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A FINANCING STRATEGY FOR SMALL OTEC PLANTS

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Abstract—The present paper proposes a financing scheme that would spread in time the capital cost of, and the technological risk associated with, small land-based Ocean Thermal Energy Conversion (OTEC) plants. It is based on the cost effectiveness of some OTEC by-products. Three separate phases are envisioned: the first one would consist in supplying air-conditioning (A/C) needs with deep cold seawater pumped through a pipe designed for larger OTEC flow rate requirements; the second one, in building a desalination unit based on some Open-Cycle OTEC hardware, with externally supplied power. The last phase represents the OTEC power plant itself. The capital expenditure would be shared relatively evenly by all phases. The philosophy of this financing strategy is that each phase has an acceptable cumulative payback period, even if the following phases cannot be implemented. Copyright © 1996 Elsevier Science Ltd

OTEC   Ocean Thermal Energy Conversion   Air conditioning   Desalination   Phased financing

OCEAN THERMAL ENERGY CONVERSION AND BY-PRODUCTS

The concept of Ocean Thermal Energy Conversion (OTEC) originated in the late 19th Century, when the French physicist d'Arsonval suggested that a working fluid could extract energy, in a closed-loop Rankine cycle, by exchanging heat with deep cold seawater and surface warm seawater. One of d'Arsonval's students, Claude, imagined and experimentally demonstrated a variation of the original Closed-Cycle (CC) OTEC concept, the Open Cycle (OC), for which the working fluid is low-pressure steam produced by the continuous flashing of some surface seawater [1, 2]. A comprehensive background on OTEC may be found in various articles, such as Refs [3 and 4].

Interest in OTEC has known highs and lows since Claude's pioneering days, depending largely on the cost and availability of other energy sources. As a matter of fact, there exists only one experimental OTEC system in operation at present [5]; its small rated capacity of 210 kW, proved adequate for the acquisition of some valuable engineering experience, but may not suffice to convince financial institutions of communities routinely consuming hundreds of megawatts. Even though OTEC remains an important candidate for renewable energy for tropical coastal regions, as illustrated in the Interlaboratory White Paper prepared for the U.S. Government in March 1990 [6], its present lack of cost effectiveness for small power outputs has led the scientific community to investigate increasingly the so-called OTEC by-products. Desalinated water is the most obvious by-product for OC-OTEC plants equipped with surface condensers. Various schemes labeled Hybrid Cycles [7], or the concept of a (second) flashing stage downstream of the OTEC power plant [8], have extended the potential of OTEC-based desalination to CC plant types. Moreover, deep seawater aquaculture raised great hopes in the seventies [9], as the nutrient-rich, biologically clean, OTEC cold water seemed to offer unmatched advantages. It also became clear that the low temperature of deep seawater, typically below 5°C, could represent an outstanding source of chilling fluid for air-conditioning purposes.

Notwithstanding the fate of OTEC-related research and development efforts today, two basic facts have emerged and gained recognition. On one hand, although the commercial future of OTEC seems to lie in large floating plants of about 50 MW, small land-based units are necessary as experimental stepping stones, while they will remain attractive to supply some remote island

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markets [10]. On the other hand, and in today's economic context, the cost effectiveness of OTEC power produced by small plants cannot be achieved unless some valuable OTEC by-products may be taken into account. Those constraints have led the authors to formulate a Multiple-Phase Financing Concept (MPFC) as an attempt to subsidize the construction of needed small-scale experimental OTEC facilities by relying on the established cost effectiveness of selected OTEC by-products.

Before proceeding, it should be added that the strict definition of cost effectiveness applied below might be superseded, in a not-too-distant future, by wider concepts [11]: for example, environmental and health problems caused by a technology would be quantified, or a so-called cradle-to-grave energy budget would be considered. This new approach could certainly favor some renewable energy concepts, including OTEC.

THE MULTIPLE-PHASE FINANCING CONCEPT (MPFC)

The idea of sharing deep cold seawater between OTEC experimental facilities and organizations researching on OTEC by-products is not new: the Hawaii Ocean Science and Technology (HOST) Park at Keahole Point, Hawaii, was conceived upon this principle. A 1 m (40″) diameter high density polyethylene (HDPE) Cold Water Pipe (CWP), carrying water from depths of about 700 m, was designed by Makai Ocean Engineering and installed in 1987 [12]. This project was a remarkable success from an ocean engineering viewpoint. The cold seawater has been used by a few firms involved in specialized mariculture, as well as by the Net Power Producing Experiment (NPPE), a 210 kW demonstration OC-OTEC facility sponsored by the U.S. Department of Energy and the State of Hawaii, at a rate of 0.4 m^3/s (6,500 gpm) [5].

Some specific features of HOST Park must be critically examined for future projects, however. On one hand, the massive sponsorship effort carried by the State of Hawaii and by the U.S. Department of Energy, not to forget funds provided by the Japanese Government for OTEC-related studies, should not be taken for granted in the near future in times of severe budget constraints and of lukewarm political commitment to renewable energies. Thus, it is important to design future experimental projects for which some cost effectiveness may be achieved, in a quasi-commercial sense: this may lead to private sponsorship with, perhaps, some limited public participation, or incentives such as fiscal aid. Secondly, HOST Park clearly views mariculture as the most important OTEC by-product. It has been established, though, that promising as they may be, most deep seawater mariculture research and development activities are far from having matured into cost effective operations yet [10]. On the other hand, as the NPPE reaches its expected goal of 40 kW net (210 kW gross) power output, the next step to advance OTEC technology will require a plant for which financing is problematic because its capacity, of the order of a few megawatts, would not allow the substantial economies of scale expected in the 50–100 MW range. Thus, a net OTEC capacity of about 1 MW corresponds to a capital investment of the order of $\$30 \times 10^6$ [8].

The dual need for substantial OTEC development capital and for cost effectiveness represents no small dilemma. One scenario is now proposed, called the Multiple-Phase Financing Concept (MPFC), whereby a solution is envisioned. It consists in identifying several phases of the project, which would be spread in time and share the following desirable criteria: (1) whenever a new phase is completed, the *overall* project remains cost effective; (2) each new phase largely makes use of the hardware deployed through previous phases. Cost effectiveness may be defined in a soft way: e.g. the payback period, over which the net present value of the project is zero, should not exceed a chosen limit, say 20 or 30 years. Criterion (1) gives the sponsor some flexibility whether to proceed or not with subsequent phases, depending on market conditions and technological improvements. Criterion (2) represents a means to share effectively the financing burden of an experimental or pre-commercial OTEC plant and to also acquire operational experience incrementally with only a few critical components at a time.

Before considering a specific example, let us define the phases of a potential MPFC scheme more precisely. It is assumed that the ultimate goal is to have an OTEC plant in operation which produces power and some other selected items. The choice of the initial step consists in using deep cold seawater as a chilling fluid for space air-conditioning needs. The cost effectiveness of this air-conditioning method has been established if the seafloor slope is steep enough to reach cold seawater with a short CWP (for example, a 3 km distance from shore giving access to 1000 m deep seawater defines an excellent site) [13, 14].

Nevertheless, caution must be exercised in matching cold seawater requirements for air-conditioning and OTEC: to ensure the large flow rate necessary to sustain an OTEC plant, when the ultimate MPFC phase is implemented, the CWP must be sized properly from the outset. Consequently, the (first) air-conditioning phase would be completed, and cost effective, with a CWP substantially larger than the conduit required by the A/C load alone. This investment penalty could be partially compensated by decreased pumping power requirements since fluid velocities in the oversized pipe would be low throughout the first phase. In short, the CWP should be initially selected with OTEC in mind, whereas the air-conditioning load should be both large enough to ensure the cost effectiveness of the first phase, and small enough to represent only a fraction of the (full) cold water flow rate required through subsequent phases.

The OTEC hardware share absorbed by the first phase amounts to at least 25% in the form of the costly CWP for smaller power outputs [15, 16]. Some other items, such as pumps, or discharge conduit, may have to be replaced or extended later, as their compatibility with the operational conditions of following phases may be questionable.

The definition and implementation of a second phase would naturally depend on technical and economic considerations, perhaps different from today's. At present, it appears that desalination, using the temperature difference between surface and deep-ocean water masses, is both a desirable and technically feasible process. Power to run this second phase facility would remain external (e.g. diesel generator) rather than be produced by OTEC yet, a "half-way" scheme that has already been envisioned [17]. Some operational experience has been available since 1990 with a system consisting of externally-driven low-pressure steam OC-OTEC components, the Heat and Mass Transfer Scoping Test Apparatus, run at Keahole Point, Hawaii [18]. Further investigation of OTEC-based desalination has also been performed more recently in parallel with the NPPE [5]. The hardware involved includes warm seawater intake and effluent discharge systems, a flash evaporator, a surface condenser and a vacuum compressor. While in operation, such a desalination unit would utilize the overall available seawater temperature difference of 20°C or so, and could produce about 70% of the maximum theoretical output per unit flow rate of cold seawater. It has been argued that multistage-flashing desalination systems could be even more efficient [7].

The capital investment burden of this desalination phase is estimated to represent, typically, 30% of the total for a two-stage OTEC plant (OTEC power *and* desalination modules). The fate of the desalination hardware, when the third (and last) OTEC power phase gets under way, depends on many factors. If the OTEC power module is based on the Closed Cycle, the desalination unit could be salvaged in its entirety as a downstream second stage, though the seawater temperature difference available for desalination, from the OTEC effluents, would approximately drop by a factor of two to about 10°C. If the OTEC power module is based on the Open-Cycle, the previous strategy remains valid, though alternatively, it is conceivable to retrofit the desalination hardware into a single-stage OTEC power module, if the incorporation of a large turbo-generator into—and the fine tuning of—the retrofitted system does not prove too difficult.

MPFC EVALUATION FOR A SMALL OPEN-CYCLE OTEC PLANT

The MPFC is applied to a previously proposed 1.8 MW (gross) land-based Open-Cycle OTEC plant [8]. This design utilizes the largest commercially available HDPE CWP, with a diameter of 1.6 m (63″), and also relies on a state-of-the-art low-pressure axial steam turbine (1.3 m, or 52″, rotor blade length). The seafloor bathymetry is typical of good land-based OTEC sites, such as Keahole Point, Hawaii, with a CWP length of the order of 3 km. Deep and surface seawater layers have average temperatures of 4 and 26°C, respectively.

As suggested earlier, cost effectiveness may be assessed by a payback period, over which the net present value of the project is zero. Simple algorithms for the payback period are presented in the Appendix. The inflation and discount rates, i and d in Equation (A-3), are chosen as 5% and 9.5%, respectively. To evaluate the revenues and operating expenses of the project, two values for the basic costs of electricity and fresh water were selected: 10 ¢/kWh and 20 ¢/kWh, 0.5 \$/m^3 (1.875 \$/kgal)

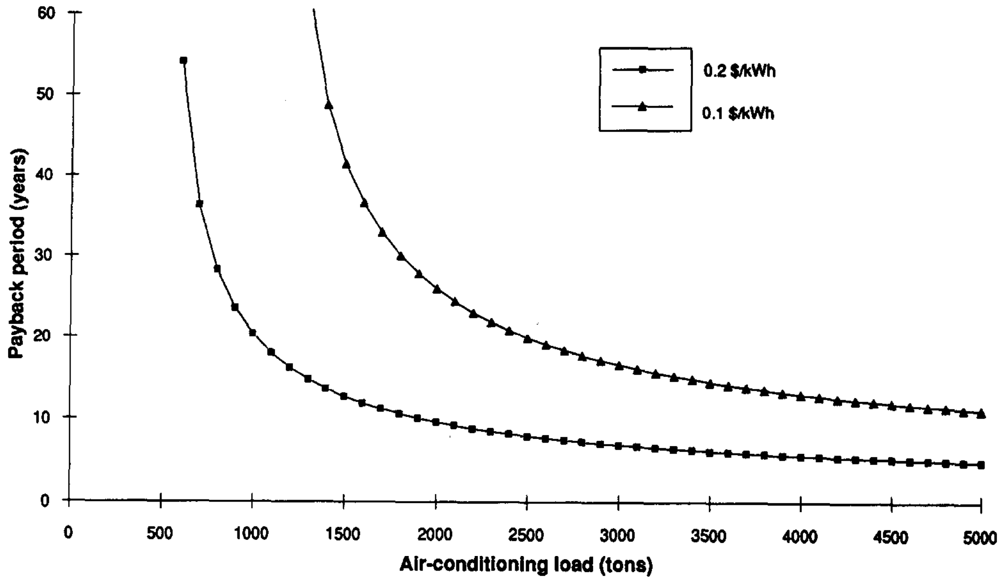

Fig. 1. Payback period as a function of air-conditioning load for Phase 1 of a sample OTEC plant project utilizing a 1.6 m diameter CWP.

and 1.0 $/m^3 (3.75 $/kgal), respectively. The lower bounds are representative of Hawaii, where it is recommended that a pilot plant be built, and the higher bounds of a "prototypical" developing island nation.

Additional assumptions regarding the first air-conditioning phase are now given. The capital cost of the deployed 1.6 m diameter (oversized) CWP system is estimated to be of the order of $\$10 \times 10^6$, on the basis of costs reported for a similar project [19]. The A/C-load-dependent air-conditioning hardware is estimated at $1,850 per ton* (for reference, the air-conditioning of one hotel room in a tropical climate roughly corresponds to one ton). The power saved by eliminating the mechanical chiller is calculated on the basis of a compression work of 0.9 kW per ton. The air-conditioning demand, or usage factor, is chosen to be 60%. A 1°C temperature rise is anticipated through the supply piping system on land, from 4°C to 5°C, and the seawater return temperature is 13°C. The supply pump is chosen to be 70% efficient, with an overall head of 30 m. Under these conditions, a flow of 110 kg/s of cold seawater and a pumping power of 461 kW are required for every 1,000 ton load.

The payback period as a function of air-conditioning load, N_1, is shown on Fig. 1. The two curves permit the quantification of the air-conditioning load appropriate for the application of the MPFC: for example, if a 30-year payback is acceptable, a 1,800 (750) ton load would suffice for the lower (higher) electricity price. The sensitivity of these first-phase results to the price of electricity is a direct consequence of the large energy conservation potential offered by using thermal energy (via deep cold seawater) rather than higher grades of energy (electrical, mechanical,...) to chill A/C fluids; in fact, if A/C hardware were compatible with seawater, one could use deep cold seawater directly in the coils.

We may now tackle the question of initiating the following phases of the project. As suggested before, the second phase may be a desalination unit equivalent to an "OC-OTEC plant without turbogenerator". For the size envisioned here, and for the full cold seawater flow available, an additional capital requirement of $\$10 \times 10^6$ is estimated. Figure 2 shows a heat-and-mass balance diagram of this potential desalination facility (the diverted cold seawater, 231 kg/s, corresponds to 2,100 tons of air-conditioning). More details may be found in Ref. [17]. The nominal fresh water output is 61 kg/s with an electricity consumption of 620 kW. In general, the air-conditioning cold seawater needs of Phase 1 do not seem to affect the desalinated water production by more than about 10%, for practical loads less than 6,000 tons. A desalination capacity factor of 90% and annual operation and maintenance expenses of $200,000 were also assumed.

*One A/C ton is numerically equal to 3.5 kW (200 Btu/min) and is defined as the continuous cooling rate provided by the melting of one short ton (906 kg) of ice over a period of 24 hours.

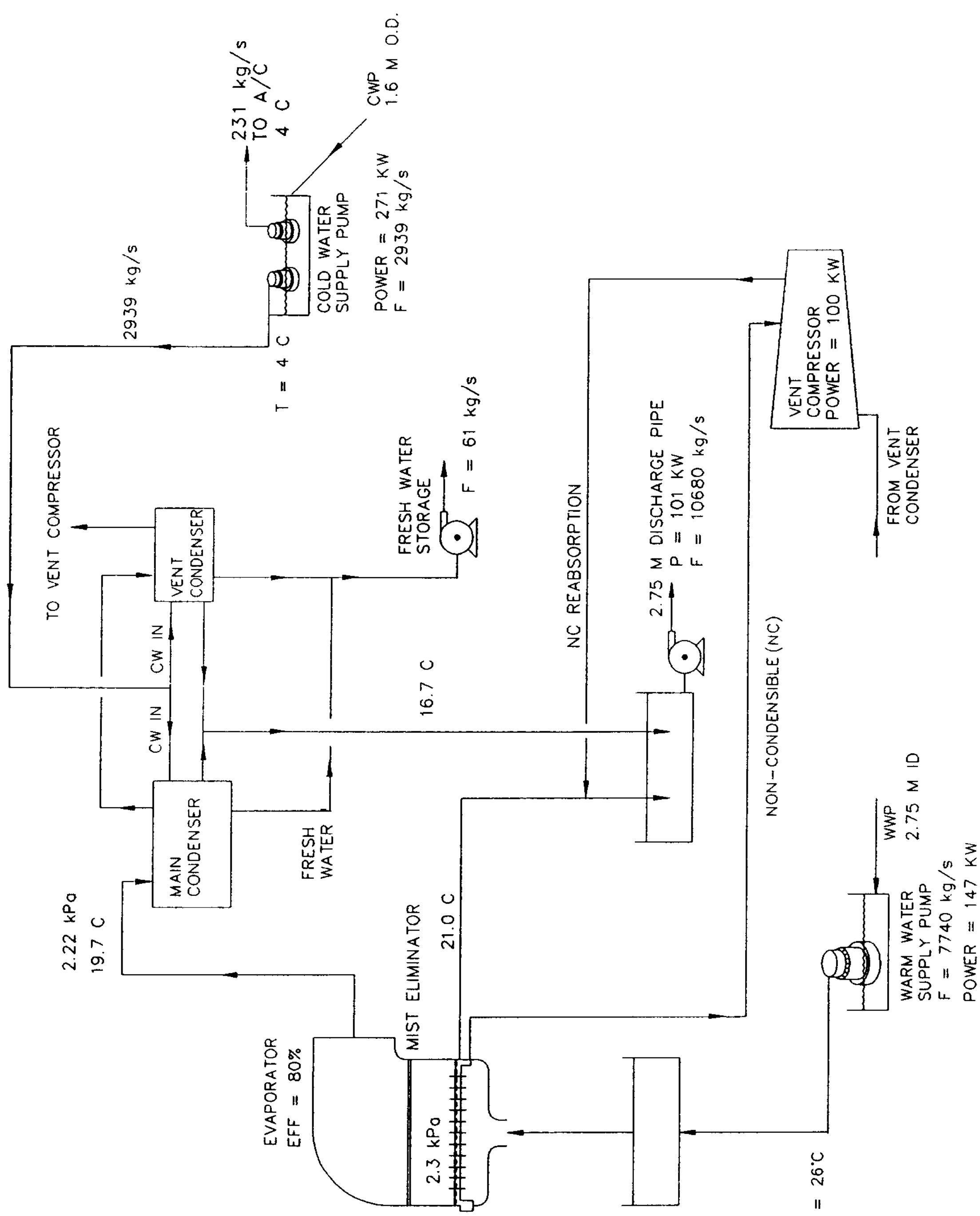

Fig. 2. Schematic heat-and-mass balance diagram for the 61 kg/s desalination plant defining Phase 2.

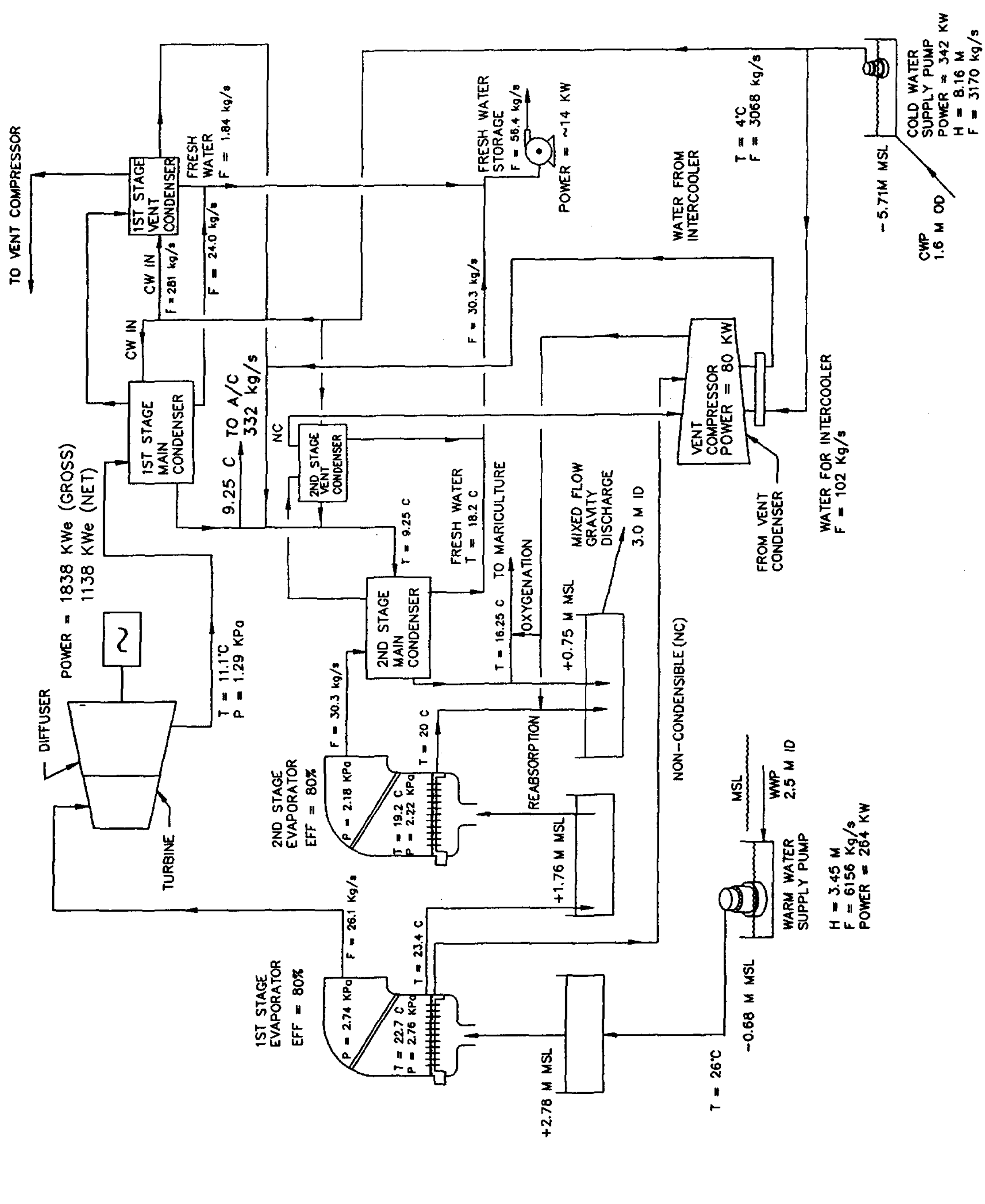

Fig. 3. Schematic heat-and-mass balance diagram for the final two-stage 1.8 MW (gross) OC-OTEC plant defining Phase 3.

Eventually, the building of the 1.8 MW (gross) OC-OTEC facility, or Phase 3, could be undertaken. Figure 3 shows the schematic heat-and-mass balance for the two-stage plant. For this example, it was estimated that the cold seawater input to the air-conditioning facility should be effected downstream of the power plant: thus, the chilling temperature rises by about 5°C; concurrently, the return seawater from the A/C facility warms up from 13°C to about 15.5°C, which remains acceptable by industry standards [20]; an approximate 50% increase in the flow required for A/C must be provided. The combined desalinated water output from both stages drops by about 10 kg/s, to 55 kg/s. Net power production is of the order of 1,140 kW. The availability of the OTEC power system was chosen to be 80%, as well as annual operation and maintenance expenses of \$500,000. A capital increment of $\$10 \times 10^6$ is deemed necessary (it is expected that the total investment through the three phases should exceed the capital required for the same two-stage plant if it were built all at once, because of a lack of thorough hardware and operational compatibility between phases; here, the additional overall capital penalty amounts to about 10%).

In the calculations below, the A/C load is assumed to be fixed from the onset of the project, since the example treated here is for illustrative purposes. There would be no difficulty, however, applying the formulas in the Appendix to scenarios where the A/C load is modified from phase to phase, as long as the impact of such modification on incremental capital and net revenue is properly accounted for.

The time at which to implement the second phase should be discussed at this point. It is shown in the Appendix that if the third phase were dropped out of consideration, there could be an optimal time v_{opt}, as given by Equation (A5), to let elapse between the A/C and desalination phases. It is also shown that if all three phases remained projected and that they were to be equally spaced, there could be an optimal "period" between them v_{per}, as given by Equation (A7). Figure 4 shows a plot of v_{opt} (label "Phase 2"), v_{per} (label "Phase 2 (periodic)") and $2v_{per}$ (label "Phase 3 (periodic)") as a function of A/C load when the price of electricity is 10 ¢/kWh. Since v_{per} is less than v_{opt}, it might be wise to decide whether or not to complete the project through the last and third phase in a time frame no longer than v_{per}, of the order of 5 years. If the third phase is dropped, one could just wait v_{opt} years before implementing the second desalination phase. No equivalent figure is proposed when the price of electricity is 20 ¢/kWh, simply because the optimum phase spacings v_{opt} and v_{per} do not exist then: this means that the MPFC is not necessary from a purely economic standpoint because the overall completed three-phase project would be more cost effective than the first (or first two) phase(s) alone. The MPFC could nevertheless be envisioned, in cases of more costly electricity, for the purpose of risk spreading: therefore, arbitrarily fixed implementation

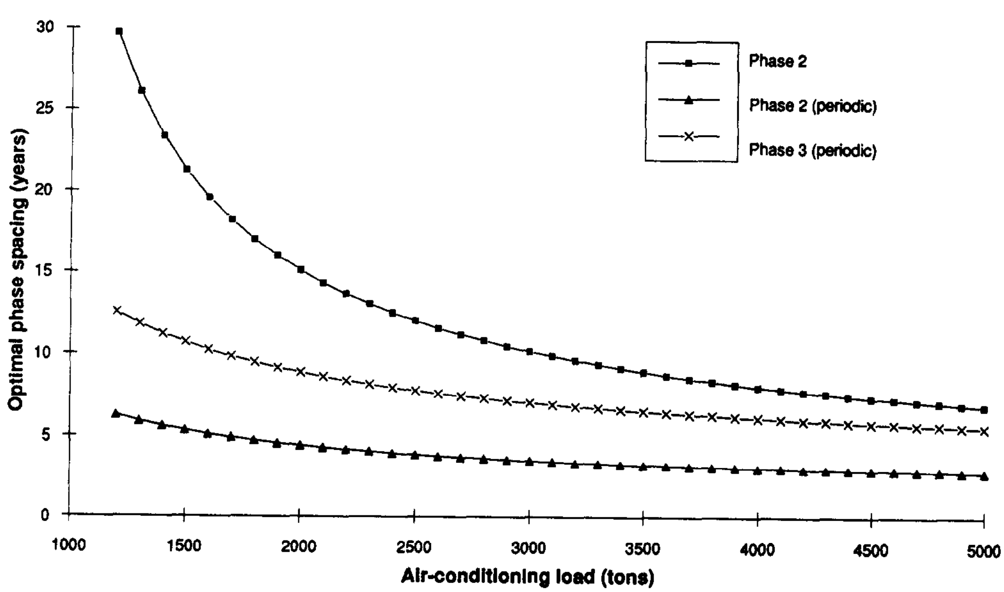

Fig. 4. Optimal phase spacings for a two-phase or three-phase-periodic implementation of the sample OTEC plant project (10 ¢/kWh electricity).

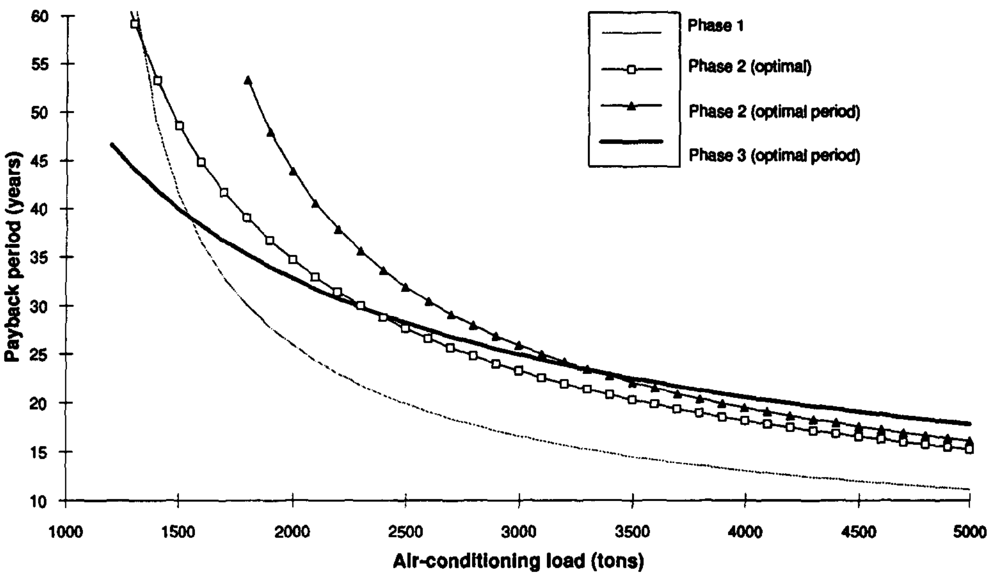

Fig. 5. Payback period as a function of air-conditioning load for the sample OTEC plant project (10 ¢/kWh electricity).

periods of 5 and 10 years, respectively, were considered, with second and third phase overall payback periods still given in each case by Equations (A4) and (A6).

Figure 5 shows the effect of implementing the second and third phases of the project upon overall project payback period, when the present price of electricity is 10 ¢/kWh. For practical project lives (e.g. 20–30 years), the first phase alone is much more cost effective. As shown mathematically, the payback period for the first two phases is minimal for $\nu = \nu_{opt}$, so that the curve labeled "Phase 2 (optimal)" lies below the curve labeled "Phase 2 (optimal period)" corresponding to $\nu = \nu_{per}$. Under the optimal periodic scheme, completing the project with Phase 3 is an improvement in cost effectiveness at A/C loads of less than about 3,300 tons. Coincidentally, an A/C load of 2,300 tons corresponds to a payback period of 30 years whether one terminates the project with desalination ["Phase 2 (optimal)"] or with the two-stage OC-OTEC plant.

Figure 6a and b are similar to Fig. 5, when the present price of electricity is 20 ¢/kWh, and for fixed implementation periods of 5 and 10 years, respectively. It is straightforward to interpret these

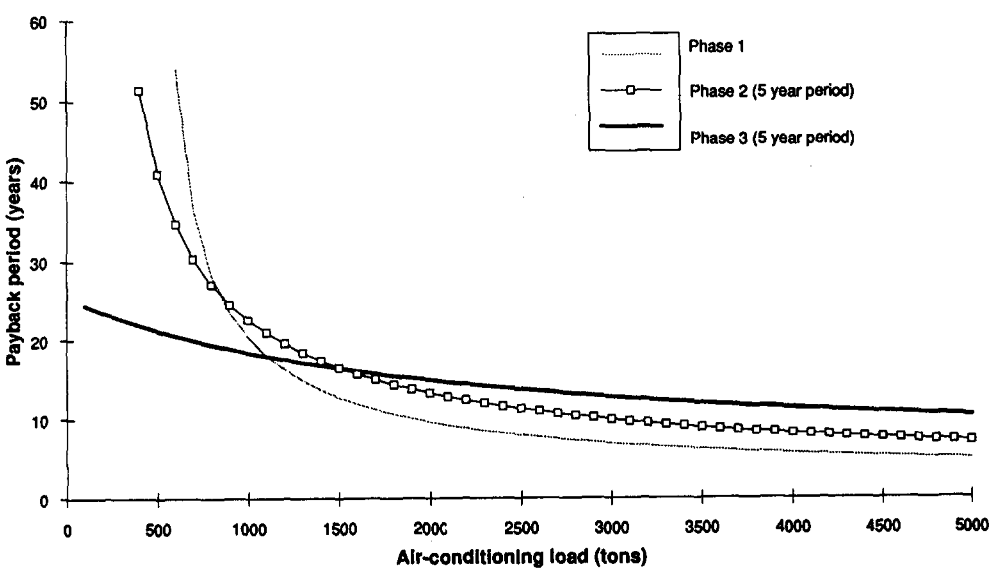

Fig. 6a. Payback period as a function of air-conditioning load for the sample OTEC plant project (20 ¢/kWh electricity, 5-year phase spacing).

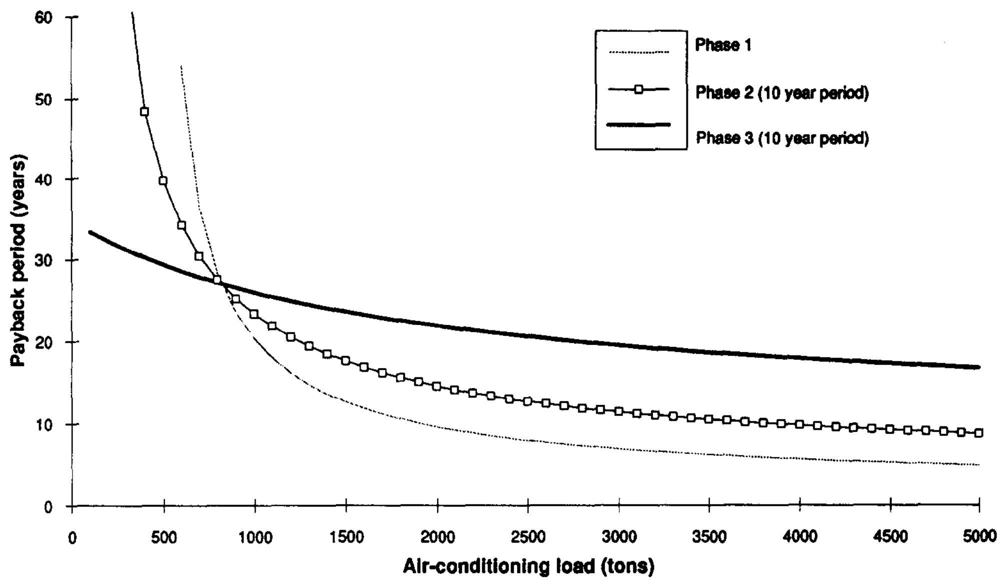

Fig. 6b. Payback period as a function of air-conditioning load for the sample OTEC plant project (20 ¢/kWh electricity, 10-year phase spacing).

figures: for example, Fig. 6b shows that, for A/C loads of the order of 800 tons, implementation of all phases at 10 year intervals would not affect the overall payback period of about 27 years.

CONCLUSIONS

OTEC is a capital intensive energy production technology for which building small pilot plants is necessary to accumulate operational and design experience. At the power outputs under consideration (≈ 1.5 MW net), no economy of scale should be anticipated. On the other hand, massive public funding much beyond existing levels is unlikely. Under these circumstances, the large capital investment required to build small OTEC plants may not be available. The present paper offers a possible strategy to finance such plants incrementally, in successive phases sharing some of the key OTEC hardware components. Each phase would implement a technology utilizing the ocean thermal resource, such as air-conditioning or desalination. At any time in the project life, overall cost effectiveness should be ensured in some sense. This investment concept was illustrated by the example of a conceptual two-stage 1.8 MW (gross) OC-OTEC pilot plant in Hawaii.

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APPENDIX

Payback period and phase spacing

Let i and d be the inflation and discount rates, respectively. $A(1)$ is a *net* revenue at the end of the first year of the life of the project. At any year j, the current revenue $A(j)$ is $A(1)\,(1+i)^{j-1}$ while the present worth of $A(j)$ is $A(j)\,(1+d)^{-j}$. If n is a given period of time, it follows that the present worth of the corresponding cumulative revenue stream S, is:

$$S = \frac{A(1)}{(1+d)} \sum_{j=1}^{n} \left(\frac{1+i}{1+d}\right)^{j-1}. \tag{A1}$$

If $d = i$, S is simply $nA(1)/(1 + \mathrm{d})$; in the more general case when $d \neq i$, the geometric series in Equation (A1) can be explicitly written, which yields:

$$S = \frac{A(1)}{(d-i)} \left\{1 - \left(\frac{1+i}{1+d}\right)^{n}\right\}. \tag{A2}$$

The payback period N may be understood as the particular value of n such that S is equal to the initial investment capital C. For $d = i$, the result is immediate, i.e. $N = C(1 + \mathrm{d})/A(1)$. When $d \neq i$, we equate S, in Equation (A2), to C and obtain, after some elementary algebra:

$$N = \frac{\mathrm{Log}\left\{1 - \dfrac{C(d-i)}{A(1)}\right\}}{\mathrm{Log}\left\{\dfrac{1+i}{1+d}\right\}}. \tag{A3}$$

Assuming that the MPFC comprises three phases, let the *present-day* capital and first-year net revenue increments be C_i and A_i, $i = 1, 2, 3$, respectively. Focusing henceforth on cases where $d \neq i$, Equation (A3) yields the payback period N_1 for the first phase by substituting C_1 and A_1, for C and $A(1)$. Subsequent phases, however, require caution because the time at which they are implemented, or phase spacing, affects cost effectiveness. Thus, if Phase 2 is implemented v years into the life of the project, the overall two-phase payback period is:

$$N_2 = v + \frac{\mathrm{Log}\left[1 - \dfrac{(C_1 + C_2)(d-i) - A_1\left\{1 - \left(\dfrac{1+i}{1+d}\right)^{v}\right\}}{A_1 + A_2}\right]}{\mathrm{Log}\left\{\dfrac{1+i}{1+d}\right\}}. \tag{A4}$$

In N_2 above, the second term represents the payback period estimated on the Phase 2 capital $(C_1 + C_2)$ minus the amortization realized over v years (i.e. the net present value of the cumulative Phase 1 revenue during that time). It turns out that there is an optimal spacing value v_{opt}, for which N_2 is minimal. Expressing the condition $\delta N_2/\delta v = 0$ yields, after some elementary algebra:

$$v_{\mathrm{opt}} + \frac{\mathrm{Log}\left[1 - \dfrac{(C_1 + C_2)(d-i) - A_2}{2A_1}\right]}{\mathrm{Log}\left\{\dfrac{1+i}{1+d}\right\}}. \tag{A5}$$

If Phase 3 is implemented μ years following Phase 2, simple considerations of capital amortization over the first $v + \mu$ years lead to the overall payback period N_3:

$$N_3 = v + \mu + \frac{\mathrm{Log}\left\{1 - \dfrac{(C_1 + C_2 + C_3)(d-i) - Y}{A_1 + A_2 + A_3}\right\}}{\mathrm{Log}\left\{\dfrac{1+i}{1+d}\right\}}, \tag{A6}$$

where

$$Y = A_1\left\{1-\left(\frac{1+i}{1+d}\right)^{v+\mu}\right\} + A_2\left(\frac{1+i}{1+d}\right)^{v}\left\{1-\left(\frac{1+i}{1+d}\right)^{\mu}\right\}.$$

Of the two necessary conditions for the existence of an overall extremum (hopefully a minimum) of N_3, $\delta N_3/\delta v = 0$ and $\delta N_3/\delta\mu = 0$, only the latter yields a given value of μ (at fixed v) while the former simply shows that N_3 is increasing monotonically with v (for a fixed μ): in other words, if v has been chosen, there is an "optimum" spacing μ between Phases 2 and 3, but there is no clear criterion for the original choice of v. In particular, the Phase 2 optimum v_{opt} may be inadequate, i.e. the best value for a two-phase project may be a poor choice for a three-phase project.

Thus, it is proposed here to look for a so-called *periodic* scenario, for which $\mu = v$, and to look for a minimum of N_3 under this condition: instead of writing $\delta N_3(v,\mu)/\delta v = \delta N_3(v,\mu)/\delta\mu = 0$, we now have $\delta N_3(v,v)/\delta v = 0$. This yields an optimal periodic phase spacing v_{per} which minimizes N_3, after some elementary algebra:

$$v_{per} = \frac{\mathrm{Log}\left[\dfrac{3A_2 + \sqrt{9A_2^2 - 16(A_1+A_2)\{2(C_1+C_2+C_3)(d-i) - 4A_1 - 2A_2 - 2A_3\}}}{8(A_1+A_2)}\right]}{\mathrm{Log}\left\{\dfrac{1+i}{1+d}\right\}}. \quad \text{(A7)}$$

In all the above formulas, it is clear that logarithmic arguments should be strictly positive and square-root arguments non-negative. Besides such mathematical necessary conditions, all phase spacings and payback periods should be positive. Failure of such conditions may indicate cases where the MPFC is not justifiable on purely economic grounds, e.g. no payback period exists, a particular phase is not economical, etc... It remains possible in some such cases, however, to impose an MPFC scheme on the basis of risk spreading alone.