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Closed strings in Ramond-Ramond backgrounds

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Abstract: We write the IIB Green-Schwarz action in certain general classes of curved backgrounds threaded with Ramond-Ramond fluxes. The fixing of the kappa symmetry in the light-cone gauge and the use of supergravity Bianchi identities simplify the task. We find an expression that truncates to quartic order in the spacetime spinors and relays interesting information about the vacuum structure of the worldsheet theory. The results are particularly useful in exploring integrable string dynamics in the context of the holographic duality.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background, Supergravity Models, Superspaces.

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Contents

1. Introduction and results

In many realizations of the holographic duality [1, 2, 3], where a perturbative string theory is found dual to strongly coupled dynamics in a field theory or in another string theory, the closed strings on the weakly coupled side of the duality are immersed in background Ramond-Ramond (RR) fluxes. Knowledge of the couplings of the string worldsheet degrees of freedom to such fluxes is then an important ingredient to the task of exploring the underpinnings of the duality.

There are three main approaches in writing down an action of closed superstrings in an arbitrary background. In the RNS formalism, powerful computational techniques are available, yet the vertex operators sourced by RR fields involve spin fields. A second approach is the Green-Schwarz (GS) formalism with spacetime supersymmetry, generally leading to an action that is particularly useful in unraveling the semi-classical dynamics of the sigma model. On the down side, manifest Lorentz symmetry is lost with the fixing of the light-cone gauge; and, at one loop level for example, the lost symmetry results in serious complications. The third approach was developed recently [9] and involves a hybrid picture. In this strategy, part of the spacetime symmetries remain manifest yet couplings to the RR fields take relatively simple forms. The cost is the introduction of several auxiliary fields, and certain assumptions on the form of the background.

In this work, we focus on the second GS formalism with spacetime supersymmetry and on determining the component form of this action. Our interest is to eventually study, semi-classically, closed string dynamics in general backgrounds that arise readily in the study of D-branes. Other recent approaches involve specializing to backgrounds with a large amount of symmetry in writing the corresponding worldsheet theory; for example, AdS spaces have been of particular interest (e.g. [10]–[13]). We would then like to extend the scope of this program by considering generic D-brane configurations with much less symmetry.

Most of the difficulties involved in writing down the string action in general form are due to the fact that superspace for supergravity, while still being a natural setting for the theory, can be considerably complicated [14]: a large amount of superfluous symmetries need to be fixed and computations are often prohibitively lengthy.

The task is significantly simplified by the use of the method of normal coordinate expansion [15, 16] in superspace. This was developed for the Heterotic string in [17], and, along with the use of computers for analytical manipulations, makes determining the type-IIA and IIB sigma models straightforward as well. The additional complications that arise in these cases — and that are absent in the Heterotic string case — are due entirely to the presence of the RR fields.

In this paper, we concentrate on the IIB theory. In [18], part of this action, to quadratic order in the spinors, was derived starting from the supermembrane action and using T-duality.[1] In this work, starting from IIB superspace directly and using the method of normal coordinate expansion, we compute the full form of the IIB worldsheet action in the light-cone gauge relevant to most backgrounds of interest. In the subsequent subsection, we present all the results of this work in a self-contained format. The details of deriving the action are then left for the rest of the paper and need not be consulted.

1.1 The results

The class of background fields we focus on is inspired by [17] and by the need to apply our results to settings that arise in the context of the holographic duality. In particular, fields generated by electric and magnetic D-branes of various configurations share certain general features of interest. We list all the conditions we impose on the background fields so that our form of the IIB action is valid:

- The supergravity fermions are to vanish. In particular, the gaugino and gravitino have no condensates.
- We choose a certain space direction that, along with the time coordinate, we will associate with the light-cone gauge fixing later. We refer to the other eight spatial directions as being transverse. With this convention, we demand that all background fields depend only on the transverse coordinates.

[1] See also [19] for a derivation of the action to quadratic order using T-duality.

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- Tensor fields can have indices in the transverse directions; and in the two light-cone directions only if the light-cone coordinates appear in pairs; e.g. a 3-form field strength F_{pqr} can have nonzero components F_{abc} or F_{-+a} where a, b, and c are transverse directions; and $+$ and $-$ are light-cone directions. But components such as F_{-ab} are zero.

For example, if we were to consider a background consisting of a number of static Dp branes, we choose the light-cone directions *parallel* to the worldvolume of the branes. All conditions listed above are then satisfied.

Under these assumptions, and once the κ symmetry is fixed, the IIB action takes the form

$$\mathcal{S} = \mathcal{S}^{(0)} + \mathcal{S}^{(2)} + \mathcal{S}^{(4)}\,, \tag{1.1}$$

where the superscripts denote the number of spinors in each expression. Hence, the action truncates to quartic order in the fermions.

The first term is the standard bosonic part[2]

$$\mathcal{S}^{(0)} = \int d^2\sigma \left[\frac{1}{2}\sqrt{-h}\,h^{ij}V_i^aV_{aj} + 2\sqrt{-h}\,h^{ij}V_i^+V_j^- + \frac{1}{2}\varepsilon^{ij}V_i^aV_j^b b_{ab}^{(1)} + 2\varepsilon^{ij}V_i^+V_j^- b^{(1)-+}\right]. \tag{1.2}$$

In this expression, and throughout, the $'+'$ and $'-'$ flat tangent space labels refer to the light-cone directions as in $x^\pm \equiv (x^0 \pm x^1)/2$, with x^0 and x^1 being respectively the time and some chosen space direction defining the light-cone. We then denote eight flat transverse tangent space indices by $a, b, \ldots$ All tensors are written with their indices in the tangent space by using the vielbein; i.e. for the NSNS B-field, we have $b_{ab}^{(1)} = e_a^m e_b^n b_{mn}^{(1)}$. We define

$$V_i^a \equiv \partial_i x^m e_m^a\,, \qquad V_i^\pm \equiv \partial_i x^m e_m^\pm\,. \tag{1.3}$$

Curved spacetime indices are then labeled by $m, n, \ldots$. Note that we write the action in the Einstein frame; in section 4.1 we cast part of the action into the string frame to compare with the literature.

Note also that while we fix the kappa symmetry, we do not fix the light-cone gauge in V_i^+ and h_{ij} so as to allow for different choices. One conventional choice in flat space is $h_{\tau\tau} = +1$, $h_{\sigma\sigma} = -1$, $h_{\tau\sigma} = 0$, $V_\sigma^+ = 0$, and $V_\tau^+ = p^+$.

Next, we represent the two spacetime spinors by a single Weyl — but otherwise complex — 16 component spinor θ. The 16×16 gamma matrices are denoted by σ^a and the conventions for the spinor representation we have adopted are summarized in appendix A. At quadratic order in θ, the action takes the form

$$\mathcal{S}^{(2)} = \int d^2\sigma \left(\mathcal{I}_{\rm kin} + V^{+i}\,V_c^j\,\mathcal{I}_{ij}^c\right) + \text{c.c.}\,, \tag{1.4}$$

where 'c.c' stands for complex conjugate. And we define separately the kinetic piece and

[2]The signature of the metric we use is $(+ - - - - \cdots)$. See appendix A for the details.

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the piece that involves no derivatives of the fermions

$$\begin{aligned}
\mathcal{I}_{\rm kin} &= -i\,\omega\sqrt{-h}\,h^{ij}V_i^+\bar\theta\sigma^-D_j\theta + i\,\omega\,\varepsilon^{ij}V_i^+\,\theta\sigma^-D_j\theta\,; && (1.5)\\
\mathcal{I}_{ij}^c &= i\,\frac{\omega}{2}\,\sqrt{-h}\,h_{ij}\,P^c\,\bar\theta\sigma^-\theta - i\,\frac{\omega}{2}\varepsilon_{ij}\,P_a\,\bar\theta\sigma^{-ca}\bar\theta - \\
&\quad - i\omega\sqrt{-h}\,h_{ij}\,F^{-+}{}_a\bar\theta\sigma^{-ca}\bar\theta + i\frac{\omega}{4}\sqrt{-h}\,h_{ij}\,F_{ab}{}^c\bar\theta\sigma^{-ab}\bar\theta + \\
&\quad + i\omega\varepsilon_{ij}F^{-+c}\bar\theta\sigma^-\theta - i\omega\varepsilon_{ij}F^{-+}{}_a\bar\theta\sigma^{-ca}\theta - i\frac{\omega}{4}\,\varepsilon_{ij}F^c{}_{ab}\bar\theta\sigma^{-ab}\theta + \\
&\quad + \frac{\omega}{4}\varepsilon_{ij}G^{-+c}{}_{ab}\theta\sigma^{-ab}\theta\,. && (1.6)
\end{aligned}$$

The covariant derivative is defined as

$$D_j\theta \equiv \partial_j\theta^\alpha - \frac{1}{4}\partial_j x^m\omega_{m,ab}\,\sigma^{ab}\theta\,. \tag{1.7}$$

The various background fields appearing in (1.6) are:

- The IIB dilaton

$$\omega \equiv e^{\phi/2}\,. \tag{1.8}$$

- The field strengths for the IIB scalars

$$\begin{aligned}
P_a &\equiv \frac{e^\phi}{2}\left(iD_a\chi - e^{-\phi}D_a\phi\right)\,; && (1.9)\\
Q_a &\equiv \frac{\bar P_a - P_a}{4\,i} = -\frac{e^\phi}{4}D_a\chi\,, && (1.10)
\end{aligned}$$

  with χ being the IIB axion.

- The 3-form field strength

$$F_{abc} \equiv \frac{e^{\phi/2}}{2}(1+e^{-\phi}+i\chi)\mathcal{F}_{abc} + \frac{e^{\phi/2}}{2}(-1+e^{-\phi}+i\chi)\bar{\mathcal{F}}_{abc}\,; \tag{1.11}$$

  with

$$\mathcal{F}_{abc} \equiv \frac{h^{(1)}_{abc}}{2} + i\frac{h^{(2)}_{abc}}{2}\,, \tag{1.12}$$

  where $h^{(1)}$ and $h^{(2)}$ are, respectively, the field strengths associated with fundamental string and D-string charges.

- And the five-form self-dual field strength G_{abcde}.

At quartic order in the spinors, the action involves many more terms. We may organize these in eight different parts:

$$\mathcal{S}^{(4)} = \int d^2\sigma\,\sqrt{-h}\,h^{ij}V_i^+V_j^+\left[\mathcal{I}_{FF}+\mathcal{I}_{FG}+\mathcal{I}_{GG}+\mathcal{I}_{DF}+\mathcal{I}_{FP}+\mathcal{I}_{DG}+\mathcal{I}_R+\mathcal{I}_{PP}\right] + \text{c.c.} \tag{1.13}$$

according to field content. Amongst these, we encounter two qualitatively different types of terms: ones involving the form $\theta\bar\theta\theta\bar\theta$; and ones involving the structure $\theta\theta\theta\bar\theta$ (or its complex conjugate). The fermions count a U(1) charge which is part of the symmetry of the supergravity theory. We then get the expressions of the first type

$$\begin{aligned}
\mathcal{I}_{GG} = {} & \frac{23\,\omega}{4608}\left(\bar\theta\,\sigma^-\,\theta\right)^2 G^{-+abc}\;G^{-+}{}_{abc} + \\
& + \frac{\omega}{9216}\bar\theta\,\sigma^{-ab}\,\theta\,\bar\theta\,\sigma^-{}_{ab}\,\theta\,G^{-+cde}G^{-+}{}_{cde} - \\
& - \frac{\omega}{384}\bar\theta\,\sigma^-\,\theta\,\bar\theta\,\sigma^{-ab}\,\theta\left[\frac{1}{12}G^{-+cde}\,G_{cdeab} + G^{-+c}{}_{ad}\,G^{-+d}{}_{cb}\right] + \\
& + \frac{\omega}{1536}\bar\theta\,\sigma^{-ab}\,\theta\,\bar\theta\,\sigma^{-cd}\,\theta\left[G^{-+e}{}_{ab}\,G^{-+}{}_{ecd} - \frac{1}{24}G_{abefg}\,G^{efg}{}_{cd}\right] - \\
& - \frac{\omega}{256}\bar\theta\,\sigma^{-ac}\,\theta\,\bar\theta\,\sigma^{-b}{}_c\,\theta\left[G^{-+}{}_{ade}\,G^{-+de}{}_b - \frac{1}{72}G_{adefg}\,G^{defg}{}_b\right]
\end{aligned} \tag{1.14}$$

$$\begin{aligned}
\mathcal{I}_{FF} = {} & \frac{13\,\omega}{24}\left(\bar\theta\,\sigma^-\,\theta\right)^2 F^{-+a}\,\bar F^{-+}{}_a + \frac{25\,\omega}{768}\,\bar\theta\,\sigma^{-ab}\,\theta\,\bar\theta\,\sigma^-{}_{ab}\,\theta\,F^{-+c}\,\bar F^{-+}{}_c + \\
& + \frac{\omega}{256}\,\bar\theta\,\sigma^-\,\theta\,\bar\theta\,\sigma^{-ab}\,\theta\left[93\,F^{-+}{}_a\,\bar F^{-+}{}_b - \frac{43}{2}\,F^{-+c}\,\bar F_{cab} - \frac{17}{24}\,F_{acd}\,\bar F^{cd}{}_b\right] - \\
& - \frac{\omega}{96}\,\bar\theta\,\sigma^{-ac}\,\theta\,\bar\theta\,\sigma^{-b}{}_c\,\theta\left[17\,F^{-+}{}_a\,\bar F^{-+}{}_b + \frac{5}{8}\,F^{-+d}\,\bar F_{dab} - \frac{7}{16}\,F_{ade}\,\bar F^{de}{}_b\right] + \\
& + \frac{\omega}{1536}\,\bar\theta\,\sigma^{-ab}\,\theta\,\bar\theta\,\sigma^{-cd}\,\theta\Big[23\,F^{-+}{}_d\,\bar F_{cab} - 7\,F^{-+}{}_a\,\bar F_{bcd} - \\
& \qquad\qquad - \frac{1}{2}F_{ace}\,\bar F^e{}_{bd} - \frac{13}{4}\,F_{abe}\,\bar F^e{}_{cd}\Big]
\end{aligned} \tag{1.15}$$

$$\mathcal{I}_{DG} = \frac{i}{192}\,\omega\,D_c\,G^{-+c}{}_{ab}\,\bar\theta\,\sigma^-\,\theta\,\bar\theta\,\sigma^{-ab}\,\theta \tag{1.16}$$

$$\mathcal{I}_{PP} = \frac{15\,\omega}{256}\,\bar\theta\,\sigma^-\,\theta\;\bar\theta\,\sigma^{-ab}\,\theta\;\;P_a\,\bar P_b - \frac{\omega}{48}\,\bar\theta\,\sigma^{-ac}\,\theta\,\bar\theta\,\sigma^{-b}{}_c\,\theta\,P_a\,\bar P_b \tag{1.17}$$

$$\begin{aligned}
\mathcal{I}_R = {} & -\frac{5\,\omega}{32}\left(\bar\theta\,\sigma^-\,\theta\right)^2 R^{-+-+} + \frac{\omega}{192}\,\bar\theta\,\sigma^{-ab}\,\theta\bar\theta\,\sigma^-{}_{ab}\,\theta\,R^{-+-+} - \\
& - \frac{\omega}{96}\,\bar\theta\,\sigma^{-ac}\,\theta\,\bar\theta\,\sigma^{-b}{}_c\,\theta\left[R^{-+}{}_{ab} + \frac{1}{2}R_{adb}{}^d\right] + \\
& + \frac{\omega}{384}\,\bar\theta\,\sigma^{-ab}\,\theta\,\bar\theta\,\sigma^{-cd}\,\theta\left[R_{acbd} + \frac{1}{2}R_{abcd}\right]
\end{aligned} \tag{1.18}$$

R_{abcd} being the Riemann tensor in the Einstein frame.

The expressions of the second type are

$$\begin{aligned}
\mathcal{I}_{FG} = {} & i\frac{\omega}{48}\,\theta\,\sigma^{-ab}\,\theta\;\bar\theta\,\sigma^-\,\theta\,\bar F^{-+e}\,G^{-+}{}_{abe} + \\
& + i\frac{\omega}{32}\theta\sigma^{-ab}\theta\bar\theta\sigma^{-cd}\theta\Big[\frac{1}{3}\bar F_{ebd}G^{-+e}{}_{ac} - \frac{1}{3}\bar F^{-+}{}_d G^{-+}{}_{abc} - \frac{5}{12}\bar F_{ecd}G^{-+e}{}_{ab} - \frac{1}{12}\bar F_{efd}G^{ef}{}_{abc} + \\
& \qquad + \frac{1}{12}\bar F_{eab}G^{-+e}{}_{cd} - \bar F^{-+}{}_b G^{-+}{}_{acd} + \frac{1}{3}\bar F^{-+}{}_e G^e{}_{abcd}\Big] + \\
& + i\frac{\omega}{48}\,\theta\,\sigma^{-ac}\,\theta\,\bar\theta\,\sigma^{-b}{}_c\,\theta\left[\bar F^{-+}{}_d\,G^{-+d}{}_{ab} + \frac{1}{4}\,\bar F_{deb}\,G^{-+de}{}_a - \frac{1}{4}\,\bar F_{dea}\,G^{-+de}{}_b\right] +
\end{aligned}$$

$$+i\frac{\omega}{1152}\,\theta\,\sigma^{-ab}\,\theta\,\bar{\theta}\,\sigma^{-}{}_{ab}\,\theta\,\bar{F}_{cde}\,G^{-+cde} \tag{1.19}$$

$$\begin{aligned}\mathcal{I}_{FP} = {}& \frac{\omega}{8}\,F^{-+}{}_{a}\,\bar{P}_b\,\bar{\theta}\,\sigma^{-}\,\theta\,\theta\,\sigma^{-ab}\,\theta - \frac{\omega}{8}\,F^{-+}{}_{a}\,\bar{P}_b\,\bar{\theta}\,\sigma^{-ac}\,\theta\,\theta\,\sigma^{-b}{}_{c}\theta + \\ & +\frac{\omega}{96}\,F_{acd}\,\bar{P}_b\,\bar{\theta}\,\sigma^{-cd}\,\theta\,\theta\,\sigma^{-ab}\,\theta - \\ & -i\,\frac{\omega}{6}\,\theta\,\sigma^{-ac}\,\theta\,\bar{\theta}\,\sigma^{-b}{}_{c}\,\theta\,Q_b\,\bar{F}^{-+}{}_{a} - i\,\frac{\omega}{24}\,\theta\,\sigma^{-ab}\,\theta\,\bar{\theta}\,\sigma^{-cd}\,\theta\,Q_c\,\bar{F}_{abd}\end{aligned} \tag{1.20}$$

$$\mathcal{I}_{DF} = \frac{\omega}{12}\,\theta\,\sigma^{-ac}\,\theta\,\bar{\theta}\,\sigma^{-b}{}_{c}\,\theta\,D_b\,\bar{F}^{-+}{}_{a} + \frac{\omega}{48}\,\theta\,\sigma^{-ab}\,\theta\,\bar{\theta}\,\sigma^{-cd}\,\theta\,D_c\,\bar{F}_{abd} \tag{1.21}$$

To make contact with the literature, we write the equations of motion satisfied by the background fields in the conventions we have adopted. Labeling flat tangent space indices that span all ten spacetime directions (including the light-cone) by $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \ldots$, we have [20]

$$D^{\hat{a}}P_{\hat{a}} = -4\,i\,Q^{\hat{a}}P_{\hat{a}} + \frac{1}{6}F_{\hat{a}\hat{b}\hat{c}}F^{\hat{a}\hat{b}\hat{c}}\,; \tag{1.22}$$

$$D^{\hat{c}}F_{\hat{a}\hat{b}\hat{c}} = -2\,i\,Q^{\hat{c}}F_{\hat{a}\hat{b}\hat{c}} - \bar{F}_{\hat{a}\hat{b}\hat{c}}P^{\hat{c}} + \frac{i}{6}G_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}F^{\hat{c}\hat{d}\hat{e}}\,; \tag{1.23}$$

$$D^{\hat{e}}G_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} = -\frac{i}{18}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}\hat{f}\hat{g}\hat{h}\hat{i}\hat{j}}F^{\hat{e}\hat{f}\hat{g}}\,\bar{F}^{\hat{h}\hat{i}\hat{j}} \tag{1.24}$$

$$R_{\hat{a}\hat{b}} = -2\bar{P}_{(\hat{a}}\,P_{\hat{b})} - \bar{F}_{(\hat{a}}{}^{\hat{c}\hat{d}}F_{\hat{b})}{}^{\hat{c}\hat{d}} + \frac{1}{12}\eta_{\hat{a}\hat{b}}\bar{F}_{\hat{c}\hat{d}\hat{e}}F^{\hat{c}\hat{d}\hat{e}} - \frac{1}{96}G_{\hat{a}}{}^{\hat{c}\hat{d}\hat{e}\hat{f}}G_{\hat{b}\hat{c}\hat{d}\hat{e}\hat{f}}\,; \tag{1.25}$$

The five-form field strength $G_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}$ is self-dual in this scheme. Our conventions conform, for example, to those in [23] with the identifications $\chi + i\,e^{-\phi} \to \lambda$, $h^{(1)} \to H^{(1)}$, $h^{(2)} \to H^{(2)}$.

In the rest of the paper, we describe how to arrive at the expressions presented. The current section was organized such that the details of these derivations are not needed to make use of the results. Section 2 summarizes the basic superspace formalism we use. Section 3 outlines the strategy and techniques that simplify the computations. Section 4 presents yet more details; in particular, in section 4.1, we write the action in the string frame to quadratic order in the spinors, compare with the literature, and confirm that our results are consistent with other recent attempts at determining this action (see however minor note in section 4.1 with regards to the U(1) charge). Section 5 includes concluding remarks about future directions. And appendix A collects some of the conventions we use throughout the paper.

Note added. The original version of this paper [27] presented the action to quartic order but without the terms involving the spinor structure $\theta\theta\theta\bar{\theta}$. In that version, it was erroneously argued — as pointed out by [28] — that these terms would vanish. In this work, this argument has been corrected and the additional terms are now presented in equations (1.19)–(1.21). Furthermore, the entire computation has been reworked and organized so as to make the calculation of all the terms sensitive to the same potential human errors. Since we are now able to check against the literature for the terms quadratic in the fermions, this organization of the computation provides an indirect check of the quartic terms as well. The entire computational scheme has been coded on Mathematica 5.0.

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2. Preliminaries

2.1 IIB supergravity in superspace

The fields of IIB supergravity are

$$\left\{e_m^{\hat{a}},\, \tau = e^{-\phi} + i\chi,\, b^{(1)}_{mn} + i\, b^{(2)}_{mn},\, b_{mnrs};\, \psi_m,\, \lambda\right\}\;; \tag{2.1}$$

these are respectively the vielbein, a complex scalar comprised of the dilaton and the axion, two two-form gauge fields, a four-form real gauge field, a complex left-handed gravitino, and a complex right-handed spinor. The gauge fields have the associated field strengths defined as

$$h^{(1)} = db^{(1)}\,, \qquad h^{(2)} = db^{(2)}\,, \qquad g = db\,. \tag{2.2}$$

An elaborate superspace formalism can be developed for this theory. It involves the standard supergravity superfields [20]

$$\left(E_M^A, \Omega_{MA}^B\right) \to \left(T_{BC}^A, R_{ABC}^D\right)\,. \tag{2.3}$$

In addition, one needs five other tensor superfields

$$\left\{P_A, Q_A, \hat{\mathcal{F}}_{ABC}, \mathcal{G}_{ABCDE}, \Lambda_A\right\}. \tag{2.4}$$

Throughout, we accord to the standard convention of denoting tangent space superspace indices by capital letters from the beginning of the alphabet. In this setting of $\mathcal{N} = 2$ chiral supersymmetry, an index such as A represents a tangent space vector index $\hat{a}$ spanning all ten dimensions, and two spinor indices α and $\bar{\alpha}$. Hence, superspace is parameterized by coordinates

$$z^A \in \left\{x^{\hat{a}}, \theta^\alpha, \theta^{\bar{\alpha}}\right\}\,. \tag{2.5}$$

Here, θ^α and $\theta^{\bar{\alpha}} \equiv \bar{\theta}^\alpha$ have same chirality and are related to each other by complex conjugation. In this manner, unbarred and barred Greek letters from the beginning of the alphabet will be used to denote spinor indices. More details about the conventions we adopt can be found in appendix A.

The two superfields P_A and Q_A are the field strengths of a matrix of scalar superfields

$$\mathcal{V} = \begin{pmatrix} u & v \\ \overline{v} & \overline{u} \end{pmatrix}, \tag{2.6}$$

with

$$u\overline{u} - v\overline{v} = 1\,. \tag{2.7}$$

This matrix describes the group $\mathrm{SU}(1,1) \sim \mathrm{SL}(2,\mathbb{R})$, which later gets identified with the S-duality group of the IIB theory. The scalars parameterize the coset space $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$, with the additional $\mathrm{U}(1)$ being a space-time dependent symmetry with an associated gauge field. We then define

$$\mathcal{V}^{-1}d\mathcal{V} \equiv \begin{pmatrix} 2iQ & P \\ \overline{P} & -2iQ \end{pmatrix}, \tag{2.8}$$

with

$$Q = \overline{Q} \tag{2.9}$$

being the U(1) gauge field mentioned above. All fields in the theory carry accordingly various charge assignments under this U(1). This is a powerful symmetry that can be used to severely restrict the superspace formalism. We also introduce the superfield strength $\hat{F}$

$$\left(\bar{\hat{\mathcal{F}}}, \hat{\mathcal{F}}\right) = \left(\bar{\hat{F}}, \hat{F}\right) \mathcal{V}^{-1}, \tag{2.10}$$

which transforms under the $\mathrm{SU}(1,1)$ as a singlet.

All these fields are associated with a myriad of Bianchi identities. As is typical in supergravity theories, there is an immense amount of superfluous symmetries in the superspace formalism. Some of these can be fixed conventionally; and using the Bianchi identities, relations can be derived between the various other components. We will be very brief in reviewing this formalism, as our focus will be the string sigma model. Instead of reproducing the full set of equations that determine the IIB theory, we present only those statements that are of direct relevance to the worldsheet theory. Throughout this work, we accord closely to the conventions and notation of [20]; the reader may refer at any point to [20] to complement his/her reading.

From the point of view of the IIB string sigma model, the following combination of the scalars turns out to play an important role

$$\omega = u - \overline{v}. \tag{2.11}$$

Requiring κ symmetry on the worldsheet leads to the condition

$$\omega = \bar{\omega}. \tag{2.12}$$

This is a choice that is unconventional from the point of view of the supergravity formalism, but is natural from the perspective of the string sigma model.

We parameterize the scalar superfields as [21, 22]

$$u = \frac{1+\bar{W}}{\sqrt{2(W+\bar{W})}} e^{-2i\theta}, \tag{2.13}$$

$$v = -\frac{1-W}{\sqrt{2(W+\bar{W})}} e^{2i\theta}, \tag{2.14}$$

with the three variables W, $\bar{W}$ and θ parameterizing the $\mathrm{SU}(1,1)$. The gauge choice (2.12) then corresponds to

$$\theta = 0, \tag{2.15}$$

This leads to

$$\omega = \sqrt{\frac{2}{W+\bar{W}}}. \tag{2.16}$$

And

$$Q_A = \frac{\bar{P}_A - P_A}{4i}. \tag{2.17}$$

Finally, the field strengths are given in terms of W by

$$P = \frac{dW}{W+\bar{W}}\,, \qquad Q = \frac{i}{4}\frac{d(W-\bar{W})}{W+\bar{W}}\,. \tag{2.18}$$

To make contact with the IIB theory's field content (2.1), we need to specify the map between the superfields (2.3) and (2.4) and the physical fields. Each superfield involves an expansion in the fermionic superspace coordinates θ. At zeroth order in this expansion, we have

$$W|_0 = \tau = e^{-\phi} + i\chi\,. \tag{2.19}$$

Similarly, the zeroth components of the Λ superfield is

$$\Lambda_\alpha|_0 = \lambda_\alpha\,. \tag{2.20}$$

In the Wess-Zumino gauge, the supervielbein's zeroth component is

$$E^\alpha_m|_0 = \psi^\alpha_m\,. \tag{2.21}$$

At this point, we can simplify the discussion significantly by choosing to set all background fermionic fields to zero

$$\lambda_\alpha \to 0\,, \qquad \psi^\alpha_m \to 0\,. \tag{2.22}$$

This identifies the class of backgrounds which is of most interest to us and that arises most frequently in the literature. Given this, the zeroth components of the other fields are

$$\hat{\mathcal{F}}_{\hat{a}\hat{b}\hat{c}}|_0 = \mathcal{F}_{\hat{a}\hat{b}\hat{c}} \equiv \frac{h^{(1)}_{\hat{a}\hat{b}\hat{c}}}{2} + i\frac{h^{(2)}_{\hat{a}\hat{b}\hat{c}}}{2}\,, \tag{2.23}$$

$$\mathcal{G}_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}|_0 = G_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\,. \tag{2.24}$$

We also define $\hat{F}_{\hat{a}\hat{b}\hat{c}}|_0 \equiv F_{\hat{a}\hat{b}\hat{c}}$. And, for completeness, we write the full form of the supervielbein

$$E^A_M|_0 = \begin{pmatrix} \hat{e}^{\hat{a}}_m & 0 & 0 \\ 0 & \delta^\alpha_\mu & 0 \\ 0 & 0 & -\delta^{\bar{\alpha}}_{\bar{\mu}} \end{pmatrix}; \tag{2.25}$$

with the zeroth components of the connection

$$\Omega^B_{cA}|_0 = \omega^B_{c,A} + \mathrm{U}(1)\ \text{connection}\,; \tag{2.26}$$

$$\Omega^B_{\alpha,A}|_0 = \Omega^B_{\bar{\alpha},A}|_0 = 0\,; \tag{2.27}$$

and the other combinations of indices being zero.

In addition, we will need the zeroth components of the Riemann and torsion superfields, as well as various spinorial components of all the superfields. To make things even worse, various first and second order spinorial derivatives of the superfields will also be needed; i.e. some of the higher order terms in the superfield expansions appear in the sigma model. These can be systematically, albeit sometimes tediously, obtained by juggling the superspace Bianchi identities. We will present the relevant pieces as we need them, instead of cataloging an incomplete set of lengthy equations out of context.

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2.2 The IIB string worldsheet in superspace

The action of the IIB string in a background represented by the superfields listed above was written in [24]

$$I = \int d^2\sigma \left\{ \frac{1}{2}\sqrt{-h}h^{ij}\Phi V_i^{\hat{a}}V_j^{\hat{b}}\eta_{\hat{a}\hat{b}} + \frac{1}{2}\varepsilon^{ij}V_i^B V_j^A \mathcal{B}_{AB} \right\} , \tag{2.28}$$

with[3]

$$V_i^A \equiv \partial_i z^M E_M^A = \left\{ V_i^{\hat{a}}, V_i^{\alpha}, V_i^{\bar{\alpha}} \right\} , \tag{2.29}$$

and

$$d\mathcal{B} = \hat{\mathcal{F}} + \bar{\hat{\mathcal{F}}} , \tag{2.30}$$

$$\Phi = \omega = \bar{\Phi} . \tag{2.31}$$

The last statement is needed to assure that the action is κ symmetric. The task is to expand this action in component form. This is generally a messy matter, which, however, can be achieved using the algorithm of normal coordinate expansion.

2.3 The method of normal coordinate expansion in superspace

Normal coordinate expansion, as applied to bosonic sigma models, was first developed in [15]. In these scenarios, the method helped to unravel some of the dynamics of highly non-linear theories approximately, as expanded near a chosen point on the target manifold. In the superspace incarnation, the technique is most powerful when used to expand an action only in a submanifold of the target superspace. In particular, expanding in the fermionic variables only, with the space coordinate left arbitrarily, the expansion truncates by virtue of the Grassmanian nature of the fermionic coordinates; leading to an exact expression for the action in component form. This can also be applied of course to the action or equations of motion for the background superfields as well, and the technique has been demonstrated in this context in many examples. As for the IIB sigma model, the expansion has been applied in [25], to expand however the action in all of superspace, leading to a linearized approximate form that can be used to study quantum effects. Our interest is to get to an exact expression for (2.28) in component form, by fixing the κ symmetry and leaving the space coordinates arbitrary. This approach was applied to the Heterotic string in [17]. There, the absence of RR fields made the discussion considerably simpler. Our approach will probe in this respect a new class of couplings by the use of this method. However, many simplifications and techniques we will use are direct generalizations of the corresponding methods applied in [17]. First, we briefly review the normal coordinate expansion method in superspace. The reader is referred to [16, 17] for more information.

The superspace coordinates are written as

$$Z^M = Z_0^M + y^M . \tag{2.32}$$

[3]Note that the index $\hat{a}$ here runs over all ten spacetime directions including the light-cone.

We choose

$$Z_0^M = (x^m, 0) \;, \qquad y^M = (0, y^\mu) \;, \tag{2.33}$$

hence expanding only in the fermionic submanifold. The action is then given by

$$I[Z] = e^{\Delta} I[Z_0] \,, \tag{2.34}$$

with the operator Δ defined by

$$\Delta \equiv \int d^2\sigma y^A(\sigma) \hat{D}_A(\sigma) \,, \tag{2.35}$$

and $\hat{D}_A$ being the supercovariant derivative. This derivative is notationally distinguished from D_A appearing elsewhere in this work in that it involves the standard connection *and* the U(1) connection. And we use the supervielbein to translate between tangent space and superspacetime indices

$$\hat{D}_A \equiv E_A^N(Z_0) \hat{D}_N \,, \qquad y^N \equiv y^A E_A^N \,. \tag{2.36}$$

For our choice of expansion variables, we then have

$$y^{\hat{a}} = 0 \,, \qquad y^\alpha = y^\mu \delta_\mu^\alpha \equiv \theta^\alpha \,, \qquad y^{\bar{\alpha}} = y^{\bar{\mu}} \delta_{\bar{\mu}}^{\bar{\alpha}} \equiv \theta^{\bar{\alpha}} \,. \tag{2.37}$$

The power of this technique is that it renders the process of expansion *algorithmic*. A set of rules can be taught say to any well-trained mammal; in principle, human intervention (for that matter the same mammal may be used again) is needed only at the final stage when Bianchi identities may be used to determine some of the expansion terms. The rules are as follows:

- Due to the definition of the normal coordinates, we have

$$\Delta y^A = 0 \,. \tag{2.38}$$

- Using super-Lie derivatives, it is straightforward to derive

$$\Delta V_i^A = \hat{D}_i y^A + V_i^C y^B T_{BC}^A \,. \tag{2.39}$$

- And the following identity is needed beyond second order

$$\Delta \left(\hat{D}_i y^A \right) = y^B V_i^D y^C R_{CDB}^A \,. \tag{2.40}$$

- Finally, when we apply Δ to an arbitrary tensor with tangent space indices, we get simply

$$\Delta X_{BC..}^{DE..} = y^A D_A X_{BC..}^{DE..} \,. \tag{2.41}$$

In the next section, we outline the process of applying these rules to (2.28).

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3. Unraveling the action

There are three sets of difficulties that arise when attempting to apply the normal coordinate expansion to (2.28). First, a priori, we need to expand to order 2^5 in θ before the expansion truncates. This problem is remedied simply by fixing the κ symmetry with the light-cone gauge, truncating the action to quartic order in θ, as we will show below. The second problem is that the expansion terms will need first and second order fermionic derivatives of the superfields. This requires us to play around with some of the Bianchi identities to extract the additional information. The process is somewhat tedious, but straightforward. The third problem is computational. Despite the simplifications induced by the light-cone gauge choice, and the algorithmic nature of the process, it turns out that the task is virtually impossible to perform by a human hand, while still maintaining some level of confidence in the result. On average 10^4 terms arise at various stages of the computation. The use of the computer for these analytical manipulations greatly simplifies the problem. However, we find that, even with this help, the complexity is such that computing time may be of order of many months, unless the task is approached with a set of somewhat smarter computational steps and unless one makes use of the simplifications that arise from the conditions imposed on the background fields as listed in the Introduction. We do not present all the messy details of these nuances, concentrating instead on the general protocol.

At zeroth order, the action is simply

$$I^{(0)} = I|_0 = \int d^2\sigma \left\{ \frac{1}{2}\sqrt{-h}h^{ij}\omega V_i^{\hat{a}} V_{j\hat{a}} + \frac{1}{2}\varepsilon^{ij} V_i^{\hat{b}} V_j^{\hat{a}} b^{(1)}_{\hat{a}\hat{b}} \right\} . \tag{3.1}$$

Note that this is written with respect to the Einstein frame metric.

At first order in Δ, the action becomes

$$\begin{aligned} I^{(1)} = \Delta I = \int d^2\sigma \Big\{ & \frac{1}{2}\sqrt{-h}h^{ij}(\Delta\Phi)V_i^{\hat{a}}V_j^{\hat{b}}\eta_{\hat{a}\hat{b}} + \sqrt{-h}h^{ij}\Phi(\Delta V_i^{\hat{a}})V_j^{\hat{b}}\eta_{\hat{a}\hat{b}} + \\ & + \frac{1}{2}\varepsilon^{ij}V_i^B V_j^A y^C \mathcal{H}_{CAB} \Big\} , \end{aligned} \tag{3.2}$$

with

$$\mathcal{H} \equiv d\mathcal{B} . \tag{3.3}$$

This result is not evaluated at for $\theta \to 0$ yet as further powers of Δ will hit it.

3.1 Fixing the κ symmetry

Matters are simplified if we analyze the form of the action we expect from this expansion once the κ symmetry is fixed. This will help us avoid manipulating many of the terms that will turn out to be zero in the light-cone gauge anyways. To fix the κ symmetry, we define

$$\sigma^{\pm} \equiv \frac{1}{2}\left(\sigma^0 \pm \sigma^{\hat{a}}\right) , \tag{3.4}$$

where $\hat{a}$ is some chosen direction in space. For conventions on spinors, the reader is referred to appendix A and [20]. We choose the spacetime fermions to satisfy the condition[4]

$$\sigma^+\theta = \sigma^+\bar{\theta} = 0 \,. \tag{3.7}$$

Consider first all even powers of θ. These will necessarily come in one of the following bilinear combinations

$$A^{ab} \equiv \theta\sigma^{-ab}\theta \,, \qquad \bar{A}^{ab} = \bar{\theta}\sigma^{-ab}\bar{\theta} \,; \tag{3.8}$$

$$B \equiv \bar{\theta}\sigma^{-}\theta \,, \qquad B^{ab} \equiv \bar{\theta}\sigma^{-ab}\theta \,, \qquad B^{abcd} \equiv \bar{\theta}\sigma^{-abcd}\theta \,. \tag{3.9}$$

In these expressions, condition (3.7) has been used, and the Latin indices a, b, c, d are transverse to the light cone directions. Furthermore, because of the self duality condition

$$\tilde{\sigma}^{(5)} = \sigma^{(5)} \tag{3.10}$$

we have $B^{abcd} = 0$.

Given the symmetry properties of the gamma matrices (see appendix A), we also know

$$\bar{A}^{ab} = -A^{ab} \,, \qquad \bar{B} = B \,, \qquad \bar{B}^{ab} = -B^{ab} \,, \qquad \bar{B}^{abcd} = B^{abcd} \,. \tag{3.11}$$

3.2 The expected form of the action

First, we note that, given that all background fermions (λ and ψ_m) are zero, only even powers of θ can appear in the expansion. We assume that all background fields have only non-zero components that are either transverse to the light-cone directions, or that the light-cone indices in them come in pairs; and that all the fields depend only on the transverse coordinates. For example, denoting the light-cone directions by '+' and '$-$', and all transverse coordinates schematically by r, all fields can only depend on r; and a tensor $X_{abc\ldots}$ can be non-zero only if either all $a, b, c, \ldots$ are transverse; or if '+' and '$-$' come as in $X_{-+bc\ldots}$ with $b, c, \ldots$ transverse or other light-cone pairs. These conditions lead to a dramatic simplification of the expansion. In particular, given that a '$-$' index is to appear in all even powers of fermion bilinears, as in (3.8) and (3.9), we must pair each bilinear with a V_i^a to absorb the light-cone index '$-$'.

Let Θ represent either θ or $\bar{\theta}$. For example, schematically $\Theta^2 \sim \theta^2, \bar{\theta}\theta, \bar{\theta}^2$. The action consists then of terms of form $\Theta^{2n} V_i^a V_j^b$, $(D\Theta)\Theta^{2n-1} V_i^a$ and $(D\Theta)(D\Theta)\Theta^{2n}$. From the expansion algorithm outlined above, with the use of equations (2.38)-(2.41), it is easy to see that

$$\text{number of } V\text{'s} + \text{number of } D\Theta\text{'s} = 2$$

in each term. Let's then look at each class of terms separately:

[4] Alternatively, we can choose [13]

$$\sigma^{\pm} \equiv \frac{1}{2}\left(\sigma^{\hat{a}} \pm i\sigma^{\hat{b}}\right) , \tag{3.5}$$

with $\hat{a}$ and $\hat{b}$ being two arbitrary space directions. We can then impose

$$\sigma^+\theta = \sigma^-\bar{\theta} = 0 \,. \tag{3.6}$$

It can be seen that this choice leads to a more complicated expansion for the action. It may still be necessary to consider such choices for other classes of background fields than those we focus on in this work.

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- For terms of the form $\Theta^{2n} V^a V^b$, the only non-zero combinations are $\Theta^2 V_i^+ V_j^a$ and $\Theta^4 V_i^+ V_j^+$. This means in particular that the Wess-Zumino term involving $\mathcal{H}$ in (3.2) does not contribute at quartic order since we must contract $V_i^+ V_j^+$ by $\sqrt{-h} h^{ij}$.
- Terms of the form $(D\Theta)\Theta^{2n-1} V_i^a$ are zero unless $n = 1$, because, otherwise, there is shortage of Vs to absorb all light-cone indices.
- Terms of the form $(D\Theta)(D\Theta)\Theta^{2n}$ are zero for all n for the same reason as above.

Hence, the action must have the form

$$I \sim \Theta D \Theta + \Theta^2 + \Theta^4 V^+ V^+ \,, \tag{3.12}$$

with the quartic piece receiving contributions only from the first two terms of (3.2). Hence, the action truncates at quartic order in the fermions. And we focus on expanding only the relevant parts.

4. More details

4.1 The quadratic terms and comparison to literature

As we expand (2.34), the quadratic terms in θ are very simple to handle, and can be done by hand. On finds that zeroth components of $D\omega$ and $D^2\omega$ are needed. For these, we note the relation

$$d\omega = -\frac{\omega}{2}\left(P + \bar{P}\right) . \tag{4.1}$$

Using the results of [20], we get

$$\hat{D}_\alpha \omega|_0 = \hat{D}_{\bar{\alpha}} \omega|_0 = 0 \,. \tag{4.2}$$

$$\hat{D}_\alpha \hat{D}_\beta \omega|_0 = -\omega \frac{i}{24} \sigma_{\alpha\beta}^{\hat{a}\hat{b}\hat{c}} F_{\hat{a}\hat{b}\hat{c}} \,, \qquad \hat{D}_{\bar{\alpha}} \hat{D}_{\bar{\beta}} \omega|_0 = -\omega \frac{i}{24} \sigma_{\alpha\beta}^{\hat{a}\hat{b}\hat{c}} \bar{F}_{\hat{a}\hat{b}\hat{c}} \,, \tag{4.3}$$

$$\hat{D}_{\bar{\alpha}} \hat{D}_\beta \omega|_0 = -\omega \frac{i}{2} \sigma_{\alpha\beta}^{\hat{a}} P_{\hat{a}}|_0 \,, \qquad \hat{D}_\alpha \hat{D}_{\bar{\beta}} \omega|_0 = -\omega \frac{i}{2} \sigma_{\alpha\beta}^{\hat{a}} \bar{P}_{\hat{a}}|_0 \,. \tag{4.4}$$

Note that the supercovariant derivative $\hat{D}_A$ is associated with the standard supergravity superconnection *plus* the U(1) contribution, as discussed in [20]. In these equations, a Latin indices $\hat{a}, \hat{b}, \hat{c}, \ldots$ run over all ten spacetime directions, the transverse and the light-cone. In the Wess-Zumino term, we need $D_\alpha \mathcal{H}_{\beta\hat{a}\hat{b}}|_0$ and $D_{\bar{\alpha}} \mathcal{H}_{\beta\hat{a}\hat{b}}|_0$. These are found

$$\hat{D}_\alpha \mathcal{H}_{\beta\hat{a}\hat{b}}|_0 = i\frac{\omega}{2} \sigma_{\hat{a}\hat{b}\beta}{}^{\gamma} \sigma_{\alpha\gamma}^{\hat{c}} \bar{P}_{\hat{c}}|_0 \tag{4.5}$$

$$\hat{D}_{\bar{\alpha}} \mathcal{H}_{\beta\hat{a}\hat{b}}|_0 = i\frac{\omega}{24} \sigma_{\hat{a}\hat{b}\beta}{}^{\gamma} \sigma_{\bar{\alpha}\gamma}^{\hat{c}\hat{d}\hat{e}} \bar{F}_{\hat{c}\hat{d}\hat{e}} \,. \tag{4.6}$$

Putting things together, we get a kinetic part for the fermions of the form

$$-\frac{i}{2}\omega V_{\hat{a}\,i} \Theta^{ij} \sigma^{\hat{a}} \hat{D}_j \bar{\theta} + \text{c.c.} = -\frac{i}{2}\omega V_{\hat{a}\,i} \Theta^{ij} \sigma^{\hat{a}} D_j \bar{\theta} - \frac{1}{2}\omega Q_{\hat{b}} V_{\hat{a}\,i}\, V_j^{\hat{b}} \Theta^{ij} \sigma^{\hat{a}} \bar{\theta} + \text{c.c.} \,, \tag{4.7}$$

where the second term arises from the U(1) connection (the θ's are charged under this U(1) [20]), and we have defined

$$\Theta^{ij} \equiv \sqrt{-h}\, h^{ij}\theta - \varepsilon^{ij}\bar{\theta}\,. \tag{4.8}$$

To compare with the literature, we want to write the quadratic part in the string frame. Using

$$D_i\theta = \tilde{D}_i\theta + \frac{1}{8}V_{i\,\hat{b}}\left(P_{\hat{a}} + \bar{P}_{\hat{a}}\right)\sigma^{\hat{a}\hat{b}}\theta \tag{4.9}$$

we can write things in terms of the string frame covariant derivative $\tilde{D}$ with metric

$$G^{(str)}_{mn} = \omega\, g_{mn} \tag{4.10}$$

We note in particular the relation $\partial_{\hat{a}} \ln\omega = -(1/2)(P_{\hat{a}} + \bar{P}_{\hat{a}})$. Switching to the string frame rescales the vielbein and hence the various fields in the action as well

$$V_i^{\hat{a}} \to \omega^{-1/2}V_i^{\hat{a}}\,, \qquad P \to \omega^{1/2}P\,, \qquad F \to \omega^{3/2}F\,, \qquad G \to \omega^{5/2}F\,. \tag{4.11}$$

Finally, we rescale the spinors $\theta \to \omega^{-1/4}\theta$ so as to canonically normalize the kinetic term. Collecting all this together, and using the properties of our gamma matrices, we write the action as

$$\mathcal{S}_{\text{quad}} = \int d^2\sigma\ \left(\mathcal{I}_{D\theta} + \mathcal{I}_F + \mathcal{I}_G\right) + \text{c.c.} \tag{4.12}$$

with

$$\begin{aligned}\mathcal{I}_{D\theta} = {} & -\frac{i}{2}V_{\hat{a}\,i}\Theta^{ij}\sigma^{\hat{a}}\tilde{D}_j\bar{\theta} - \\ & -\frac{1}{2}V_{\hat{a}\,i}V_j^{\hat{b}}Q_{\hat{b}}\Theta^{ij}\sigma^{\hat{a}}\bar{\theta} + \frac{1}{4}V_{\hat{a}\,i}V_{\hat{b}\,j}Q_{\hat{c}}\Theta^{ij}\sigma^{\hat{a}\hat{b}\hat{c}}\bar{\theta} + \frac{1}{4}V_{\hat{b}\,i}V_j^{\hat{b}}Q_{\hat{a}}\Theta^{ij}\sigma^{\hat{a}}\bar{\theta}\end{aligned} \tag{4.13}$$

and

$$\begin{aligned}\mathcal{I}_F = {} & i\frac{\omega}{32}\,V_{\hat{a}\,j}V_{\hat{d}\,i}\Theta^{ij}\sigma^{\hat{b}\hat{c}\hat{d}}\theta\left(F_{\hat{a}\hat{b}\hat{c}} + 3\bar{F}_{\hat{a}\hat{b}\hat{c}}\right) - i\frac{\omega}{32}\,V_{\hat{a}\,i}V_{\hat{d}\,j}\Theta^{ij}\sigma^{\hat{b}\hat{c}\hat{d}}\theta\left(F_{\hat{a}\hat{b}\hat{c}} - \bar{F}_{\hat{a}\hat{b}\hat{c}}\right) + \\ & + i\frac{\omega}{96}\,V_{\hat{d}\,i}V_j^{\hat{d}}\Theta^{ij}\sigma^{\hat{a}\hat{b}\hat{c}}\theta\left(F_{\hat{a}\hat{b}\hat{c}} - \bar{F}_{\hat{a}\hat{b}\hat{c}}\right) + i\frac{\omega}{8}\,V_i^{\hat{a}}V_j^{\hat{b}}\Theta^{ij}\sigma^{\hat{c}}\theta F_{\hat{a}\hat{b}\hat{c}}\end{aligned} \tag{4.14}$$

$$\mathcal{I}_G = -\frac{\omega}{96}V_i^{\hat{a}}V_j^{\hat{b}}\Theta^{ij}\sigma^{\hat{c}\hat{d}\hat{e}}\bar{\theta}G_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\,. \tag{4.15}$$

These expressions agree with [28] except for a numerical factor in one of the terms. In [28], the first term of the second line of (4.13) appears with an additional factor of 2. This term arises from the U(1) charge associated with the spinor. We believe that the discrepancy is accounted for by a typo in [28] (perhaps related to adding the complex conjugate piece to the action). Otherwise, our expressions are identical. We conclude that the result, to quadratic order in the spinors, agrees with the literature.[5]

[5]The minor issue regarding the coefficient of the U(1) charge cannot be settled through comparison to other sources of literature because this term vanishes for cases involving AdS backgrounds.

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4.2 The quartic terms

At quartic order in θ, the action is much more difficult to find. Indeed, the use of computation by machine becomes necessary. We do not present all the details, but only some of the important relations that are needed to check the results. In this section, to avoid clutter in index notation, indices $a, b, c, \ldots$ will run over all ten spacetime directions as opposed to using $\hat{a}, \hat{b}, \hat{c}, \ldots$ as we did in the rest of the paper.

First derivatives of some of the Riemann tensor components arise; particularly, $\hat{D}_{\bar{\alpha}}\hat{R}_{\beta a\gamma_1}^{\gamma_2}$ and $\hat{D}_{\alpha}\hat{R}_{\bar{\beta}a\gamma_1}^{\gamma_2}$. Using the results of [20], it is straightforward to find

$$\hat{D}_{\alpha}\hat{R}_{\bar{\beta}a\gamma_1}^{\gamma_2}|_0 = \frac{i}{8}\sigma_{\gamma_1}^{cd\gamma_2}\left(\sigma_{a\bar{\beta}\delta}\hat{D}_{\alpha}T_{cd}^{\delta} + \sigma_{c\bar{\beta}\delta}\hat{D}_{\alpha}T_{ad}^{\delta} + \sigma_{d\bar{\beta}\delta}\hat{D}_{\alpha}T_{ca}^{\delta}\right)\Big|_0 + \frac{i}{2}\delta_{\gamma_1}^{\gamma_2}P_a\sigma_{\alpha\bar{\beta}}^{b}\bar{P}_b|_0\,; \quad (4.16)$$

$$\hat{D}_{\bar{\alpha}}\hat{R}_{\beta a\gamma_1}^{\gamma_2}|_0 = -\frac{i}{8}\sigma_{\gamma_1}^{cd\gamma_2}\left(\sigma_{a\beta\bar{\delta}}\hat{D}_{\bar{\alpha}}T_{cd}^{\bar{\delta}} + \sigma_{c\beta\bar{\delta}}\hat{D}_{\bar{\alpha}}T_{ad}^{\bar{\delta}} + \sigma_{d\beta\bar{\delta}}\hat{D}_{\bar{\alpha}}T_{ca}^{\bar{\delta}}\right)\Big|_0 - \frac{i}{2}\delta_{\gamma_1}^{\gamma_2}\bar{P}_a\sigma_{\bar{\alpha}\beta}^{b}P_b|_0\,. \quad (4.17)$$

We note the distinction between R and $\hat{R}$; the latter includes the curvature from the U(1) gauge field, as defined in [20]. To avert confusion, we also note that the covariant derivative $\hat{D}_A$ is with respect to $\hat{R}$; whereas the one appearing elsewhere in the text as D does not involve the U(1) connection. This aspect of our notation then differs slightly from that of [20].

We need a series of first spinorial derivatives of the torsion. For these, we need to use the Bianchi identity

$$\sum_{(ABC)} \hat{D}_A T_{BC}^{D} + T_{AB}^{E}T_{EC}^{D} - \hat{R}_{ABC}^{D} = 0\,, \quad (4.18)$$

where the sum is over graded cyclic permutations. We then find

$$\hat{D}_{\alpha}T_{cd}^{\delta}|_0 = R_{cd\alpha}^{\delta} - \hat{D}_d T_{\alpha c}^{\delta} - \hat{D}_c T_{d\alpha}^{\delta} + 2T_{\alpha[d}^{\bar{\beta}}T_{c]\bar{\beta}}^{\delta} - 2T_{\alpha[d}^{\beta}T_{c]\beta}^{\delta} + \delta_{\alpha}^{\delta}\bar{P}_{[c}P_{d]}\,, \quad (4.19)$$

and

$$\hat{D}_{\bar{\alpha}}T_{bc}^{\bar{\delta}}|_0 = -\hat{D}_b T_{c\bar{\alpha}}^{\bar{\delta}} - \hat{D}_c T_{\bar{\alpha}b}^{\bar{\delta}} + R_{bc\bar{\alpha}}^{\bar{\delta}} + 2T_{\bar{\alpha}[c}^{\bar{\gamma}}T_{b]\bar{\gamma}}^{\bar{\delta}} - 2T_{\bar{\alpha}[c}^{\gamma}T_{b]\gamma}^{\bar{\delta}} + \delta_{\bar{\alpha}}^{\bar{\delta}}\bar{P}_{[b}P_{c]}\,. \quad (4.20)$$

We also have

$$\hat{D}_{\alpha}T_{\bar{\beta}\bar{\gamma}}^{\delta}|_0 = -\frac{i}{24}\sigma_{\bar{\beta}\bar{\gamma}}^{d}\sigma_{d}^{\delta\beta}\sigma_{\alpha\beta}^{abc}F_{abc} + \frac{i}{24}\delta_{\bar{\beta}}^{\delta}\sigma_{\alpha\bar{\gamma}}^{abc}F_{abc} + \frac{i}{24}\delta_{\bar{\gamma}}^{\delta}\sigma_{\alpha\bar{\beta}}^{abc}F_{abc}\,. \quad (4.21)$$

In all these and subsequent equations, the right hand sides are to be evaluated as zeroth order in θ.

As if first derivatives are not enough of a mess, two derivatives of the torsion are also needed. For example, $\hat{D}_{\alpha}\hat{D}_{\beta}T_{\bar{\gamma},a}^{\delta}$ arises and is found

$$\begin{aligned}\hat{D}_{\alpha}\hat{D}_{\beta}T_{\bar{\gamma}a}^{\delta}|_0 = &-\frac{3}{16}\sigma_{\bar{\gamma}}^{de\delta}\left(-\frac{1}{32}\mathcal{K}_{ade\beta}{}^{\gamma}\hat{D}_{\alpha}\hat{D}_{\gamma}\omega + 3P_{[a}\sigma_{de]\beta}{}^{\gamma}\hat{D}_{\alpha}\hat{D}_{\gamma}\omega + 3i\sigma_{[a\beta\gamma}\hat{D}_{\alpha}T_{de]}^{\gamma}\right) - \\ &-\frac{1}{48}\sigma_{a\bar{\gamma}}{}^{cde\delta}\left(-\frac{1}{32}\mathcal{K}_{cde\beta}{}^{\gamma}\hat{D}_{\alpha}\hat{D}_{\gamma}\omega + 3P_{[c}\sigma_{de]\beta}{}^{\gamma}\hat{D}_{\alpha}\hat{D}_{\gamma}\omega + 3i\sigma_{[c\beta\gamma}\hat{D}_{\alpha}T_{de]}^{\gamma}\right), \quad (4.22)\end{aligned}$$

where we define the matrix

$$\mathcal{K}_{cde} \equiv \sigma_{cdefgh}\bar{F}^{fgh} + 3\bar{F}_{[c}{}^{fg}\sigma_{de]fg} + 52\bar{F}_{[cd}{}^{f}\sigma_{e]f} + 28\bar{F}_{cde}\,. \quad (4.23)$$

To find $\hat{D}_{\bar{\alpha}}\hat{D}_\beta T^{\gamma_2}_{\gamma_1 a}$, we use the standard statement

$$[\hat{D}_A, \hat{D}_B\} = -T^C_{AB}\hat{D}_C - \hat{R}^D_{ABC}\,. \tag{4.24}$$

And we get

$$\hat{D}_{\bar{\alpha}}\hat{D}_\beta T^{\gamma_2}_{\gamma_1 a}|_0 = -T^b_{\bar{\alpha}\beta}\hat{D}_b T^{\gamma_2}_{\gamma_1 a} + R^\delta_{\bar{\alpha}\beta\gamma_1}T^{\gamma_2}_{\delta a} + R^b_{\bar{\alpha}\beta a}T^{\gamma_2}_{\gamma_1 b} - T^\delta_{\gamma_1 a}R^{\gamma_2}_{\bar{\alpha}\beta\delta} - \hat{D}_\beta\hat{D}_{\bar{\alpha}}T^{\gamma_2}_{\gamma_1 a}\,. \tag{4.25}$$

We need $\hat{D}_\alpha\hat{D}_{\bar{\beta}}T^{\gamma_2}_{\gamma_1 a}$, which is

$$\begin{aligned}\hat{D}_\alpha\hat{D}_{\bar{\beta}}T^{\gamma_2}_{\gamma_1 a}|_0 =& -\hat{D}_\alpha\hat{D}_{\gamma_1}T^{\gamma_2}_{\bar{\beta}a} - T^b_{\bar{\beta}\gamma_1}\hat{D}_\alpha T^{\gamma_2}_{ba} - \hat{D}_\alpha R^{\gamma_2}_{\bar{\beta}a\gamma_1} - \\ &- T^{\bar{\alpha}}_{\gamma_1 a}\sigma^b_{\bar{\alpha}\bar{\beta}}\sigma_b^{\gamma_2\delta}\hat{D}_\alpha\hat{D}_\delta\omega + T^{\bar{\alpha}}_{\gamma_1 a}\delta^{\gamma_2}_{\bar{\alpha}}\delta^\delta_{\bar{\beta}}\hat{D}_\alpha\hat{D}_\delta\omega + T^{\bar{\alpha}}_{\gamma_1 a}\delta^{\gamma_2}_{\bar{\beta}}\delta^\delta_{\bar{\alpha}}\hat{D}_\alpha\hat{D}_\delta\omega\,.\end{aligned} \tag{4.26}$$

Finally, we collect the zeroth order components of some of the superfields that arise in the computation as well. These can be found in [20], but we list them for completeness:

$$T^a_{\alpha\bar{\beta}}|_0 = -i\sigma^a_{\alpha\beta}\,. \tag{4.27}$$

$$T^{\bar{\gamma}}_{a\beta}|_0 = -\frac{3}{16}\sigma_\beta^{bc\gamma}\bar{F}_{abc} - \frac{1}{48}\sigma_{abcd\beta}{}^\gamma\bar{F}^{bcd}\,. \tag{4.28}$$

$$T^\gamma_{a\beta}|_0 = i\sigma_\beta^{bcde\gamma}Z_{abcde}\,. \tag{4.29}$$

$$R_{\alpha\beta,ab}|_0 = i\frac{3}{4}\sigma^c_{\alpha\beta}\bar{F}_{abc} + \frac{i}{24}\sigma_{abcde\alpha\beta}\bar{F}^{cde}\,. \tag{4.30}$$

$$R_{\alpha\bar{\beta},ab}|_0 = -\frac{1}{24}\sigma^{cde}_{\alpha\beta}g_{abcde}\,. \tag{4.31}$$

$$H_{a\beta\gamma}|_0 = -i\omega\sigma_{a\beta\gamma}\,. \tag{4.32}$$

$$H_{a\bar{\beta}\bar{\gamma}}|_0 = -i\omega\sigma_{a\beta\gamma}\,. \tag{4.33}$$

All other components as they arise in the expansion are zero. The final result is given in (1.13).

5. Discussion

In this work, we derived the component form of the IIB worldsheet theory in backgrounds involving RR fluxes. In the light-cone gauge, the action was found to truncate to quartic order in the spacetime spinors. Terms quadratic in the fermions could be compared to results already existing in the literature; and we concluded that our computation agrees with the existing results (modulo a term we commented on in section 4.1). The quartic interactions terms are most interesting in addressing issues of integrability of the worldsheet and were computed as well. The complete results were summarized in equations (1.4) and (1.13).

The form of our action is such that the spinors θ may dynamically acquire a non-trivial vacuum configuration depending on the strengths of the various background fields. There is also an interesting coupling to the covariant derivative of the field strengths DF. And it is easy to see that many of the terms vanish when one considers center of mass motion of the closed string. An important program is then to arrange for simplified semi-classical settings and see how turning on the various couplings independently affects the vacuum of the worldsheet theory. This can help us develop intuition about the effects of RR fields on closed string dynamics. We defer such a complete analysis to an upcoming work [29].

Other future directions include writing the IIA action in a similar manner, or by using T-duality (see, for example, [18]). Furthermore, given the algebraic complexity of the computations involved in deriving some parts of our action, it can be useful to have some of the details of our results checked independently, preferably with different methods. Finally, it would be helpful to develop general computational techniques that allow us to analyze, at least semi-classically, dynamics of closed strings in arbitrary backgrounds - with the RR fields taken into account. In this regard, approximation methods such as expansion about center of mass motion — which is in some respects an extension of the normal coordinate expansion technique we used in superspace — may be used. We hope to address some of these issues in the future.

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A. Spinors and conventions

In this appendix, the indices $a, b, c, \cdots$ run over all ten spacetime directions. Our spinors are Weyl but not Majorana. They are then complex and have sixteen components. The associated 16×16 gamma matrices satisfy

$$\left\{\sigma^a, \sigma^b\right\} = 2\eta^{ab}\,, \tag{A.1}$$

with the metric

$$\eta_{ab} = \mathrm{diag}(+1, -1, -1, \ldots, -1)\,. \tag{A.2}$$

Note that the signature is different from the standard one in use in modern literature. This is so that we conform to the equations appearing in [20]. Also, the worldsheet metric h^{ij} has signature $(-,+)$ for space and time, respectively. Throughout, the reader may refer to [20] to determine more about the spinorial algebra and identities that we are using. However, we make no distinction between σ and $\hat{\sigma}$ as defined in [20] as this will be obvious from the context.

We note that σ^a, σ^{abcd} and σ^{abcde} are symmetric; while σ^{ab} and σ^{abc} are antisymmetric; and σ^{abcde} is self-dual.

With the choice given in (3.4), we then have

$$\sigma^+\sigma^- + \sigma^-\sigma^+ = 1\,. \tag{A.3}$$

And complex conjugation is defined so that

$$\overline{\sigma^a} = \sigma^a\,. \tag{A.4}$$

Conjugation also implies

$$\overline{\theta_1\theta_2} = \bar{\theta}_2\bar{\theta}_1\,. \tag{A.5}$$

Finally, antisymmetrization is defined as

$$\sigma^{ab} \equiv \sigma^{[a}\sigma^{b]}\,, \tag{A.6}$$

with a conventional 2! hidden by the braces.

Using the completeness relation and the algebra above, we have, for any matrix $Q_{\alpha\beta}$ with lower indices

$$Q_{\alpha\beta} = \frac{1}{16}\left(\mathrm{Tr}[Q\sigma_a]\sigma^a_{\alpha\beta} - \frac{1}{3!}\,\mathrm{Tr}[Q\sigma_{abc}]\sigma^{abc}_{\alpha\beta} + \frac{1}{5!}\,\mathrm{Tr}[Q\sigma_{abcde}]\sigma^{abcde}_{\alpha\beta}\right)\,. \tag{A.7}$$

This allows us, for example, to rearrange certain combinations such as

$$(\bar{\theta}\sigma^{-(r)}\theta)(\bar{\theta}\sigma^{-(s)}\theta) = \frac{1}{2}\frac{\mathrm{sgn}(r)}{16^2}(\bar{\theta}\sigma^-\sigma^{bc}\bar{\theta})(\theta\sigma^-\sigma^{ef}\theta)\,\mathrm{Tr}[\sigma^{bc}\sigma^{(s)}\sigma^{ef}\sigma^{(r)}]\,, \tag{A.8}$$

$$\mathrm{sgn}(r) \equiv \begin{cases} +1 & \text{for } r=0 \\ -1 & \text{for } r=2\ ; \\ +1 & \text{for } r=4 \end{cases} \tag{A.9}$$

this identity arises repeatedly in the computations. Finally, to avert confusion, we also note the summation convention used

$$U^A V_A = U^a V_a + U^\alpha V_\alpha - U^{\bar{\alpha}} V_{\bar{\alpha}}\,. \tag{A.10}$$

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