupole moments in this field

$$A^{ab} = 0, \tag{16}$$

$$\begin{aligned} A^{+-} = &4J\pi^2\alpha'^3\sqrt{g_s}\sqrt{-h}h^{ij}V_i^+V_j^+\theta\sigma^{-cb}\theta\bar{\theta}\sigma_c^{-a} \\ &\times \theta\partial_a\partial_b\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{17}$$

Once again, no higher moments arise.

(iv) Finally, a dipole moment of the 4-form gauge field

$$\begin{aligned} A^{+-ab} = &192J\pi^2\alpha'^3\sqrt{g_s}\epsilon^{ij}V_i^+[2\theta\sigma^{-c[b}\theta V_j^{a]} \\ &- \theta\sigma^{-ab}\theta V_j^c]\partial_c\left(\frac{1}{r^6}\right) + \text{c.c.}; \end{aligned} \tag{18}$$

with all other components being zero.

These multipole charges are relaying information about the fermionic condensates to infinity. We want to think of these expressions throughout as sandwiched between states of the quantized zero modes of the spinors; i.e. for the axion

$$\chi \to \langle Z_1|\chi|Z_2\rangle, \tag{19}$$

where $|Z_1\rangle$ and $|Z_2\rangle$ would be one of the 256 ground states of the closed string arising from the Clifford algebra of the zero modes. Additional information may be hidden in the moments of the dilatino and gravitino bulk fields. At the linearized level, these additional couplings on the world sheet do not affect our results. However, a complete picture of all the information about the closed string state as seen from far away requires knowledge of these additional couplings.

Our next task will be to explore particular interesting arrangements of closed strings that fit the general requirements we outlined at the beginning of this section.

III. EXAMPLES

In this section, we look at four different configurations of closed strings whose spacetime imprints share the general features required of the bulk fields in the previous

[3]However, in certain cases, one may also read off the g^{0a}, g^{1a}, and g^{01} components from these expressions. The subtlety arises because of certain assumptions about the structure of the background fields made in [18] in attaining Eq. (B8). Particularly, one must be careful in this formalism while trying to extract the spins of the closed string states from moments of the metric.

section. Focusing on specific examples allows us to write more explicit formulae for the moments in certain common scenarios. This will also help us in acquiring some basic physical intuition about the role of these moments in describing closed string states.

For the different setups we consider, we will need to fix the world-sheet reparametrization and Weyl scale invariance by fixing the world-sheet metric as in $h_{00} = -h_{11} = 1$ and $h_{01} = 0$,[4] along with the coordinates $x^\pm(\sigma)$ while assuring that the Virasoro constraints are satisfied

$$(\dot{x} \pm x')^2 = 0. \tag{20}$$

We will then need to determine the Jacobian (2). Finally, we will put things together carefully so as not to violate the assumed isometries of the bulk spacetime. Note that one is to arrange the world-sheet configuration as a source *in flat space*; the back reaction of the spacetime onto the source is subleading to this computation. Hence, the Virasoro constraint above does not include the spinors as these arise in terms coupled to field strengths and the connection.

A. Standard ground state

Our first example is the simplest possible setup. We consider ''point''-like closed string states i.e. the 256 ground states of the closed string; we pick one species, and consider a uniform density beam of these massless particles projected along the x^1 direction. For each particle, one has

$$x^+ = p^+ \frac{\alpha'}{\sqrt{g_s}}\sigma^0, \qquad x^- = 0, \tag{21}$$

with $x^a =$ constant. This setup requires us to consider the Green's function in 9 space dimensions, instead of 8, as given by (B1). Hence, the measure of the world-sheet action takes the form

$$\int d^8x dx^1 \delta^8(x) \sum_n \delta(x^1 - n\epsilon) \int d^2\sigma, \tag{22}$$

where we have laid the particles in the beam with a small spacing ϵ along x^1. In a semiclassical treatment, the ground state is not pointlike but has a small size, say ϵ (of order the string scale). Hence, we write

$$\int d\sigma \to \epsilon. \tag{23}$$

Integrating over x^1, we would need to deal with the new Green's function

$$\sum_n \frac{1}{((x^1 - n\epsilon)^2 + x^a x^a)^{7/2}} \to \frac{16}{15}\frac{G_8}{\epsilon}, \tag{24}$$

where G_8 is the eight dimensional Green's function $1/r^6$, and the arrow implies turning the sum into an integral in the limit $\epsilon \to 0$. One also needs a factor of $7\Omega_8/6\Omega_7 = 15/16$. Putting things together, the two ϵ factors from (23) and (24) cancel and $\mathcal{L}_M$ in (1) acquires an additional factor of

$$J = \frac{1}{p^+}\frac{g_s^{3/4}}{\alpha'^{3/2}}. \tag{25}$$

Otherwise, one may now make use of the eight dimensional Green's function in moment computations.

In this scenario, one finds the following simplified expressions for the moments from (9)–(18):

(i) The dipole moment of the metric is

$$g_{ab} = 0; \tag{26}$$

the quadrupole moment becomes

$$\begin{aligned} g^{ab} = \frac{1}{96}\pi^2\left(p^+ \frac{\sqrt{\alpha'}}{g_s^{1/4}}\right)\alpha'^3\sqrt{g_s}[16\bar{\theta}\sigma^{-c(a}\theta\bar{\theta}\sigma^{b)-d}\theta \\ - 16\eta^{d(a}\bar{\theta}\sigma^{b)-e}\theta\bar{\theta}\sigma_e^{-c}\theta \\ - 4\eta^{ab}\bar{\theta}\sigma^{-ec}\theta\bar{\theta}\sigma_e^{-d}\theta]\partial_c\partial_d\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{27}$$

(ii) No moments for the dilaton

$$\phi = 0; \tag{28}$$

and none for the axion field

$$\chi = 0. \tag{29}$$

(iii) No dipole moment for the 2-form

$$A_{ab} = 0; \tag{30}$$

but quadrupole moments may exist

$$\begin{aligned} A^{ab} = 2\pi^2\left(p^+ \frac{\sqrt{\alpha'}}{g_s^{1/4}}\right)\alpha'^3\sqrt{g_s}\theta\sigma^{-ad}\theta\bar{\theta}\sigma^{-bc} \\ \times \theta\partial_c\partial_d\left(\frac{1}{r^6}\right) + \text{c.c.}; \end{aligned} \tag{31}$$

$$\begin{aligned} A^{+-} = -4\pi^2\left(p^+ \frac{\sqrt{\alpha'}}{g_s^{1/4}}\right)\alpha'^3\sqrt{g_s}\theta\sigma^{-bc}\theta\bar{\theta}\sigma^{-ac} \\ \times \theta\partial_a\partial_b\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{32}$$

(iv) And finally no moments for the 4-form

$$A^{+-ab} = 0. \tag{33}$$

Note that these equations are written in the Einstein frame: a scaling $\alpha' \to \alpha'\sqrt{g_s}$ yields to the canonical factor g_s^2 of

[4]Note that in our conventions the signature of the metric is mostly negative.

the string frame (after rescaling χ as well). Hence, we see from these expressions that we have the possibility of nonzero RR electric and magnetic quadrupole moments in the 2-form gauge field.

B. Wrapped ground state

Our next example is that of a closed string wrapping the x^1 direction, which is also the longitudinal direction of the light-cone gauge. Hence, we might want to think of the x^1 direction as being compact. Since the Green's function in question does not probe this direction, compactifying it is harmless. Because of the Virasoro constraints (20), one needs

$$x^+ = p^+(\sigma^0 + \sigma^1), \qquad x^- = p^+(\sigma^0 - \sigma^1) \quad \text{or} \quad 0, \tag{34}$$

with $x^a = 0$. One then easily finds that all moments (9)–(18) vanish. A wrapped BPS string does not carry any multipole moments.

C. Wrapped ground state with a parallel wave

A more interesting setup would be to consider the wrapped state of the previous section

$$x^1 = wR_s\sigma^1; \tag{35}$$

where w is the wrapping number and R_s is the radius of compactification; but to add along the string a wave of mode number n

$$x^2 = A\cos[n(\sigma^0 + \sigma^1)], \tag{36}$$

parallel to the wrapping. To assure that the spacetime retains the needed isometries, we would need that the amplitude of the wave A is small enough $A \ll R_s$, and that observations are made over time scales large compared to the period of oscillation; alternatively, we would increase $n \gg 1$. This would allow us to average over the wave profile. We may even consider a more general profile

$$x^2 = f(\sigma^0 + \sigma^1), \tag{37}$$

as long as f is such that the aforementioned assumptions may still be made. Note that such states with unidirectional momentum modes are $\frac{1}{2}$ BPS. We first solve the Virasoro constraint (20)

$$x^0 = wR_s\sigma^0 + F(\sigma^0 + \sigma^1), \tag{38}$$

where F' can easily be found

$$F' = -wR_s + \sqrt{w^2R_s^2 + f'^2}. \tag{39}$$

We then have

$$V_0^+ = V_1^+ = \tfrac{1}{2}(F' + wR_s). \tag{40}$$

Even before averaging over the profile f, one can easily see that all quadrupole moments vanish since $V_i^+V^{+i} = 0$. This arises from the fact that the momentum modes run parallel to the wrapping. Hence, a $\frac{1}{2}$ BPS state consisting of a wrapped closed string with a plane wave moving parallel to the wrapping direction gives no quadrupole moments.

As for dipole moments, we see from (9), (12)–(15), and (18) that these all involve the factors $h^{ij}V_i^aV_j^+$ or $\varepsilon^{ij}V_i^aV_j^+$. From (40), we immediately see that all dipole moments vanish as well. In reaching this result, the fact that the wave on the string is moving parallel to the winding direction is crucial. Hence, this system does not carry any dipole charges either.

D. Wrapped ground state with an antiparallel wave

The obvious next step is to put the wave on the wrapped string of the previous section in the opposite direction to the wrapping

$$x^2 = A\cos[n(\sigma^0 - \sigma^1)]. \tag{41}$$

Or in general

$$x^2 = f(\sigma^0 - \sigma^1). \tag{42}$$

We again solve the Virasoro constraint with

$$x^0 = wR_s\sigma^0 + F(\sigma^0 - \sigma^1) \tag{43}$$

for F again given by

$$F' = -wR_s + \sqrt{w^2R_s^2 + f'^2}. \tag{44}$$

We define the shorthand

$$K \equiv \tfrac{1}{2}\sqrt{w^2R_s^2 + f'^2}. \tag{45}$$

We then find, unlike in (40)

$$V_0^+ + V_1^+ = wR_s, \tag{46}$$

for any profile f, with

$$V_0^+ = K. \tag{47}$$

The Jacobian of the measure is

$$dtdx = 2wR_sKd^2\sigma = 2d^2\sigma/J. \tag{48}$$

We now need to insert these expressions into (9)–(18). We focus next on the needed averaging process. We encounter two types of factors to average over

$$\left\langle \frac{f'}{2K} \right\rangle \equiv D \tag{49}$$

in dipole moments, and

$$\left\langle \frac{2K - wR_s}{2K} \right\rangle \equiv Q \tag{50}$$

in quadrupole moments.

The averages are to be taken over time scales much larger than the oscillation period of the profile, which is set by the string scale. From the viewpoint of the bulk spacetime, superstringy time scales of observation are indeed natural and needed.[5]

We look at two cases:

(i) For $wR_s \gg f'$, one has

$$D \simeq -\frac{1}{2}\frac{\langle f'^3\rangle}{(wR_s)^3} \ll 1, \tag{51}$$

and

$$Q \simeq \frac{\langle f'^2\rangle}{2w^2R_s^2} \ll 1; \tag{52}$$

where we have used the fact that f must be periodic $f(2\pi) = f(0)$. These expressions are finite but parametrically small with the average amplitude of the profile.

(ii) For $wR_s \ll f'$, one gets

$$D \simeq \langle \mathrm{sgn}(f')\rangle < 1, \tag{53}$$

and

$$Q \simeq 1. \tag{54}$$

which are finite and may be of order 1.

Both cases are to be arranged so that the perturbations along the strings are of negligible amplitude compared to the scale set by R_s. For case one, one needs to have low mode numbers; for case two, one needs large mode numbers (and the averaging process gets favored even more by the supergravity side). In either case, one is arranging for profiles that appear homogeneous over long enough time scales, yet give the strings a "thickness" in the x^2 direction. Furthermore, in both cases, all profiles f that are even functions yield zero dipole moments.

We then find the following moments:

(i) A dipole moment for the metric

$$\begin{aligned} g^{ab} = {}& i\pi^2 D\alpha'^3\sqrt{g_s}[-16\eta^{2(b}\theta\sigma^{a)-c}\theta \\ &- 16\eta^{2(b}\bar\theta\sigma^{a)-c}\theta + 2\eta^{ab}\theta\sigma^{-c2}\theta \\ &+ 2\eta^{ab}\bar\theta\sigma^{-c2}\theta]\partial_c\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{55}$$

And a nonzero quadrupole moment as well

$$\begin{aligned} g^{ab} = {}& \frac{\pi^2}{48}Q\alpha'^3\sqrt{g_s}[16\bar\theta\sigma^{-c(a}\theta\bar\theta\sigma^{b)-d}\theta \\ &- 4\eta^{ab}\bar\theta\sigma^{-ce}\theta\bar\theta\sigma_e^{-d}\theta \\ &- 16\eta^{d(b}\bar\theta\sigma^{a)-e}\theta\bar\theta\sigma_e^{-c}\theta \qquad (56)\\ &+ 8\eta^{cd}\bar\theta\sigma^{-ea}\theta\bar\theta\sigma_e^{-b}\theta + \eta^{ab}\eta^{cd}\bar\theta\sigma^{-ef}\theta\bar\theta\sigma_{ef}^{-}\theta] \\ &\times \partial_c\partial_d\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{57}$$

(ii) Dipole moments for the dilaton and axion

$$\begin{aligned} \phi = {}& 8i\pi^2 D\alpha'^3\sqrt{g_s}[-\bar\theta\sigma^{-b2}\bar\theta \\ &+ \eta^{b2}\bar\theta\sigma^{-}\theta]\partial_b\left(\frac{1}{r^6}\right) + \text{c.c.}; \end{aligned} \tag{58}$$

$$\begin{aligned} \chi = {}& 8\pi^2 D\frac{\alpha'^3}{\sqrt{g_s}}[-\bar\theta\sigma^{-b2}\bar\theta + \eta^{b2}\bar\theta\sigma^{-}\theta]\partial_b\left(\frac{1}{r^6}\right) \\ &+ \text{c.c.} \end{aligned} \tag{59}$$

(iii) Dipole moments for the 2-form

$$\begin{aligned} A^{ab} = {}& -24i\pi^2 D\alpha'^3\sqrt{g_s}[2\eta^{2[b}\bar\theta\sigma^{a]-c}\theta \\ &+ 2\eta^{2[b}\theta\sigma^{a]-c}\theta + \eta^{c2}\bar\theta\sigma^{-ab}\theta \\ &+ \eta^{c2}\theta\sigma^{-ab}\theta]\partial_c\left(\frac{1}{r^6}\right) + \text{c.c.}; \end{aligned} \tag{60}$$

$$\begin{aligned} A^{+-} = {}& -96i\pi^2 D\alpha'^3\sqrt{g_s}[\bar\theta\sigma^{-b2}\theta - \theta\sigma^{-b2}\theta \\ &- \eta^{b2}\bar\theta\sigma^{-}\theta]\partial_b\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{61}$$

And a nonzero electric quadrupole moment as well

$$A^{ab} = 0; \tag{62}$$

$$\begin{aligned} A^{+-} = {}& -8\pi^2 Q\alpha'^3\sqrt{g_s}\theta\sigma^{-bc}\theta\bar\theta\sigma_c^{-a}\theta\partial_a\partial_b\left(\frac{1}{r^6}\right) \\ &+ \text{c.c.} \end{aligned} \tag{63}$$

(iv) Finally, an electric dipole moment of the 4-form

$$\begin{aligned} A^{+-ab} = {}& -384\pi^2 D\alpha'^3\sqrt{g_s}(-2\eta^{2[a}\theta\sigma^{b]-c}\theta \\ &+ \eta^{c2}\theta\sigma^{-ab}\theta)\partial_c\left(\frac{1}{r^6}\right) + \text{c.c.} \end{aligned} \tag{64}$$

We see that one may have a myriad of multipole charges for wrapped strings with antiparallel modes on the world sheet—in contrast to the case where the modes are running parallel to the wrapping.

IV. POLARIZATION STATES

Next, we would like to consider as a concrete example an explicit condensate of the spinors on the world sheet. To do this, one needs to describe the state space of zero modes of the spinors in the standard way. We first decompose our Weyl spinors θ into Majorana-Weyl spinors θ_1 and θ_2

$$\theta \equiv \theta_1 + i\theta_2, \qquad \bar\theta \equiv \theta_1 - i\theta_2. \tag{65}$$

We define states $|A\rangle$ and $|\alpha\rangle$ where $A = 2, \ldots, 9$ describes the polarizations of the vector state, and $\alpha = 1, \ldots, 16$ labels the polarizations of the spinor; the latter satisfying

[5]For short wavelengths or large mode numbers, we may write for example $\langle F\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} d\sigma F$.

$(\sigma^+)^\alpha_\beta|\alpha\rangle = 0$, leaving 8 polarizations. Requiring that the rotation operator $\bar{\theta}\sigma^{-ab}\theta$ acts on these states properly, we write

$$\theta_1^\alpha|A\rangle_1 = \frac{\sqrt{\Gamma}}{2}(\sigma^A)^{\alpha\beta}|\beta\rangle_1, \tag{66}$$

$$\theta_1^\alpha|\beta\rangle_1 = \frac{\sqrt{\Gamma}}{2}(\sigma_a^+)^\alpha_\beta|a\rangle_1, \tag{67}$$

where Γ is fixed by the quantization condition $\{\theta^\alpha, (\sigma^-\bar{\theta})_\beta\} = \Gamma\delta^\alpha_\beta$ of the spinors

$$\Gamma \equiv \frac{1}{p^+}. \tag{68}$$

For the closed string, the state space is spanned by a direct product of two copies of such states

$$|A, B\rangle, \qquad |A, \alpha\rangle, \qquad |\alpha, A\rangle, \qquad |\alpha, \beta\rangle, \tag{69}$$

yielding the usual 256 states of the IIB supergravity multiplet. We normalize these as in[6]

$$\langle A|B\rangle = -\eta^{AB}, \qquad \langle\alpha|\beta\rangle = \delta_{\alpha\beta}. \tag{70}$$

The task then is to take the expectation value of the expressions in the previous sections in the desired states. This is straightforward but rather tedious—requiring extensive use of Fierz identities. We have tabulated the needed background information in Appendix C, which includes all possible VEVs that can arise in any such computation.

As a simple demonstration, focus on the simplest state: a dilatonic state, described by $|A, A\rangle$ where A is being summed over. Using the catalog in Appendix C, one finds that all moments in (9)–(18) vanish except two:

(i) The axion dipole moment:

$$\chi = -\frac{512\pi^2}{p^+}J\frac{\alpha'^3}{\sqrt{g_s}}\sqrt{-h}h^{ij}V_i^+V_j^a\partial_a(r^{-6}); \tag{71}$$

(ii) And the quadrupole moment of the metric

$$g_{ab} = -\frac{2^{10}\pi^2}{(p^+)^2}J\Gamma^2\alpha'^3\sqrt{g_s}\sqrt{-h}h^{ij}V_i^+V_j^+\partial_a\partial_b(r^{-6}). \tag{72}$$

Depending on the embedding in the target space, one can get nonzero moments in the axion field or metric. For example, for a wrapped configuration with an antiparallel wave in the dilaton state polarized in the x^2 direction, we would have an RR axion dipole moment given by

$$\chi = \frac{3072\pi^2}{p^+}\frac{\alpha'^3}{\sqrt{g_s}}D\frac{x^2}{r^8}, \tag{73}$$

with D defined in (51) and (52).

V. DISCUSSION AND RESULTS

In this work, we considered a class of IIB string configurations with nonzero vacuum expectation values for the world-sheet spinors. By focusing on the moments of the bosonic fields, we unraveled a map between world-sheet states and the multipole charges as seen at infinity. Our conclusions may be summarized as follows:

(i) Spinor VEVs may source dipole and quadrupole moments in general but no higher moments.

(ii) For the beam of ground state particles, and for strings wrapping a cycle of the geometry with or without a parallel wave along it, all moments from spinor VEVs vanish.

(iii) For wrapped strings with waves along it arranged antiparallel to the winding, we find in general nonzero dipole and quadrupole moments of both RR and NSNS types.

(iv) We presented a catalog for computing moments for BPS as well as non-BPS string configurations, provided the bulk spacetime retains certain isometries.

These results fix the asymptotic tails of bulk fields in new geometries that one is to look for and that may exist in one to one correspondence with IIB string states of different zero mode polarizations. Consider, for example, the case of a singly charged hole. Perhaps the spinor VEVs are to react on the geometry so as to generate new fuzz-ball solutions; yet, the area of the fuzz ball should be proportional to the log of the degeneracy of the ground state, i.e. log of 256 for a singly charged hole. This implies that the area of the ''stretched horizon'' is of order the string scale. Hence, the corresponding supergravity solutions would be expected to break down in the region of most interest. The more interesting case is that of a hole with two charges, such as one corresponding to a IIB string with an antiparallel wave. Here, we may expect the existence of new smooth geometries with dipole and quadrupole moments.[7] However, spinor VEVs can also have an imprint onto the tail of the dilatino and gravitino, which we did not consider. Adding the effect of the couplings of these supergravity fields to the formalism greatly complicates matters, and would be needed to a proper accounting of the entropy. Hence, a complete treatment is deferred to a future work. Another direction of pursuit would involve non-BPS con-

[6]Note that the signature of our metric is mostly negative.

[7]In a recent work, Taylor provides explicit solutions for strings with classical bosonic and fermionic world-sheet profiles [19]. This work explores only NSNS couplings by working in the world-sheet picture as opposed to the Green-Schwarz formalism we have adopted. The hallmark of spinor condensates appearing as multipole moments in the bulk fields is seen in that work as well.

figurations, with both right and left moving waves on the string.

Another interesting aspect of these results has to do with open/close string duality. For example in the case of Eq. (73), we identified a certain IIB string state that is endowed with D-instaton dipole moment. One may then ask whether there is a cartoon involving, say, a D-instaton and an anti-D-instanton that mimics the corresponding IIB string state. Indeed, in the context of matrix theory [20,21], the fundamental string is realized through the degrees of freedom of wound D-strings. In this spirit, it is easiest to attempt to construct such a picture of the closed string states with D-instantons only, guided by the required multipole moments of the axion. The phenomena of D-brane polarization à la Myers [22] would generate the expected multipole moments for the other brane charges for free (i.e. because of T duality). We hope to report on this in the near future.

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APPENDIX A: SPINOR CONVENTIONS

Our spinors are Weyl but not Majorana. They are complex and have 16 components. The associated 16×16 gamma matrices satisfy

$$\{\sigma^a, \sigma^b\} = 2\eta^{ab}, \tag{A1}$$

with the metric

$$\eta_{ab} = \mathrm{diag}(+1, -1, -1, \ldots, -1). \tag{A2}$$

Note that the signature is different from the standard one in use in modern literature. This is so that we conform to the equations appearing in [23]. Also, the world-sheet metric h^{ij} has signature $(-, +)$ for space and time, respectively. Throughout, the reader may refer to [23] to determine more about the spinorial algebra and identities that we are using. However, we make no distinction between σ and $\hat{\sigma}$ as defined in [23] as this will be obvious from the context.

We note that σ^a, σ^{abcd} and σ^{abcde} are symmetric; while σ^{ab} and σ^{abc} are antisymmetric; and σ^{abcde} is self-dual. We then have

$$\sigma^+\sigma^- + \sigma^-\sigma^+ = 1. \tag{A3}$$

And complex conjugation is defined so that

$$\overline{\sigma^a} = \sigma^a. \tag{A4}$$

Conjugation also implies

$$\overline{\theta_1\theta_2} = \bar{\theta}_2\bar{\theta}_1. \tag{A5}$$

Finally, antisymmetrization is defined as

$$\sigma^{ab} \equiv \sigma^{[a}\sigma^{b]}, \tag{A6}$$

with a conventional 2! hidden by the braces. For more details about the conventions and the formalism used, the reader is referred to [18].

APPENDIX B: THE WORLD SHEET AND BULK ACTIONS

In this appendix, we collect some basic background information and conventions used throughout the paper. In computing moments, we encounter the Green's function equation

$$\nabla_d^2 G = -(d-2)\Omega_{d-1}\delta^d(x) \Rightarrow G = \frac{1}{r^{d-2}}, \tag{B1}$$

where

$$\Omega_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)}. \tag{B2}$$

The linearized IIB supergravity action we use is written as

$$\mathcal{L} = \frac{1}{2\kappa^2}\sqrt{|g|}\bigg[R + \frac{1}{2}g_s^2(\partial\chi)^2 + \frac{1}{2}(\partial\phi)^2 + \frac{1}{3}\bar{F}_{abc}F^{abc} + \frac{1}{96}G_{abcde}G^{abcde}\bigg], \tag{B3}$$

where the gravitational coupling is

$$2\kappa^2 = (2\pi)^7\alpha'^4. \tag{B4}$$

The 3-form flux is complex and has NSNS and RR pieces $H^{(1)}_{abc}$ and $H^{(2)}_{abc}$, respectively

$$F_{abc} = \frac{1}{2\sqrt{g_s}}H^{(1)}_{abc} + i\frac{\sqrt{g_s}}{2}H^{(2)}_{abc}, \tag{B5}$$

written to linear order in the fields. We also write this in terms of the gauge fields

$$H^{(i)} = dA^{(i)}. \tag{B6}$$

The self-dual 5-form field strength is written as

$$G = dA, \tag{B7}$$

with respect to a 4-form gauge field A at infinity. At the linearized order, no distinction is needed between tangent space and spacetime indices. We use latin letters to label coordinates $a, b, c, \ldots$; these indices span over the eight dimensional transverse subspace eluded to in the main text.

The Green-Schwarz world-sheet action in the light-cone gauge is [18]

$$I_{WS} = \int d^2\sigma\mathcal{L}_s \quad \text{with } \mathcal{L}_s \equiv \mathcal{L}^{(0)} + \mathcal{L}^{(2)} + \mathcal{L}^{(4)}, \tag{B8}$$

where the superscripts identify the number of spinors in each term. We have

$$\mathcal{L}^{(0)} = -T\frac{\omega}{2}\sqrt{-h}h^{ij}V_i^a V_{aj} - 2T\omega\sqrt{-h}h^{ij}V_i^+ V_j^- - \frac{T}{2}\varepsilon^{ij}V_i^a V_j^b A_{ab}^{(1)} - 2T\varepsilon^{ij}V_i^+ V_j^- A^{(1)-+}, \tag{B9}$$

where the string tension is

$$T = \frac{1}{2\pi\alpha'}. \tag{B10}$$

Note that the world sheet is written in the Einstein frame, hence the additional $\omega \to \sqrt{g_s}$ dressing the tension. We also define

$$V_i^a \equiv \partial_i x^m e_m^a, \qquad V_i^\pm \equiv \partial_i x^m e_m^\pm. \tag{B11}$$

The term quadratic in the spinors is given by

$$\mathcal{L}^{(2)} = iT\omega\sqrt{-h}h^{ij}V_i^+\bar\theta\sigma^- D_j\theta - iT\omega\varepsilon^{ij}V_i^+\theta\sigma^- D_j\theta - TV^{+i}V_c^j I_{ij}^c + \text{c.c.}, \tag{B12}$$

where θ is a Weyl 16-component spinor and the σ's are gamma matrices defined in Appendix A. We also have defined

$$\begin{aligned} I_{ij}^{\,c} = {} & i\frac{\omega}{2}\sqrt{-h}h_{ij}P^c\bar\theta\sigma^-\theta - i\frac{\omega}{2}\varepsilon_{ij}P_a\bar\theta\sigma^{-ca}\bar\theta - i\omega\sqrt{-h}h_{ij}F_a^{-+}\bar\theta\sigma^{-ca}\bar\theta + i\frac{\omega}{4} \\ & \times\sqrt{-h}h_{ij}F_{ab}^c\bar\theta\sigma^{-ab}\bar\theta + i\omega\varepsilon_{ij}F^{-+c}\bar\theta\sigma^-\theta + i\omega\varepsilon_{ij}F_a^{-+}\bar\theta\sigma^{-ca}\theta + i\frac{\omega}{4}\varepsilon_{ij}F_{ab}^c\bar\theta\sigma^{-ab}\theta \\ & - \frac{\omega}{4}\varepsilon_{ij}G_{ab}^{-+c}\theta\sigma^{-ab}\theta, \end{aligned} \tag{B13}$$

where we see the couplings of the spinor bilinears on the world sheet to the supergravity fields. The covariant derivative appearing (B12) is written as

$$D_j\theta \equiv \partial_j\theta^\alpha - \tfrac{1}{4}\partial_j x^m\omega_{m,ab}\sigma^{ab}\theta \tag{B14}$$

in terms of the spacetime connection $\omega_{m,ab}$.

We summarize the meaning of the various fields appearing in these expressions:

(i) The dilaton is written as

$$\omega \equiv e^{\phi/2}. \tag{B15}$$

(ii) The field strength for the IIB scalars

$$P_a \equiv \frac{e^\phi}{2}(iD_a\chi - e^{-\phi}D_a\phi); \tag{B16}$$

with χ being the IIB axion.

(iii) The complex 3-form field strength

$$F_{abc} \equiv \frac{e^{\phi/2}}{2}(1 + e^{-\phi} + i\chi)\mathcal{F}_{abc} + \frac{e^{\phi/2}}{2}(-1 + e^{-\phi} + i\chi)\bar{\mathcal{F}}_{abc}; \tag{B17}$$

with

$$\mathcal{F}_{abc} \equiv \frac{H_{abc}^{(1)}}{2} + i\frac{H_{abc}^{(2)}}{2}, \tag{B18}$$

where $H^{(1)}$ and $H^{(2)}$ are, respectively, the field strengths associated with the fundamental string and the D-string charges.

(iv) And the five-form self-dual field strength G_{abcde}.

The terms quartic in the spinors may also involve terms quadratic in the supergravity fields; we write

$$\mathcal{L}^{(4)} = \sqrt{-h}h^{ij}V_i^+V_j^+[\mathcal{L}_{FF} + \mathcal{L}_{FG} + \mathcal{L}_{GG} + \mathcal{L}_{DF} + \mathcal{L}_{FP} + \mathcal{L}_{DG} + \mathcal{L}_R + \mathcal{L}_{PP}] + \text{c.c.} \tag{B19}$$

where we have defined the various pieces as

$$\begin{aligned} \mathcal{L}_{GG} = {} & -\frac{23\omega T}{4608}(\bar\theta\sigma^-\theta)^2G^{-+abc}G_{abc}^{-+} - \frac{\omega T}{9216}\bar\theta\sigma^{-ab}\theta\bar\theta\sigma_{ab}^-\theta G^{-+cde}G_{cde}^{-+} \\ & + \frac{\omega T}{384}\bar\theta\sigma^-\theta\bar\theta\sigma^{-ab}\theta\left[\frac{1}{12}G^{-+cde}G_{cdeab} + G_{ad}^{-+c}G_{cb}^{-+d}\right] - \frac{\omega T}{1536}\bar\theta\sigma^{-ab}\theta\bar\theta\sigma^{-cd}\theta\left[G_{ab}^{-+e}G_{ecd}^{-+} - \frac{1}{24}G_{abefg}G_{cd}^{efg}\right] \\ & + \frac{\omega T}{256}\bar\theta\sigma^{-ac}\theta\bar\theta\sigma_c^{-b}\theta\left[G_{ade}^{-+}G_b^{-+de} - \frac{1}{72}G_{adefg}G_b^{defg}\right] \end{aligned} \tag{B20}$$

$$\begin{aligned} \mathcal{L}_{FF} = {} & -\frac{13\omega T}{24}(\bar\theta\sigma^-\theta)^2F^{-+a}\bar F_a^{-+} - \frac{25\omega T}{768}\bar\theta\sigma^{-ab}\theta\bar\theta\sigma_{ab}^-\theta F^{-+c}\bar F_c^{-+} \\ & - \frac{\omega T}{256}\bar\theta\sigma^-\theta\bar\theta\sigma^{-ab}\theta\left[93F_a^{-+}\bar F_b^{-+} - \frac{43}{2}F^{-+c}\bar F_{cab} - \frac{17}{24}F_{acd}\bar F_b^{cd}\right] \\ & + \frac{\omega T}{96}\bar\theta\sigma^{-ac}\theta\bar\theta\sigma_c^{-b}\theta\left[17F_a^{-+}\bar F_b^{-+} + \frac{5}{8}F^{-+d}\bar F_{dab} - \frac{7}{16}F_{ade}\bar F_b^{de}\right] \\ & - \frac{\omega T}{1536}\bar\theta\sigma^{-ab}\theta\bar\theta\sigma^{-cd}\theta\left[23F_d^{-+}\bar F_{cab} - 7F_a^{-+}\bar F_{bcd} - \frac{1}{2}F_{ace}\bar F_{bd}^e - \frac{13}{4}F_{abe}\bar F_{cd}^e\right] \end{aligned} \tag{B21}$$

$$\mathcal{L}_{DG} = -\frac{i}{192}\omega T D_c G_{ab}^{-+c}\bar{\theta}\sigma^-\theta\bar{\theta}\sigma^{-ab}\theta \tag{B22}$$

$$\mathcal{L}_{PP} = -\frac{15\omega T}{256}\bar{\theta}\sigma^-\theta\bar{\theta}\sigma^{-ab}\theta P_a\bar{P}_b + \frac{\omega T}{48}\bar{\theta}\sigma^{-ac}\theta\bar{\theta}\sigma_c^{-b}\theta P_a\bar{P}_b, \tag{B23}$$

$$\begin{aligned}\mathcal{L}_R = {}& \frac{5\omega T}{32}(\bar{\theta}\sigma^-\theta)^2 R^{-+-+} - \frac{\omega T}{192}\bar{\theta}\sigma^{-ab}\theta\bar{\theta}\sigma_{ab}^-\theta R^{-+-+} + \frac{\omega T}{96}\bar{\theta}\sigma^{-ac}\theta\bar{\theta}\sigma_c^{-b}\theta\left[R_{ab}^{-+} + \frac{1}{2}R_{adb}^d\right] \\ & - \frac{\omega T}{384}\bar{\theta}\sigma^{-ab}\theta\bar{\theta}\sigma^{-cd}\theta\left[R_{acbd} + \frac{1}{2}R_{abcd}\right]\end{aligned} \tag{B24}$$

R_{abcd} being the Riemann tensor in the Einstein frame. And

$$\begin{aligned}\mathcal{L}_{FG} = {}& -i\frac{\omega T}{48}\theta\sigma^{-ab}\theta\bar{\theta}\sigma^-\theta\bar{F}^{-+e}G_{abe}^{-+} - i\frac{\omega T}{32}\theta\sigma^{-ab}\theta\bar{\theta}\sigma^{-cd}\theta\left[\frac{1}{3}\bar{F}_{ebd}G_{ac}^{-+e} - \frac{1}{3}\bar{F}_d^{-+}G_{abc}^{-+} - \frac{5}{12}\bar{F}_{ecd}G_{ab}^{-+e}\right. \\ & \left. - \frac{1}{12}\bar{F}_{efd}G_{abc}^{ef} + \frac{1}{12}\bar{F}_{eab}G_{cd}^{-+e} - \bar{F}_b^{-+}G_{acd}^{-+} + \frac{1}{3}\bar{F}_e^{-+}G_{abcd}^e\right] \\ & - i\frac{\omega T}{48}\theta\sigma^{-ac}\theta\bar{\theta}\sigma_c^{-b}\theta\left[\bar{F}_d^{-+}G_{ab}^{-+d} + \frac{1}{4}\bar{F}_{deb}G_a^{-+de} - \frac{1}{4}\bar{F}_{dea}G_b^{-+de}\right] - i\frac{\omega T}{1152}\theta\sigma^{-ab}\theta\bar{\theta}\sigma_{ab}^-\theta\bar{F}_{cde}G^{-+cde}\end{aligned} \tag{B25}$$

$$\mathcal{L}_{FP} = -\frac{\omega T}{8}F^{-+}{}_a\bar{P}_b\bar{\theta}\sigma^-\theta\theta\sigma^{-ab}\theta + \frac{\omega T}{8}F^{-+}{}_a\bar{P}_b\bar{\theta}\sigma^{-ac}\theta\theta\sigma^{-b}{}_c\theta - \frac{\omega T}{96}F_{acd}\bar{P}_b\bar{\theta}\sigma^{-cd}\theta\theta\sigma^{-ab}\theta \tag{B26}$$

$$\mathcal{L}_{DF} = -\frac{\omega T}{12}\theta\sigma^{-ac}\theta\bar{\theta}\sigma^{-b}{}_c\theta D_b\bar{F}^{-+}{}_a - \frac{\omega T}{48}\theta\sigma^{-ab}\theta\bar{\theta}\sigma^{-cd}\theta D_c\bar{F}_{abd}. \tag{B27}$$

In computing moments, most of these quartic terms do not contribute.

APPENDIX C: SPINOR VEVS

In this appendix, we collect all VEVs needed to compute the couplings on the world sheet for all possible 256 states of the state space of spinor zero modes.

1. Spinor bilinears of the form $\bar{\theta}\theta$

$$\langle A, B|\bar{\theta}\sigma^{-ab}\theta|C, D\rangle = 2\Gamma(-\eta^{Ab}\eta^{BD}\eta^{aC} - \eta^{bB}\eta^{aD}\eta^{AC} + \eta^{aB}\eta^{bD}\eta^{AC} + \eta^{aA}\eta^{BD}\eta^{bC});$$

$$\langle A, B|\bar{\theta}\sigma^-\theta|C, D\rangle = 4\Gamma\eta^{BD}\eta^{AC} \qquad \langle A, \alpha|\bar{\theta}\sigma^{-ab}\theta|C, \beta\rangle = -\frac{\Gamma}{2}(4\delta_{\alpha\beta}[-\eta^{Ab}\eta^{aC} + \eta^{aA}\eta^{bC}] + \sigma_{\alpha\beta}^{ab}\eta^{AC});$$

$$\langle A, \alpha|\bar{\theta}\sigma^-\theta|C, \beta\rangle = -2\Gamma\delta_{\alpha\beta}\eta^{AC} \qquad \langle\alpha, \beta|\bar{\theta}\sigma^{-ab}\theta|\gamma, \omega\rangle = \frac{\Gamma}{2}(\delta_{\beta\omega}\sigma_{\alpha\gamma}^{ab} + \delta_{\alpha\gamma}\sigma_{\beta\omega}^{ab}) \qquad \langle\alpha, \beta|\bar{\theta}\sigma^-\theta|\gamma, \omega\rangle = 0.$$

2. Terms quartic in the spinors of the form $\bar{\theta}\theta\bar{\theta}\theta$

Because of the ambiguity in ordering the bilinears in such quartic terms, we employ the standard prescription of symmetrizing the bilinears before quantization; i.e. we write for example

$$:\bar{\theta}\sigma^{-ab}\theta\bar{\theta}\sigma^{-cd}\theta: = \tfrac{1}{2}\bar{\theta}\sigma^{-ab}\theta\bar{\theta}\sigma^{-cd}\theta + \tfrac{1}{2}\bar{\theta}\sigma^{-cd}\theta\bar{\theta}\sigma^{-ab}\theta.$$

One then gets:

$$\langle A,B|{:}\bar\theta\sigma^{-ab}\theta\bar\theta\sigma^{-cd}\theta{:}|C,D\rangle = 8\Gamma^2(\eta^{Bc}\eta^{bC}\eta^{Ad}\eta^{aD} + \eta^{Bc}\eta^{AC}\eta^{bd}\eta^{aD} + \eta^{Ab}\eta^{cC}\eta^{Bd}\eta^{aD} + \eta^{bB}\eta^{Ac}\eta^{Cd}\eta^{aD} + \eta^{AB}\eta^{bc}\eta^{Cd}\eta^{aD} + \eta^{Bc}\eta^{aC}\eta^{bd}\eta^{AD} + \eta^{aB}\eta^{cC}\eta^{bd}\eta^{AD} + \eta^{ac}\eta^{BC}\eta^{Ad}\eta^{bD} + \eta^{ac}\eta^{bC}\eta^{Ad}\eta^{BD} + \eta^{aA}\eta^{cC}\eta^{bd}\eta^{BD} + \eta^{aA}\eta^{BC}\eta^{bd}\eta^{cD} + \eta^{bB}\eta^{Ac}\eta^{ad}\eta^{CD} + \eta^{aA}\eta^{bc}\eta^{Bd}\eta^{CD} + \eta^{AB}\eta^{bc}\eta^{aC}\eta^{dD} + \eta^{Ab}\eta^{Bc}\eta^{aC}\eta^{dD} + \eta^{bB}\eta^{ac}\eta^{AC}\eta^{dD} + \eta^{aB}\eta^{Ac}\eta^{bC}\eta^{dD} + \eta^{aA}\eta^{bB}\eta^{cC}\eta^{dD} + 2\eta^{bc}\eta^{BC}\eta^{ad}\eta^{AD} + 2\eta^{AB}\eta^{ac}\eta^{bd}\eta^{CD} + 2\eta^{ac}\eta^{AC}\eta^{bd}\eta^{BD})|_{[a,b],[c,d]}.$$

The notation $|_{[a,b],[c,d]}$ signifies antisymmetrization of the enclosed expression for $[a,b]$ and $[c,d]$ separately.

$$\langle A,\alpha|{:}\bar\theta\sigma^{-ab}\theta\bar\theta\sigma^{-cd}\theta{:}|C,\beta\rangle = \Gamma^2(-8\delta_{\alpha\beta}\eta^{Ab}\eta^{cC}\eta^{ad} + 8\delta_{\alpha\beta}\eta^{ac}\eta^{bC}\eta^{Ad} + \tfrac{3}{2}\eta^{cC}\eta^{Ad}\sigma^{-ab}_{\alpha\beta} + \tfrac{3}{2}\eta^{Ab}\eta^{aC}\sigma^{-cd}_{\alpha\beta} + \tfrac{1}{2}\eta^{ac}\eta^{bd}\sigma^{-AC}_{\alpha\beta} - \tfrac{1}{2}\eta^{AC}\sigma^{-abcd}_{\alpha\beta} + \eta^{cC}\eta^{bd}\sigma^{-aA}_{\alpha\beta} + \eta^{bC}\eta^{ad}\sigma^{-Ac}_{\alpha\beta} + \eta^{Ab}\eta^{ad}\sigma^{-cC}_{\alpha\beta} + \eta^{bc}\eta^{Ad}\sigma^{-aC}_{\alpha\beta} + \eta^{aC}\eta^{Ad}\sigma^{-bc}_{\alpha\beta} - \eta^{aA}\eta^{Cd}\sigma^{-bc}_{\alpha\beta})|_{[a,b],[c,d]}$$

$$\langle \alpha,\beta|{:}\bar\theta\sigma^{-ab}\theta\bar\theta\sigma^{-cd}\theta{:}|\gamma,\omega\rangle = \frac{\Gamma^2}{2}\left(-\frac{1}{2}\sigma^{-ab}_{\beta\omega}\sigma^{-cd}_{\alpha\gamma} - \frac{1}{2}\sigma^{-ab}_{\alpha\gamma}\sigma^{-cd}_{\beta\omega} - \delta_{\beta\omega}\sigma^{-abcd}_{\alpha\gamma} - \delta_{\alpha\gamma}\sigma^{-abcd}_{\beta\omega}\right).$$

3. Terms quartic in the spinors of the form $\theta\theta\bar\theta\bar\theta$ or c.c.

$$\langle A,B|{:}\theta\sigma^{-ab}\theta\bar\theta\sigma^{-cd}\theta{:}|C,D\rangle = 8\Gamma^2(\eta^{bc}\eta^{AC}\eta^{Bd}\eta^{aD} + 2\eta^{aB}\eta^{Ac}\eta^{Cd}\eta^{bD} + \eta^{ac}\eta^{bC}\eta^{Ad}\eta^{BD} + \eta^{Ab}\eta^{ac}\eta^{Cd}\eta^{BD} + 2\eta^{Ab}\eta^{Bc}\eta^{aC}\eta^{dD} + \eta^{aB}\eta^{bc}\eta^{AC}\eta^{dD}),$$

$$\langle A,\alpha|{:}\theta\sigma^{-ab}\theta\bar\theta\sigma^{-cd}\theta{:}|C,\beta\rangle = 0,$$

$$\langle \gamma,\omega|{:}\theta\sigma^{-ab}\theta\theta\sigma^{-cd}\theta{:}|\gamma,\omega\rangle = 0.$$

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